The Perspective. A Matter of Points of View

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Abstract

This article presents a study that preceded a lesson for the Department of Architecture of Bologna held to children between eight and fourteen years of age, , as part of an initiative by the University for the dissemination of science. The study presents some experiences of perspective and stereotomy. Among the objectives of the research is to explain to children, through playing, the illusory power of perspective and, therefore, the deceptions to which the human perception of space is subjected. For this purpose a small Ames room was built, studying its decomposition in parts, in a sort of contemporary stereotomy.
In the first part of the article, the lesson given to the children is briefly presented, as well as some insights on the theoretical issues addressed, which obviously could not be included in the lesson. In the second part, of the study, design and construction of the Ames room are described.

Keywords: Ames Room; Descriptive Geometry; Mirror; Perception; Stereotomy

Introduction

The starting point of the research was a lesson for children aged between 8 and 14 years under *Unijunior*. "Unique in Italy, *Unijunior* was born with the ambitious goal of bringing the youngest children closer to study important subjects, using simple and familiar tools to the child such as practical experience, play and entertainment. *Unijunior* stimulates the curiosity of the child by leveraging the natural instinct of exploration that spurs him to know the world, finding answers to his endless questions and fulfilling his priority needs" [1].
The occasion was an opportunity to gain some experiences of perspective and stereotomy. To illustrate the properties of the perspective and, at the same time, to illustrate some characteristics of the human perception of space, a plastic of polystyrene was designed, which was then realized by means of a numerical control wire cutting machine.
The intent is twofold: the first, to explain to children, through the design of the room, what the drawing of architecture is and how it differs from the commonly understood drawing; the second, equally important, consists in demonstrating the illusory power of perspective, which derives from the modalities of visual perception.
The first part of the article, briefly describes the lesson given to the children, with some other insights on the theoretical issues addressed, which obviously could not be included in the lesson. The second part describes the design and construction of the Ames room.

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Synopsis of the Lecture for Children

The original title of the lesson for children (*The world as it is and the world as it appears*) is inspired by methods of representation of architecture.
As is known, these methods are used to accurately describe and transmit forms in space and also allow us to study the mutual relations and the properties of these forms. The graphic methods are: representation in double orthogonal projections, axonometry and perspective. These methods have been complemented by digital representations: numerical and mathematical. We will not talk about the latter, because their peculiar characteristics do not change the substance of the topics covered [2]. The first two methods, i.e. the so-called parallel projections, serve to describe the 'world as it is'. The third method, or perspective, serves to describe the 'world as it appears'.
In fact, the architect uses the plan, the section and the elevation to design and measure space. He also uses axonometry to understand the relationship between volumes and the mechanism of relationships between shapes in space. Parallel projections serve, therefore, to control the metric and formal aspects of space in an environment that is isotropic and homogeneous as space itself.
But man does not see three-dimensional forms as they are: man sees space through the filter of perspective, that is, of projection that transforms the three-dimensional world into the images collected by the eye [3]. And so it becomes vital to be able to describe space as it actually appears to the human eye and for this purpose the architect employs perspective.
To explain what a perspective is, just think of a photograph, and the functioning of the eye is analogous to that of a camera. However, it would be wrong to think that the phenomenon of vision is limited to these passages of optical-mechanical nature, because, as we shall see shortly, it is the brain that processes the images collected by the eye and it is in the brain that the deceptions of perspective are produced.
In truth, the perspective question we have just touched is very controversial. We can say that there are two main distinct schools of thought.
The first considers perspective (linear and relief) only a scientific method to produce images (static or dynamic) that have the right to exist only as a product of human ingenuity, but which are not able to evoke the perception of space.

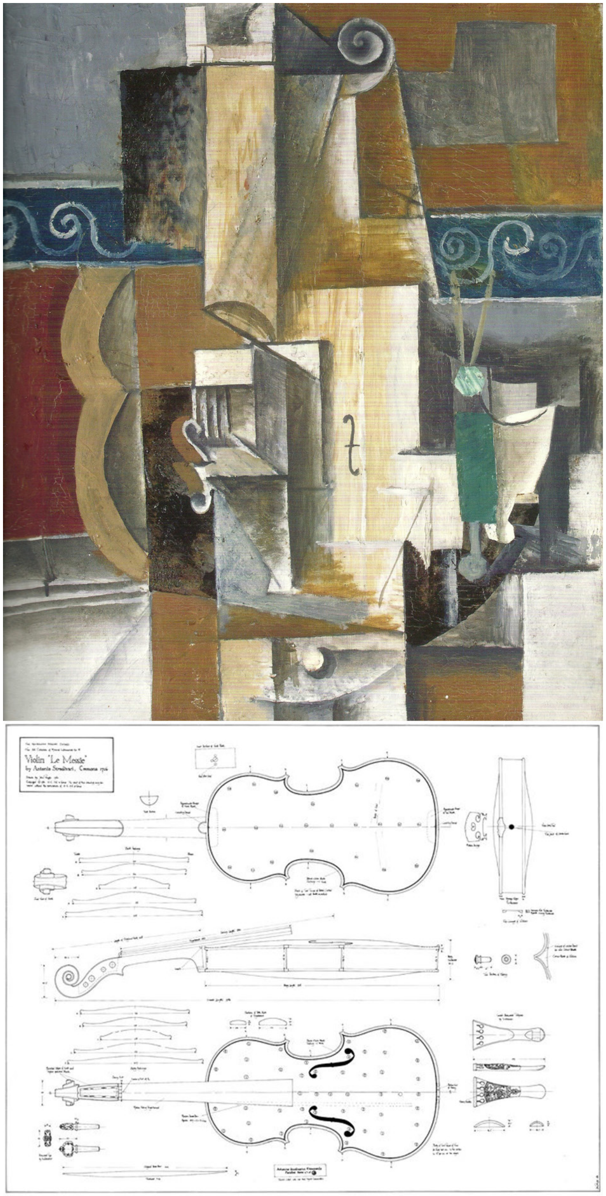


Fig. 1. Pablo Picasso, Guitar with violin, 1913. Violin by Antonio Stradivari, technical drawing.

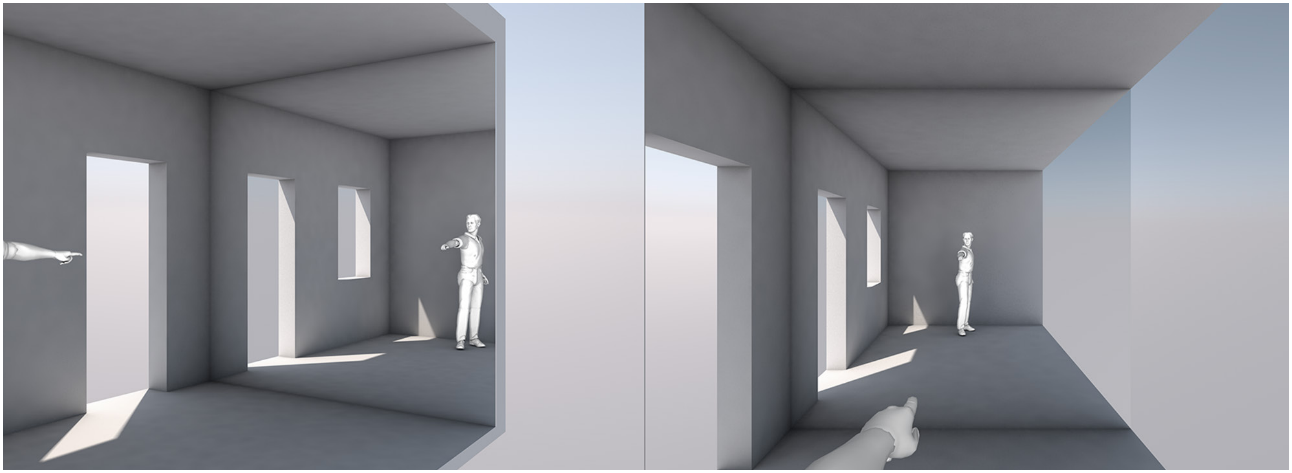
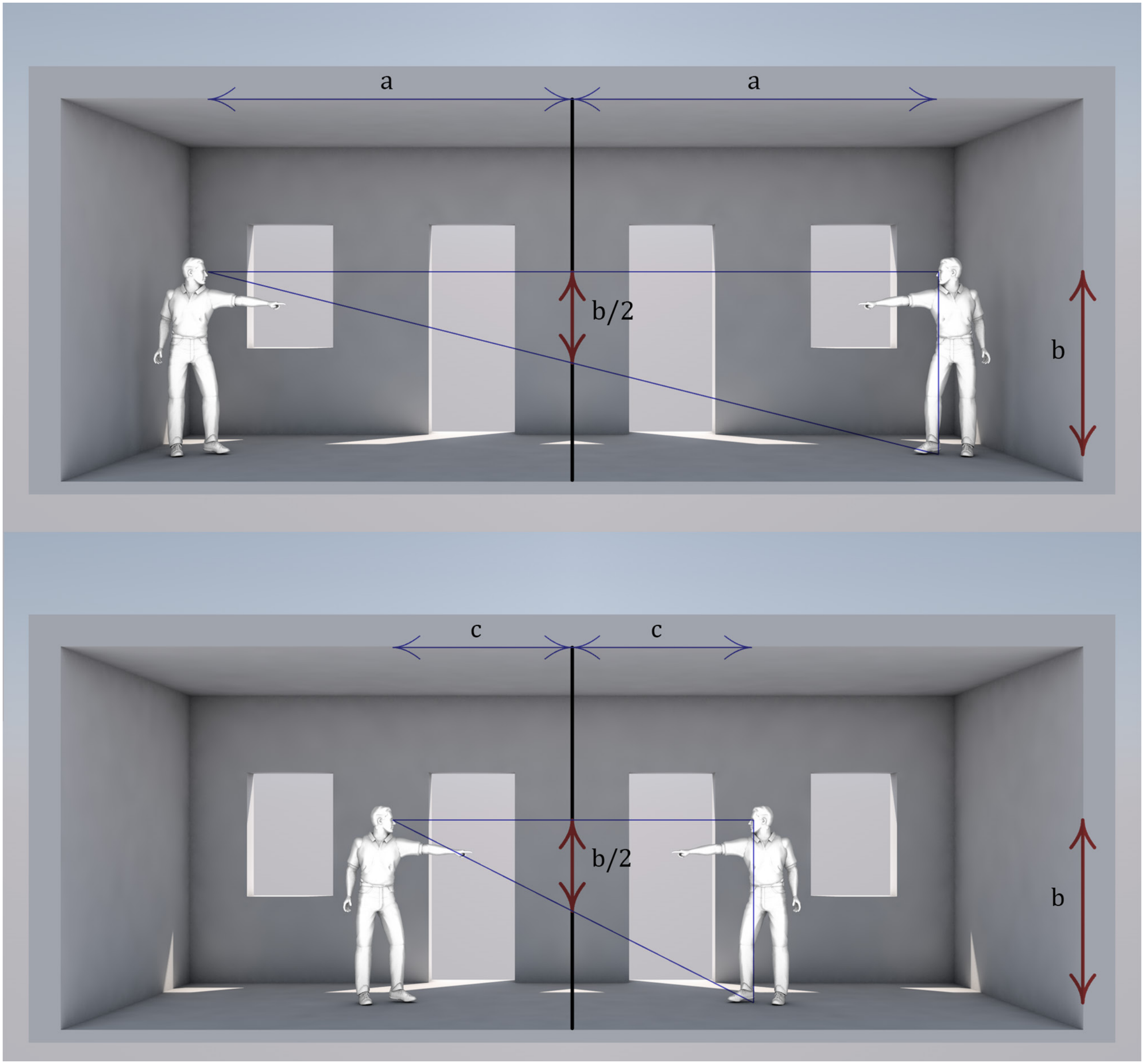


Fig. 2. The mirror image and the 'twin game'.

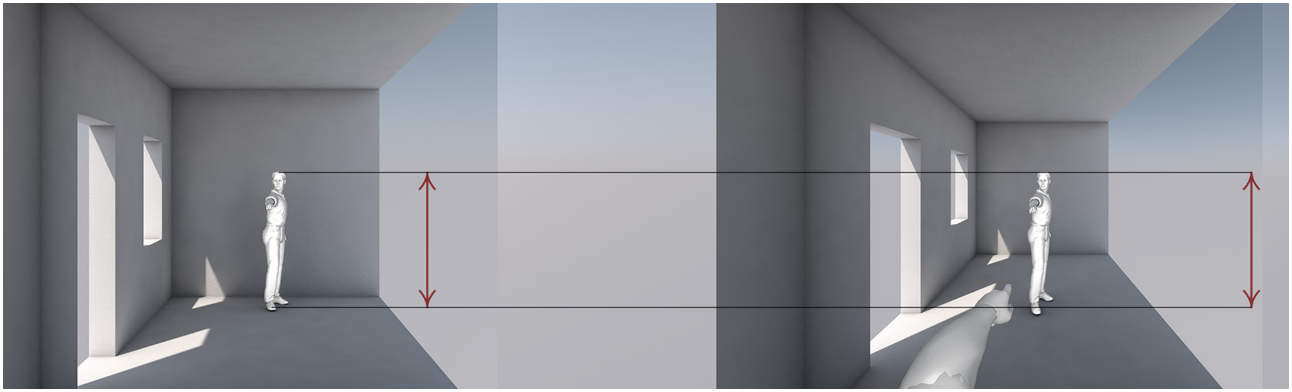
The second, on the other hand, considers perspective as a scientific method capable not only of describing the forms in space, but also of describing them in such a way as to evoke, in the observer, the natural vision.
In other words, according to the first school, the *perspectiva naturalis*, i.e. the vision, and the *perspectiva artificialis*, i.e. the perspective representation, are distinct and in conflict, because vision is conditioned by peculiarities, such as the curvature of the retina, that the *perspectiva artificialis* does not simulate (but the question would take us too far and refer to the bibliography for further study).
The second school, on the other hand, affirms that the *perspectiva artificialis*, discovered in the Italian Renaissance and developed to this day, is one and corresponds to the natural vision of man, being perfectly able to imitate it in its many forms, also through stereoscopy and the dynamics of cinematographic images.
In theory, an experiment could be carried out that would put an end to the dispute, but we are currently unable to put it into practice [4]. The experiment would consist in taking two photographs from a certain point of view. The first natural photograph should be taken by our eye and proposed in the mental processing of the observer. The second artificial one could be taken through a camera or built, geometrically, using the perspective method. The author is convinced that the overlapping of the two photographs, the natural and the artificial ones, would be perfect. Something very similar to this experiment was done by Filippo Brunelleschi in the early years of the fifteenth century [Fields 2005].
The difference between the two schools of thought is not a minor difference, as it involves a different vision of the history of representation and architecture. There is an important literature on this subject and the well-known essay *Perspective as Symbolic Form* of Erwin Panofsky of 1927 could be considered the progenitor of the long dispute [Panofsky 1961].
Today, the most recent studies on human visual perception affirm that man interacts with his surrounding reality through the five senses but sees reality through his own brain [5]. In other words, we never see the world as it is but we see it as our brain reconstructs it, comparing the images it receives from the eye with the models it has memorized in the evolutionary age [6].
The science that describes the methods of representation is descriptive geometry. This name is due to a French engineer and mathematician of the revolution, Gaspard Monge [7]. For years this discipline has been taught in schools and universities by engineers and then mathematicians. In the last forty years, however, especially in Italy, descriptive geometry is studied and taught only by architects and engineers. We do not want to elaborate

Fig. 3. The real world where our 'alter ego' is and the reflected world where our 'twin' is.



here the historical motivations of this change: let's just say that mathematicians have lost interest in the drawing and power of vision, even if, in the last years, thanks to the advent of digital technology, they show curiosity, if nothing else in the evocative power of images, whether realistic or symbolic.
What is the difference between the drawing of an architect and the drawing commonly understood as that of an artist or a painter?
To respond effectively (to children) it was decided to compare two representations of the same object. If we look at Pablo Picasso's *Guitar and Violin* painting we might think, at first glance, that the painter was not good at drawing or did not really like musical instruments [8]. Naturally, neither of the two statements are true. Picasso was a great drawer and was a great lover of music. The other figure shows the technical drawing of a violin by Antonio Stradivari (fig. 1).
The difference between the two drawings, that is between that of a painter and that of an architect (or a designer), is that the former interprets the form in a subjective way and transmits this emotion to the observer for empathy, while the latter, the technical drawing, measures the form and transmits it as an objective datum, aiming to remove any margin of ambiguity. This drawing must be transmissible and have a bijective relationship with the reality it represents: in other words, given a plan representation and elevation, to that correspond, in reality, the object represented and, vice versa, given a real object, it is possible to draw a plan and an elevation that match it.
Likewise, the fundamental characteristic of architectural drawing is to incorporate the code that allows us to move from reality to model and vice versa.
But can an architect use the free sketch to express his ideas as a painter? Yes, even the architect can use the drawing in a freer way, but only to follow with his mind, as with a pencil or computer, the representation of that space that can and must be measured and constructed.
Therefore, when designing a house, the method of double orthogonal projections is generally used to invent and measure space. It is also possible to accompany this study with axonometric representations to well define the volumes, the relationships between the parts and to better analyse the space. Furthermore, it is often advisable to construct a physical model, to check volumes and proportions in scale.
The perspective, instead, is used to study the perception of space, or to understand how that space will be seen and experienced, even from an emotional point of view.
The children seem to have appreciated the scientific part of the class in which the projective principles of drawing are described.
The operations that define drawing are the projection and the section. To get a drawing, we have to imagine that the sheet of paper is the picture plane, like the screen of the cinema or the computer, and there is a projection centre out of the picture plane. The image is obtained by projecting the object through the centre and dissecting the line star that derives from the picture. There may be two

Fig. 4. The size of our mirror image is always equal to the half of our real height.



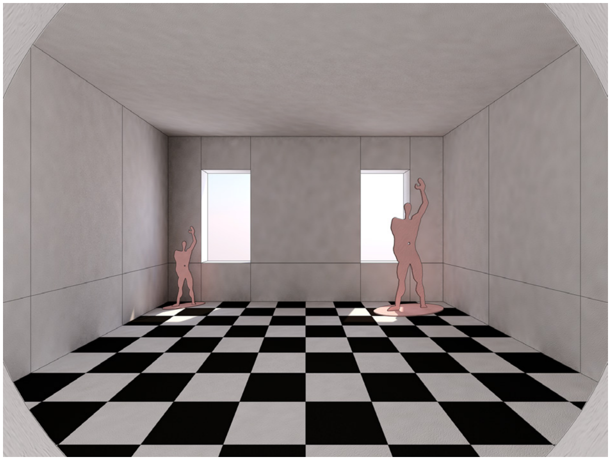


Fig. 5. Rendering of the constrained view of the Ames room digital model.

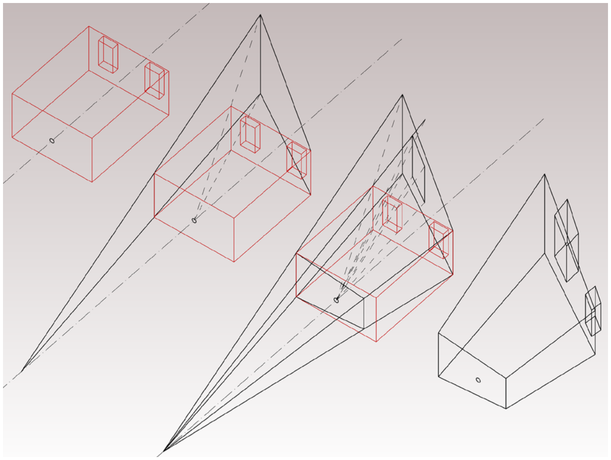
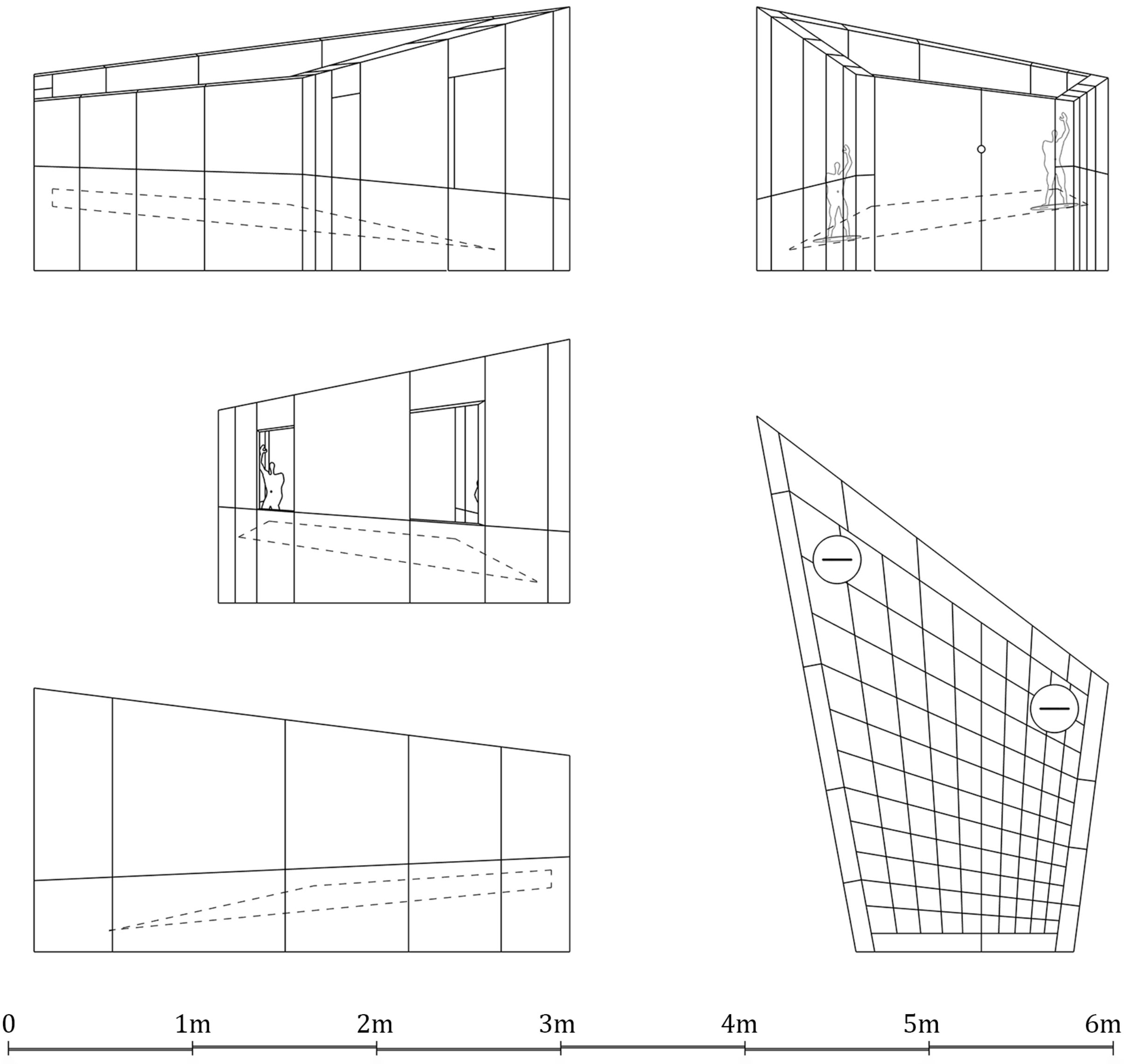


Fig. 6. Fig. 6. Projective construction of the Ames room model.

cases: in the first one the projection centre is a point that has a finite distance from the picture; in the second case, the projection centre has an infinite distance from the picture, i.e. it is very far: it is the direction of the projecting straight lines.
In the first case the drawing is a perspective, otherwise called the central projection. In the second case, i.e. the parallel projection, the drawing can be a plan, a prospect or an axonometry.
In conclusion, the former case serves to study the world as it appears and the latter serves to study the world as it is. A good architect is able to manage both situations well. However, the perspective view can be misleading, because sometimes the world is not exactly what it looks like!
What is the perspective machine we use every day?
The first answer that comes to mind might be the mobile phone camera. Well no, there is a perspective machine that we use every day from before the advent of mobile phone; this machine is the mirror (fig. 2). The mirror recreates a parallel world beyond the glass, still three-dimensional, which is exactly symmetrical of the real world. In the morning, when we wash or get dressed, we all usually mirror each other. Have you ever wondered how big your mirror image is?
Well, this image is no smaller when we move away and neither is it bigger when we approach the mirror. Our reflected image is always the same size and measures a specific quantity. To understand the problem we can imagine to observe our alter ego in front of the mirror and reconstruct the virtual world that is created beyond the mirror (fig. 3). There are two worlds: in the real one there are us and in the virtual one there is our twin. The only difference is that if we are right-handed our twin will be left-handed or vice-versa. In other words, in the symmetry of the mirror the right and the left exchange roles. If we look at the figure, it is easy to see that the distance between us and the mirror, and between this and our twin is the same and does not change with our distance from the mirror: if we approach the mirror, even our twin will approach the mirror in equal measure. If we now look at the projective triangle that forms our image, it is equally easy to see that the height of our image does not change with the change of the distance from the mirror: this height is a constant and is always half of the real one. To conclude, the measurement of our mirror image is always equal to half our height (fig. 4). To test this surprising truth, all we need to do is measure our own image on a mirror: two small signs are enough, above and below, to prove that this image will always be half our height whether we move away or approach the mirror.
The second experience, which I want to propose, to test the illusory power of the perspective vision is Ames room.

Fig. 7. The mathematical model of the Ames room.



Adalbert Ames Jnr. was an American ophthalmologist expert in optics. He is known for his experiments on visual perception which explained some fundamental principles of the visual perception. Demonstrations began in Hannover in 1938 and were carried out with the University of Princeton. These experiments are still reproduced in many departments of psychology and museums worldwide. Among these experiments, the best known and particularly peculiar is the Ames room.
The existence of this space was theorized for the first time by Fiermann von Helmholtz in 1866. He realized that objects of a multitude of different shapes and sizes can return to the eye the same image , and that a distorted room, constructed to return to the eye the same image of a rectangular room, may result in the perspective view identical to a regular room. The merit of Ames was to have built this distorted room and to have included two subjects in the room, studying its effects also on a group of volunteers. Observing the space of the room from the special hole, one gets the impression of being in front of a perfectly regular room. But if we put two subjects inside the room or facing the two windows at the back, we realize that something is wrong (fig. 5). One seems to be much bigger than the other or, conversely, if they change place, the they also change size.
We are so used to perceiving size and space in a certain way, that at first we cannot see that space is deformed and that we are not facing a regular space but a trapezoidal room. This space is specially constructed according to the projection centre which is positioned exactly in the centre of the hole. The illusion of finding ourselves in front of a perfectly regular space is disorienting; we just need to observe the space on the opposite side to immediately realize the trick.
But the most unusual feature of this experience is that even if the trick is known, the illusion does not lose its effectiveness at all: we cannot see the distortion of the room and we continue to perceive the two subjects one much smaller than the other or, vice versa, one much larger than the other. Ames is convinced that there is a memory of perception that conditions human perception, that is, the habit of living in regular spaces influences our vision and our perception. There are other theories and explanations in this regard but until now a conclusive and convincing theory that can explain this phenomenon well has not been formulated [9].
As far as we are concerned, these two experiences tell how important it is for an architect to know how to observe and represent shapes in space, both in their real form and in their appearance.

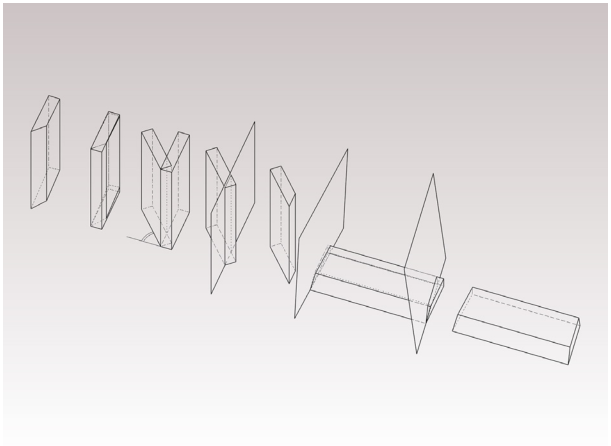
Fig. 8. Images of the constrained view of the polystyrene model of the Ames room.

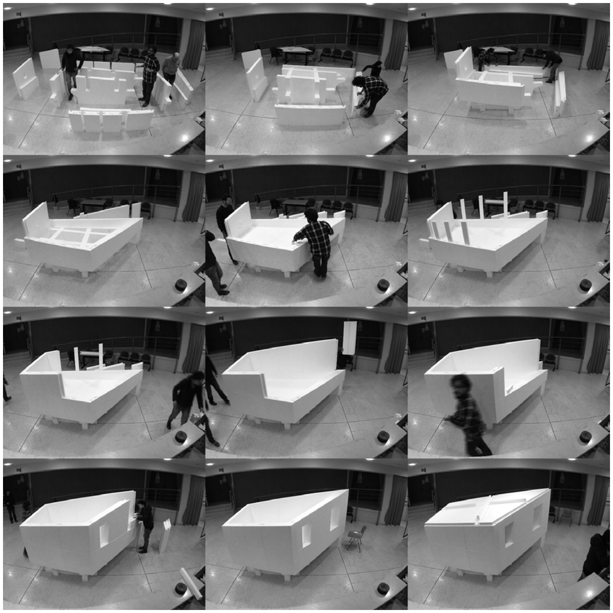
The Construction of the Ames Room

Compared to the many Ames room models built in museums, this Room was designed in parts and made of a single material. This choice, as mentioned, was dictated by the need to test the use of polystyrene and make the model safe for children.
To model the Ames room, the following procedure must be followed.
You design a regular room as you like, that is, a rectangular prismatic space that has all the walls perpendicular to each other. To make the illusion more effective you can design two regular openings on the front wall of the room. On the opposite side, on the anterior wall, chose the position of the optical hole and projection centre of the transformation. It would be a good idea to choose the height of the projection centre at the height of a man's point of view. In the case study examined this height was calculated at the height of a child.
In order to construct the transformed prism, or Ames room, it is necessary to respect the planarity of all the faces. This geometric characteristic is respected if the four faces perpendicular to the front plane, once the prism has been transformed, belong to a pyramid which has the apex on the axis perpendicular to the front wall passing through the projection centre (fig. 6). In fact, only respecting this geometric condition the four faces will all be flat: otherwise, one or more of the faces will turn into hyperbolic paraboloids.
Ultimately, in order to easily control the transformation, you can draw the pyramid axis first. The vertex of the pyramid can be chosen on this axis: the more the vertex will be close to the anterior wall, the more important the projective transformation will be and vice versa. You choose a vertical edge of the front wall to maintain a fixed position. You select an upper (or lower) vertex of the other vertical edge and project it (from the projection centre) until you meet the straight line passing through the apex of the pyramid and the upper (or lower) vertex of the anterior wall. This sets the position of the transformed vertical corner. The remaining side faces must all belong to the apex of the pyramid, or pass through that point; while the anterior and front faces are sections, always flat, of the pyramid. To construct the windows of the wall placed in front of the observer it is possible to project (from the centre of projection) on the transformed wall itself the vertices of the edges. All edges parallel to the axis of the regular prism are transformed into lines that still belong to the apex of the pyramid.
The model was designed with ashlars so that it can be assembled and disassembled (like a dry-stone wall) in a short time.
To construct the room, a mathematical model was first created and from this model the measurements of the individual ashlars were obtained (fig. 7). The scale of the model was dictated by two factors. The first was to make the children protagonists of the experience and to do so the model of the room had to be large enough. The second was dictated by reasons of external space, i.e. the model had to be small enough to easily enter inside the entrance space of the main hall of the Psychological Faculty of the University of Bologna. The final model is a room of about two meters by three, inside of which it is not possible to walk (as in the original Ames room) but it is possible to look out of the windows and observe the space directly (fig. 8).
Another fundamental choice was to build the model entirely in polystyrene in order to experiment with the construction of the ashlars using a wire cutting machine; moreover, as I already said, polystyrene is a light and safe material for children [10].
Each ashlar has been designed and modelled considering the projection centre: observing the space from the projection centre , the ashlars appear to divide the space in a regular way according to the conventional horizontal and vertical directions. In reality, the ashlars are all skewed and the faces that form such parts are not perpendicular to each other.
Initially we tried to realize the skewed segments making only two cuts. This solution, however, was immediately discarded after the first attempt. In fact, to obtain this result, the machine for processing the oblique cut had to move the two motors that carry the wire independently; doing this, however, the wire stretched too much, until it breaks. Consequently, we decided to let the motors work together and in parallel so as to avoid breaking or loosening the wire.
To obtain this result, we had to calculate the angle between the planes that form the various individual ashlars and the exact size of the regular volume that enclose each piece (fig. 9).
Starting with a piece of regular polystyrene, equal to the overall volume of the single ashlar, the various cuts were made separately, each time placing a worked piece in the

Fig. 9. Schematic illustration of the cutting stages for the construction of a polystyrene ashlar.

Fig. 10. Building of the polystyrene model at the entrance hall of the Aula Magna of Psicology faculty, University of Bologna.





machine chamber according to the calculated angles. The cuts were also designed to fit the various dry-ashlars together.
The final model is dry-mounted in about twenty minutes and can be dismantled just as easily to transport it (fig. 10). The floor has been designed as a checkerboard to accentuate the illusion of a regular space.

Conclusions

Through the game and the direct experience of space, we illustrated the illusory power of perspective and experienced the stereometric construction of a small Ames room. The children's response was positive, that is they seemed to have understood and appreciated the experiments on perspective. The lesson on *the world as it is* and *the world as it appears* will be repeated next year for the new edition of *Unijunior* 2018.
In the future, the idea is to be able to design and construct other models that can stimulate the study of visual perception and space. Regarding the question of perspective and vision, there are still open questions that would be interesting to investigate. Perspective continues to be a stimulating and mysterious theme: each time it is dealt with, it reveals its elusive and profound nature that has ancient roots. Today, living in the digital age, we have the opportunity to simulate the construction of various models and can study their potential. However, the need to physically build models that allow direct experience of those deceptions is even more surprising. Perhaps one day we will be able to definitively unravel the question of *perspective artificialis* and *naturalis*. Or the case will simply remain a matter of perspective points of view. However, we cannot forget that the experiences and research described here have been realized thanks to the geometric theory of perspective. The fact that the two experiments, the twins and the Ames room, are effective seems to bring a further element in favour of the existence of a single perspective that corresponds to the human sensible vision.

Acknowledgments

I would like to thank the architect and technical manager Davide Giaffreda and the collaborator Marika Mangano for the indispensable help in the design and construction of Ames Room's polystyrene model. The model was built entirely with the tools of the Department of Architecture; in particular, the model was realized within the Laboratory Lamo of the Department of Architecture of Cesena, scientific director Francesco Gulinello. Thanks to Fabrizio Ivan Apollonio and Riccardo Foschi for the publication of the photos. Thanks also to Valentina Orioli who has supported the initiative.

Notes

[1] The internet website of the *Unijunior* association is: <http://www.unijunior.it/> (accessed 2018, February 20).

[2] The digital methods are basically two: the method of mathematical representation and the method of numerical representation. The mathematical method describes continuously and accurately the geometric shapes in space. NURBS mathematics is the most widely implemented to describe curves and surfaces in mathematical modeling programs. On the other hand, the numerical or polygonal method describes the shapes in space in a discrete and approximate way. Polygonal shapes or mesh shapes are used to describe curves and surfaces in polygon modeling programs. Of course the two methods have advantages and disadvantages that make them suitable for some purposes. Mathematical modeling is generally used in the design phase and to accurately construct and measure shapes in space. In this sense we can say that mathematical representation is the equivalent of the two parallel projections in classical methods, namely the representation in plan and elevation and axonometry. While numerical representation is generally used to visualize and formally study shapes in space, ie to construct static and dynamic perspective and static and dynamic rendering. In this sense we can say that numerical representation is the equivalent of perspective in classical methods. Today we speak commonly of BIM or of the generative parametric representation (for example the use of visual programming languages like Grasshopper). The latter digital representations can be considered as digital representation techniques and not real methods. They do not change the geometric nature of the objects described; which can be mathematical, polygonal and hybrid. Furthermore, both techniques can be used to obtain accurate or approximate models. Unfortunately, at the state of the art there is no univocal consensus on the classification of digital methods at national and international level. The reason is naturally due to the novelty of these methods and techniques and the rapid development that these techniques are having over the years.

[3] In this context 'perceiving' has the meaning proposed by Italian Dictionary Zinagarelli 2018, that is: 'grasp the data of reality through the senses'. To avoid misunderstandings in this article I will use the terms 'to see' and 'perceive' strictly with the first meaning reported in the Italian language dictionary. It is natural that man is able to imagine and see space also in axonometry (and in double orthogonal projections). For some cultures such as Asian, in particular the Chinese and Japanese, the parallel projection method has been the main method of representing the surrounding world. And perhaps it is no coincidence that when man designs and analyses space, he is naturally led to use and prefer parallel projections.

[4] There is an episode of a British television series, *Black Mirror*, released in 2011 in which a situation is described that recalls the experiment mentioned. The third and final episode of the first season, entitled *Hazardous Memories*, is set in an alternative reality, where most people have a grain implanted behind the ear, which records everything that is done, seen or heard. This allows memories to be played in front of the owner's eyes or on a screen through a process known as 're-do', just like videos. It seems that this grain is implanted since newborns, but that a person can decide to have it removed.

[5] Here the verb 'to see' is to be understood in a broader sense.

[6] In this sense it is sufficient to think about how natural it is for man to imagine and read space in axonometry. Beau Lotto, in his essay [Lotto 2017], while not making any direct reference to the perspective, describes numerous examples that demonstrate how man reconstructs in his mind what he sees. In conclusion, even the latest theories of perception do not seem to help us on the question of perspective. Nevertheless, the scientific theses supporting the existence of a *naturalis* perspective different from the *artificialis* one are not conclusive [Gioseffi 1957].

[7] To deepen the notions and the history of Gaspard Monge and descriptive geometry refer to Cardone [Cardone 2017]. With regard to descriptive geometry and the '*scuola romana*' refer to Migliari [Migliari 2010].

[8] Picasso's painting is from 1912. While the technical drawing of the violin refers to one of the instruments construct by the well-known Italian luthier Antonio Stradivari (1644-1737).

[9] Gregory says that with the Ames room it is possible to put in place an experiment that is perhaps even more disturbing and it is able to challenge a fundamental law of physics. Simply take two objects, like two balls, and drop them. We will then see the two spheres falling at different times defying the gravitational law. Even in this case, at first glance, the impression is to be in front of objects that do not respect the same physical laws and we can not perceive that the height from which the two objects were dropped were different [Gregory 1994].

[10] The machine of the Laboratory Lamo (Laboratorio modelli di Architettura) of University of Bologna is the model 120P Box of Nettuno Sistemi; <http://www.nettunosistemi.it/120pbox.php> (accessed 2018, February 20).

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