

Fractal Structures. Understanding the Geometries of Nature

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Abstract

The observation of nature has historically guided design disciplines, from the proportional harmony of antiquity to the empirical studies of the Renaissance, which laid the groundwork for the development of biology and bionics. In the 20th century, the formulation of fractal geometry and the emergence of computer science made it possible to describe and simulate complex systems, going beyond formal imitation to investigate the processes of growth, adaptation and self-organisation characteristics of living organisms. Mandelbrot's research has enabled the quantitative interpretation of morphogenetic phenomena in the natural world, highlighting fractal rules common to both animal and plant systems. The history of architecture is full of examples of the unconscious adoption of self-similar logic, which can now be reinterpreted using algorithmic and parametric modelling tools. This article proposes a reinterpretation of organic models and construction archetypes, such as brick, within a design paradigm based on computational morphogenesis, in which natural principles are transformed into operational protocols capable of combining formal complexity, structural efficiency and construction innovation.

Keywords: computational morphogenesis, fractal geometry, computational design, brick, construction archetypes.

Introduction

Nelle discipline progettuali, l'analisi sistematica e la comprensione in design disciplines, systematic analysis and understanding of morphological structures play a crucial role in ensuring the efficiency of the proposed solutions. The outcome of any design process is, in fact, conditioned by the ability to meet operational requirements such as time constraints, material resources and manufacturing processes necessary to achieve the set objectives. Since a large part of human learning takes place through imitation, it is not surprising that the natural world has historically been the primary source of inspiration [Rossi 2001; Rossi 2006]. Since the earliest manifestations of graphic representations of natural organisms and phenomena—such as those found in cave paintings—humans have

studied nature to decipher its principles. In classical antiquity, the observation of biological models was primarily aimed at translating the harmony of natural forms into mathematical and numerical language through the study of proportional relationships between different morphological components. These investigations not only guided the aesthetic canons of art and architecture but also laid the foundations for an initial formalisation of structural correspondences. It was only with Leonardo da Vinci's Renaissance studies on the flight of birds, perhaps the first documented example of research supported by a systematic analysis of a biological system, that the connection between natural phenomena and design processes began to promote an empirical approach. However, the

ability to translate natural principles into applied technologies remained fragmented, determined by the level of development of the theoretical tools and technologies available. Although there were erratic cases of earlier applications, it was only at the beginning of the 19th century, with the advent of biology as an autonomous discipline dedicated to the study of living systems, that the relationship between design and the natural world acquired a rigorous methodological basis [Thompson D'Arcy 1917]. During the 20th century, the emergence of biotechnology in the first half and bionics in the second half led to the development of cognitive models capable of describing more complex relationships and dimensional realities in mathematical terms. This made it possible to reproduce and control biological structures and phenomena that were once thought to be undisputable. With the new millennium, the organic reference took on considerable importance in the progressive shift of interest from form to the relationships constituting generative dynamics, in an in-depth study that led over time to the replacement of mere imitation with the analysis of biological processes of growth, transformation, and responsive adaptation [Rossi 2014; Rossi 2019; Rossi, Buratti 2017].

In the field of design practice, this progress can be attributed to two distinct yet related factors. The first is the formulation and characterisation of Mandelbrot's fractal geometry, influenced by a broader epistemological shift that sees the abandonment of the classical deterministic paradigm in favour of studying the intrinsic non-linearity of natural phenomena.

The establishment of computer science represents the second factor as an autonomous scientific discipline, utilising computers not only as a calculation tool but also as a privileged means for investigating, modelling, and simulating design systems based on complex logic.

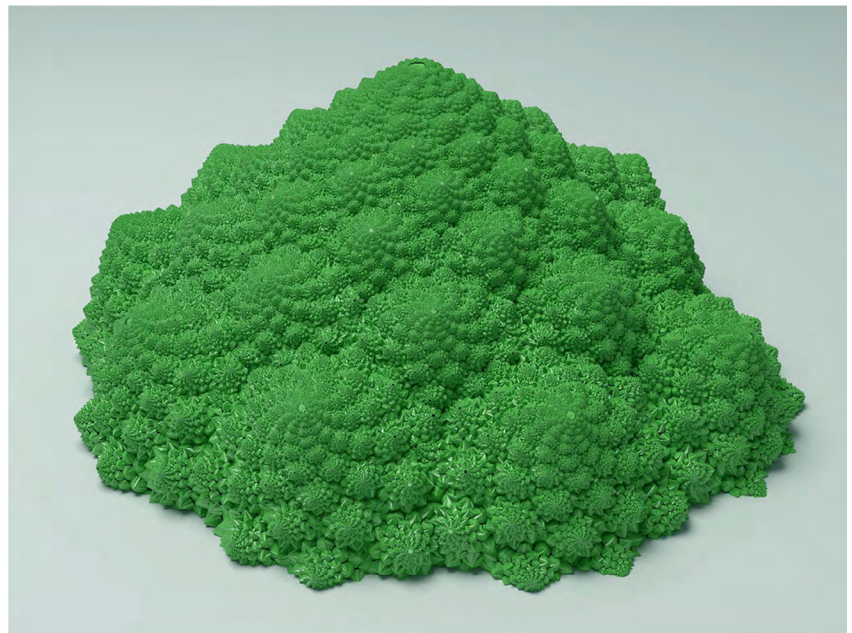
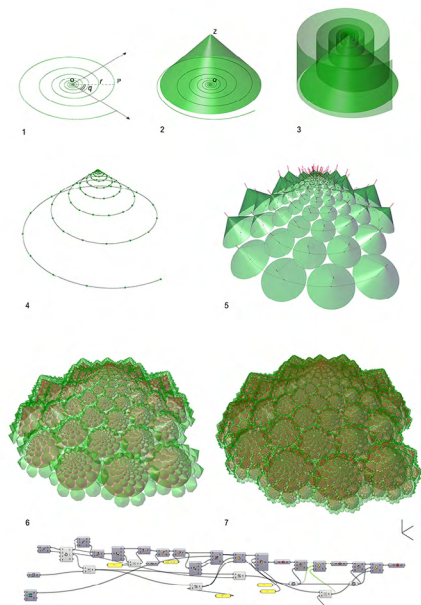
Fractal geometry: references and digital models

Fractal geometry has made it possible to describe and quantitatively characterise complex natural structures that are too intricate to be defined in Euclidean terms. Strictly speaking, fractal models can be recognised in numerous artistic and architectural expressions that developed across centuries and continents, even before Mandelbrot's studies [Mandelbrot 1975]. Traces of self-similar geometries can be found in classical Greek art

and African vernacular architecture, in the decorations of Egyptian civilisations, in pre-Columbian cultures, as well as in Islamic and Hindu religious complexes [Sala, Cappellato 2004]. They reflect the aforementioned predisposition of human beings to reproduce in construction and decoration the same principles of complexity and self-organisation observable in natural ecosystems, characterised by sensorially rich patterns, spaces connected on multiple levels of scale, and fractal dynamics of repetition and continuous variation.

It is also true that, from a scientific perspective, fractal structures were already known by the end of the 19th century and in the early decades of the 20th century. Georg Cantor (Cantor set, 1883), Giuseppe Peano (Peano curve, 1890), David Hilbert (Hilbert curve, 1891), Helge von Koch (Koch snowflake, 1904) and Wacław Sierpiński (Sierpiński carpet, 1916) had described sets and curves that challenged classical Euclidean notions of topological dimension, Lebesgue measure [1] and perimeter. These objects were united not only by recursive iteration, internal homothety and self-similarity, but also by the impossibility of being represented as the locus of the solution points of differential equations or elementary algebraic-geometric systems. Paradoxical properties such as curves of infinite length delimiting regions of finite area or disconnected but not discontinuous sets caused scepticism and mistrust among many scholars of the time, so much so that fractals were initially considered 'mathematical monsters' [2] with no counterpart in physical reality [Falconer 2003]. It is therefore thanks to Mandelbrot that these objects find an actual correspondence in the real world, even if the fractal structures observable in nature exhibit self-similar behaviour only within a defined interval. This is because, as Goethe already intuited when he said that 'nature has ensured that trees do not grow to the sky' [3], natural laws vary according to the phenomena considered. For example, at the cellular level, where the growth of living organisms is concerned, the force of gravity does not play a decisive role. In contrast, in the macroscopic world, this force profoundly affects the structure and movement of bodies. Even the arrangement of plant organs according to phyllotaxis patterns responds to a morphogenetic organisation that optimises exposure to sunlight and air circulation, ensuring physiological conditions conducive to growth. A prime example is Romanesco broccoli, frequently cited for its extraordinary geometric regularity

Fig. 1. Morphogenetic development of Roman broccoli (elaboration by the authors).



and the marked recursiveness of its structures, characteristics that make it easy to model algorithmically (fig. 1). The same logic of efficiency in the spatial distribution of branches can also be found in the morphological development of numerous tree species. It is therefore not surprising that, in the field of design, references to the organic world have taken on an increasingly important role over the years, in parallel with a shift in interest from form to the complex generative dynamics that underlie it. This in-depth study has gradually led to the imitative approach being overcome, directing research toward the analysis of growth, combination, and structural processes. One of the first examples of the conscious application of biomechanical principles in architecture is the Crystal Palace, built in London in 1851 for the Great Exhibition, also known as the first World's Fair. The design, conceived by Joseph Paxton, reinterprets *Victoria Amazonica* [4] in the design of the iron supporting elements of the building's roof arches, allowing for the creation of a highly lightweight structure that is nevertheless capable of supporting large glass surfaces. The organic metaphor also influenced American architectural thinking, particularly through the theories of Horatio Greenough, who identified the correspondence between form and function as a fundamental principle of natural organisation [Greenough 1975; Greenough 1852; Tuckermann 1853]. The form is shaped in response to the functional needs of the genus and species, through a process of adaptation that reflects the natural formal economy. These concepts found effective synthesis in Louis Sullivan's famous motto, *form ever follows function*, which formulated a principle that was to exert a profound influence on 20th-century architecture and design, constituting one of the theoretical foundations of the Modern Movement.

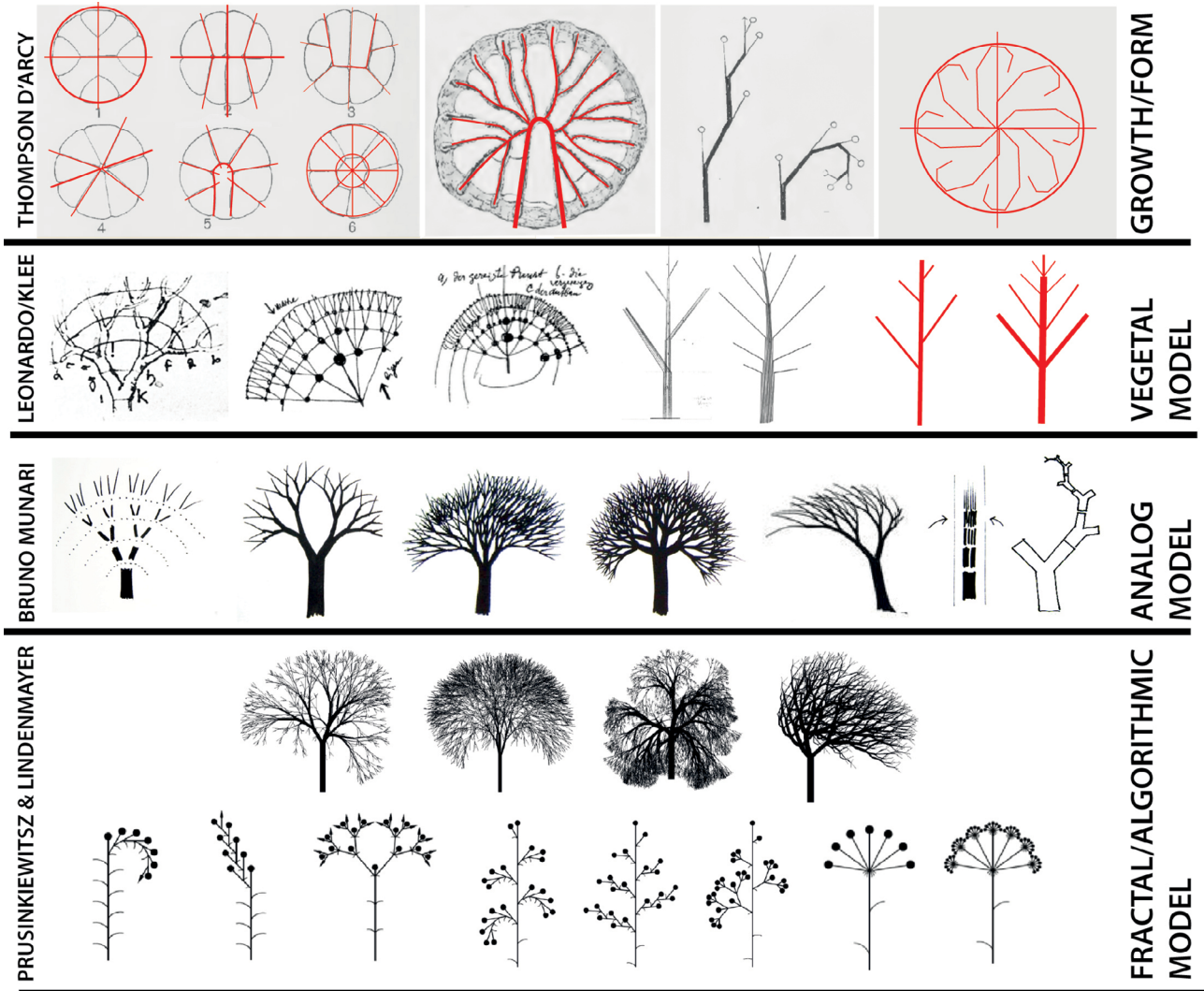
During the same period, Eugène Viollet-le-Duc produced drawings that 'rationalised' the conformation of mountains [Viollet-le-Duc 1876] and glaciers, considered by analogy as ruined buildings in need of restoration. The French architect described the heights with a geometrisation that anticipated the principle of box counting associated with IFS (Iterated Function System) fractals, a solution he considered necessary to overcome the limitations of Euclidean geometry in the description of natural phenomena, anticipating Mandelbrot by almost a century with his functional fractal generation for the digital modelling of the orography of territories. The characteristics of self-similarity and scale invariance

lend themselves to the creation of efficient algorithms that enable the development of predictive simulations of complex systems, territories, or organisms.

In this regard, it is particularly interesting to compare this method with the one proposed by Bruno Munari for graphically representing a tree, through a process that aims to understand its genesis and formal logic. The exercise involves constructing a simplified model [Munari 1978] which, although designed for educational purposes, serves as an effective analogy of an IFS (Iterated Function System) fractal structure, generated according to a deterministic algorithm [Sala, Cappellato 2004]. Although Munari does not explicitly refer to fractal mathematics, the association has been evident since 1990, when Hungarian biologist Aristid Lindenmayer and Polish computer scientist Przemysław Prusinkiewicz published the first systematic study on the digital simulation of plant morphogenesis. In this research, *L-system* –generative formalisms introduced by Lindenmayer himself in 1968– were applied to model plant growth patterns [Prusinkiewicz, Lindenmayer 1990]. The logic of generating tree morphology and annotating how external factors –primarily the action of wind– can modify development with respect to the 'ideal' growth model is the same. The analogy between Munari's drawing of a tree bent by the wind [1978] and the morphological deformation scheme subsequently published by Prusinkiewicz and Lindenmayer is particularly evident: in both cases, there is a systemic deviation of the geometries derived from environmental inputs, which corresponds to the generative computational models of *L-Systems* (fig. 2). The designer cites the source in his own way: "An old friend of mine from the provinces, a certain Leonardo, born in a small village near Florence: Vinci (postcode 50059) was an inquisitive man. He would spend hours observing plants and then drawing them and noting down everything he could understand about how they branch out" [5].

The representation reveals the growth rules that constitute the compositional pattern of the design and digital modelling. Thus, interest in fractals extends to the fields of design and architecture, and even to spatial planning, an area in which design is called upon to bring order to a complex and apparently chaotic system, in which the signs of anthropogenic transformations have become stratified [Rossi et al. 2022]. C. Alexander's *Pattern Language* emphasises the relationship between the fractal

Fig. 2. Evolution of growth and branching models: from D'Arcy Thompson's morphological growth model to L-system fractal models (elaboration by the authors).



geometry of *L-Systems* in settlements and territorial configurations that show the development of self-similar structures in the hierarchical aggregation of the archetype of housing as a large-scale design method, aimed at ensuring a balanced development between the different needs of a complex society [Alexander et al. 1987; Sala, Cappellato 2004].

A reference to the self-similarity of fractals can also be found in the rationality of medieval buildings. One example is the diverse decorative and construction solutions of the large rose windows in medieval cathedrals, which follow a combinatorial logic, as seen in Milan Cathedral [Rossi, Buratti 2022], another artificial representation of the cosmos.

Once again, once a new mathematical approach has been defined to solve an unsolved problem, its effectiveness in relation to other issues is discovered with the development of simplified solutions, consistent with the latest digital design tools. In recent decades, fractal models have acquired a fundamental role in modelling various scientific fields, including biology, economics, and the social sciences, as well as medicine, with constantly growing fields of application. Fractals, therefore, offer significant potential for the development of digital applications geared towards architectural design on all scales. This article presents some experiments aimed at morphological optimisation, seeking to reinterpret formal archetypes and historical patterns in a contemporary key, using computational tools capable of integrating geometric complexity and structural coherence.

Designing complex fractal structures: from broccoli to rose windows

The characterisation of fractals is not based on a single analytical expression, but on an algorithmic process (not necessarily numerical in nature) used to generate a surface curve. An algorithm is a systematic protocol consisting of a sequence of formally defined and unambiguously interpretable instructions, designed to guide an executing agent toward achieving a predetermined goal. When that agent is an electronic computer, the algorithm must be coded in an executable programming language.

As already mentioned, computer science has played a fundamental role in the study of fractals. Thanks to computers, Mandelbrot was able to simulate the complexity

of the recursive laws typical of nonlinear systems, discovering the rules that govern their evolution a posteriori. In this way, he demonstrated that such phenomena cannot be treated with a top-down hypothetical-deductive approach, i.e. by predicting future trends based on initial conditions, but require a bottom-up model. By defining the behaviour of individual elementary entities and exploiting computing power to simulate their collective interactions, it is therefore possible to highlight recurring patterns and compare them with natural processes.

The refinement of IT skills that characterises the new millennium has also led designers to explore the processes hidden by the interface that determine the functioning of everyday digital tools. This focus has promoted the evolution of a new type of computer-aided design, which frees designers from the restrictions imposed by traditional modelling software, thanks to the possibility of defining the process of relationships that lead to the formation of the shape itself. The morphology of an artefact thus becomes the result of the interaction between various design determinants, whether technological, economic or cultural, in a process defined as computational morphogenesis precisely because, as in the natural morphogenesis that characterises the development and growth processes of living organisms, form arises from the interaction of material capacities intrinsic to the system and exogenous environmental forces.

The examples presented in this work demonstrate how this approach enables the description and control of the complexity factors in the biological reference models.

Using *Grasshopper*, a visual algorithm editor associated with McNeel's *Rhino* software, various definitions were developed to describe organic morphologies precisely.

The first algorithm [Buratti, Rossi 2021] takes Romanesco broccoli as a reference, a vegetable that has attracted the interest of many scholars due to its peculiar self-similar morphology. The arrangement of the rosettes follows a fractal structure attributable to principles of internal homothety. Each inflorescence replicates the geometry of the whole on a smaller scale, generating an iterative sequence of cones arranged along the lateral surface of the previous one. This growth can be described using the Fibonacci sequence, whose successive ratios approximate the proportional invariance observed in the spatial distribution of the elements (fig. 2). In this way, the spatial distribution of the inflorescences is optimised, making the most of the ratio between the number and size of the

Fig. 3. a) Diagram of Pythagoras' fractal tree; b) Three-dimensional equivalent based on triangular prisms and tetrahedra; c, d) Comparison of the fractal development that governs the growth of a tree and also regulates the morphogenesis of pulmonary blood vessels (elaboration by the authors).

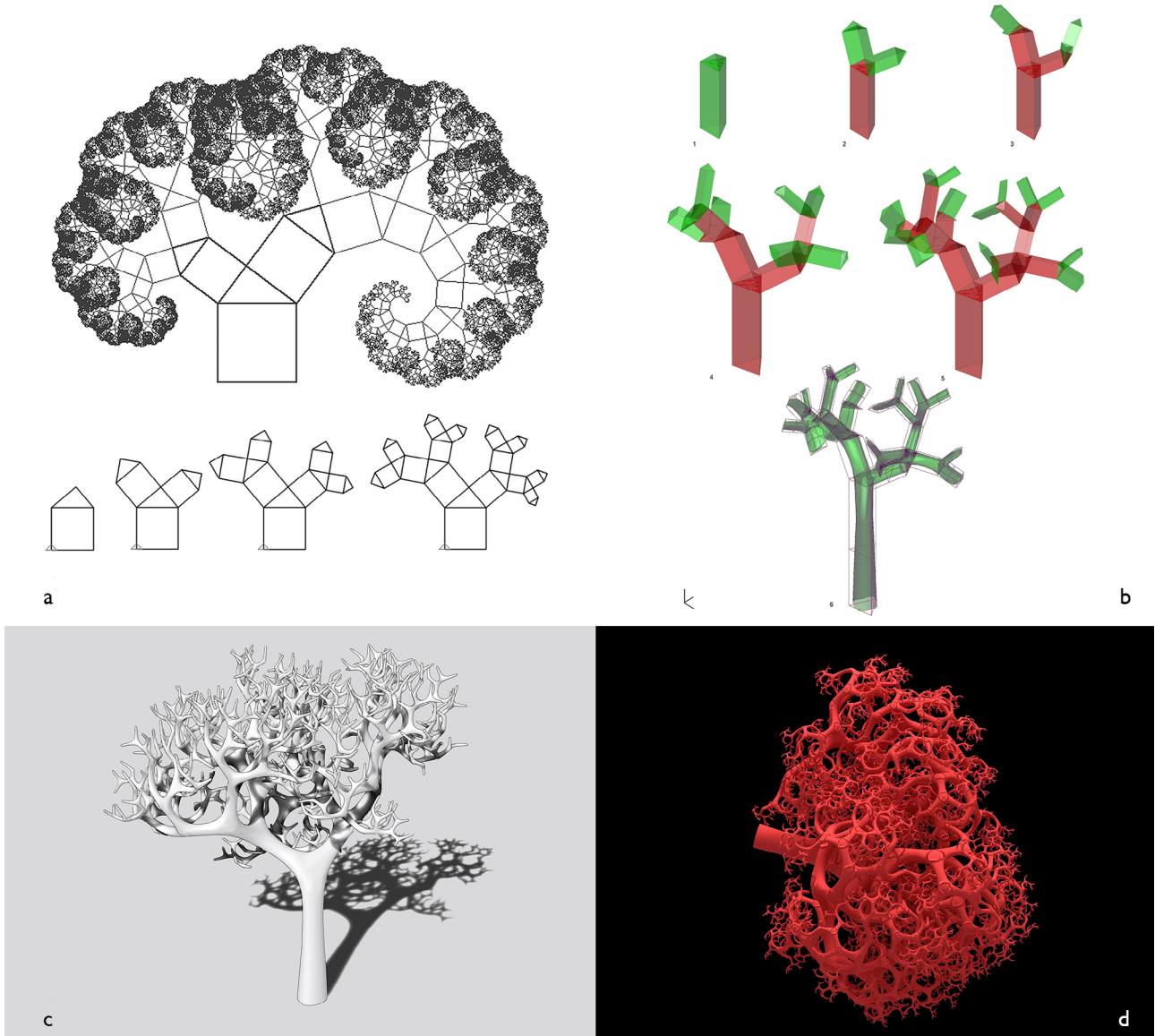
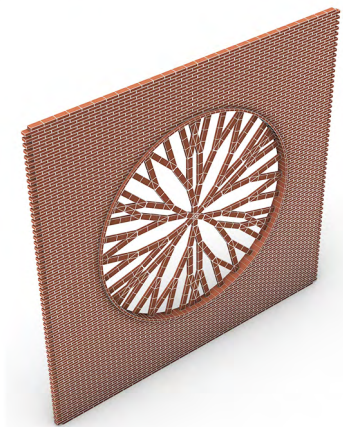
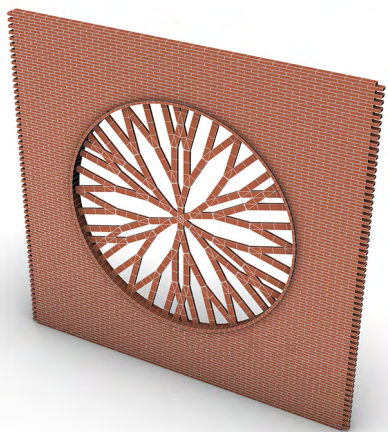
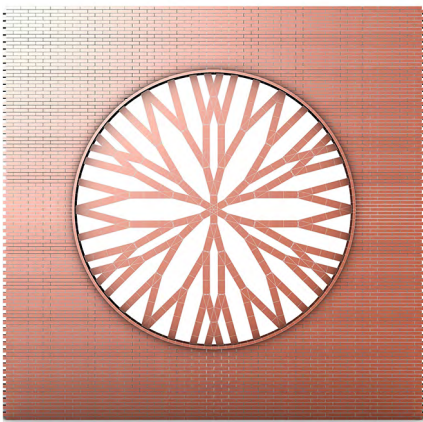
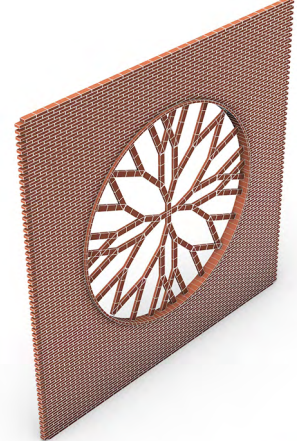
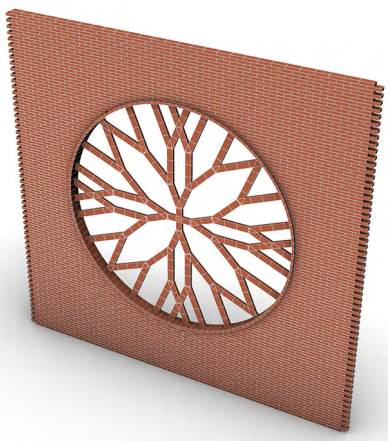
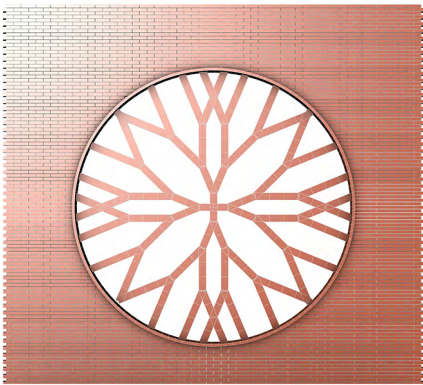
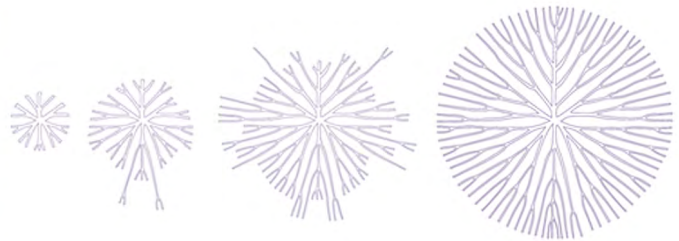
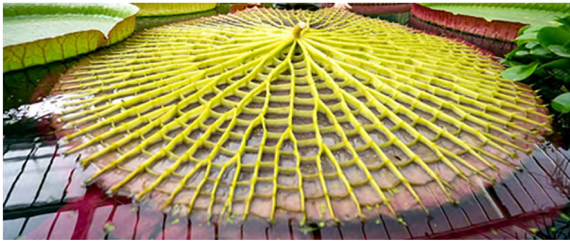


Fig. 4. Study of the morphogenetic process of *Victoria Amazonica*, characterised by radiant and branched growth that incorporates principles of structural efficiency and translates its radiant structure into ornamental building elements based on bricks (elaboration by the authors).



cones and the available surface area. One may wonder why Romanesco broccoli develops its heads from a circular base rather than other polygons capable of optimising the tessellation of the surface, such as squares, triangles or hexagons. One possible answer is that the curvature of the conical surface maximises exposure to sunlight, regardless of the angle of incidence, thereby increasing the efficiency of light absorption for photosynthesis, even in conditions of limited illumination [6].

The second algorithm [Buratti, Rossi 2021] computes three-dimensional branched structures inspired by the “Pythagorean tree” fractal (fig. 3a), based on the Pythagorean identity $a^2=b^2+c^2$ and a binary iterative process of rotations and homotheties. In three-dimensional space, the construction replaces squares and triangles with triangular prisms and tetrahedra (fig. 3b), generating self-similar geometries representative of plant branches and the pulmonary vascular system (fig. 3c, d). In this case, the repetition of the binary division, associated with the halving of the section, allows for effective occupation of space without interference between one branch and another. The reason for this is easy to understand when one considers how closely respiration and photosynthesis are linked to the efficiency of gas exchange. This increases as the available surface area increases, hence the branches, which, in the limited volume of the lungs, can generate surfaces of up to 100 m², the equivalent of a tennis court.

Subsequent experiments exploit the fractal properties of self-similarity and recursion in the modular use of bricks. Bricks are a fundamental building element whose history is intertwined with the development of architecture and civilisation. Its versatility, durability and ease of production have made it a preferred material in different ages and cultures, giving rise to a wide range of construction techniques and architectural styles that are not limited to simple masonry use but extend to the construction of vaults, arches and domes, demonstrating the flexibility and ability of brick to adapt and combine in complex geometries.

The iterative and recursive principles whereby, in a fractal, an elementary shape repeats itself according to scalable patterns, generating structures that nevertheless maintain geometric consistency, translate into constructional terms into compositional systems in which individual bricks, understood as fundamental units, are aggregated according to configurations typical of self-similar systems [Jong 2005]. The adoption of fractal patterns

not only optimises the use of space but also the distribution of structural loads, thanks to their unique hierarchical and branched nature, which, as demonstrated by numerous examples in nature, promotes efficient stress transmission [Banach, Wrobel 2014]. Furthermore, the fractal dimension, a non-integer measure that quantifies geometric complexity, can guide modular design to maximise the contact surface and cohesion between individual elements, facilitating their possible digital fabrication [Oxman 2010].

In synergy, fractal principles and bricks were initially applied to possible configurations of rose windows, radial elements of ornamental and symbolic value, which lend themselves well to algorithmic reinterpretation [Buratti, Rossi 2022]. From a geometric point of view, in fact, a rose window can be constructed as a series of recursive transformations applied to a basic module (brick). Each iteration produces a reduced or rotated copy of the module, arranged according to radial symmetries belonging to dihedral groups D_n but enriched by scalar variations and translations on multiple scale levels (figs. 4, 5).

The algorithmic process defines the primitive unit –the brick– and its connection graph, described as a set of planes oriented in space that represent possible coupling interfaces. Translation and rotation operations applied to these planes modulate the topological and metric relationships between adjacent modules. The recursive application of these rules then generates complex discrete structures, similar to those produced by *L-System* generative systems.

A second series of studies explored the possibilities offered by introducing spatial hierarchies between elements, modifying the generation process and aggregation rules to construct structures based on a series of load-bearing arches and related infills.

The adoption of discrete computational design logic, formalised in the *Wasp* tool [7] [Rossi 2017; Rossi, Tesmann 2019], made it possible to tackle the complexity determined by the use of brick in a hemispherical dome and to study the possible variants (figs. 6, 7). Although the bottom-up approach allows for the exploration of a wide range of geometric configurations, the proposed algorithm does not compute indications on the stability of the final structure. The generation of the structure based solely on local relationships does not imply that the algorithm is efficient in considering the overall morphology of the whole. This lack of control at the

Fig. 5. Another structural scheme based on *Victoria Amazonica* for the design of an architectural element that highlights the transfer of organic branching principles into a functional geometric configuration for building construction (elaboration by the authors).

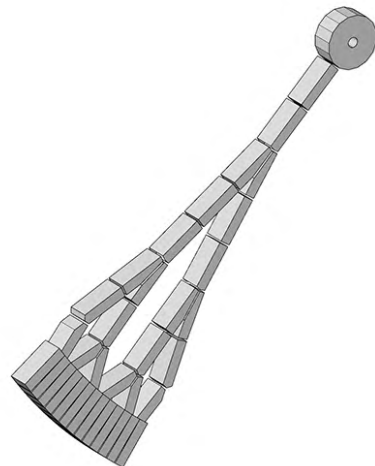
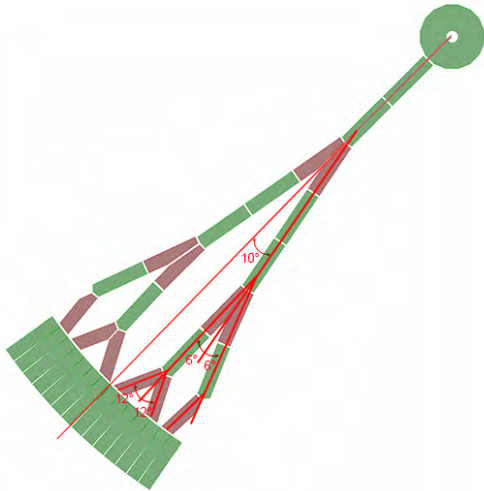
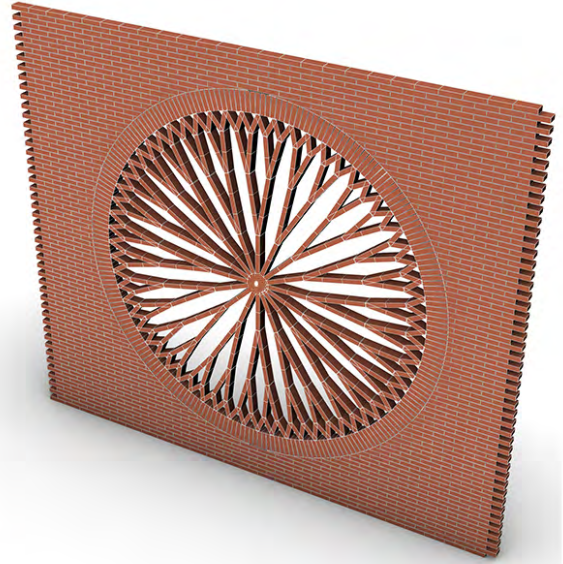
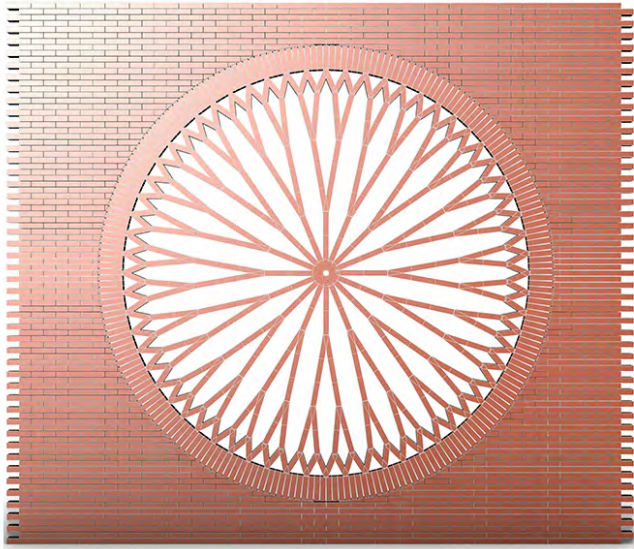
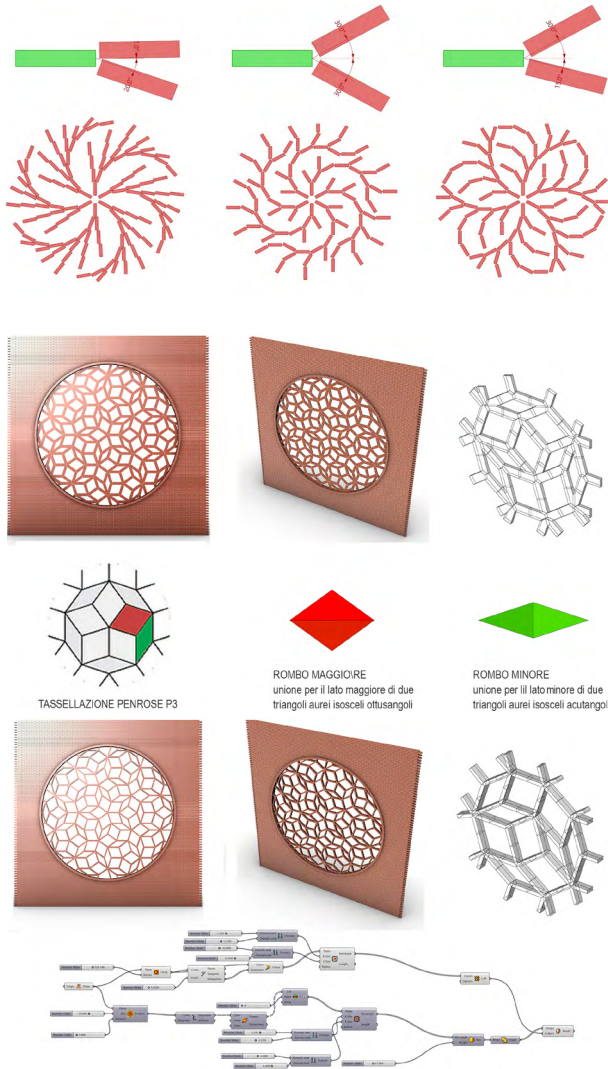


Fig. 6. Computational investigation of the application of organic growth models for structural design in architecture: transposition of biomimetic schemes for the construction of domes through the formal and structural optimisation of the use of brick (elaboration by the authors).



Fig. 7. Parametric relationships between the various brick elements (elaboration by the authors).

Fig. 8. Experimentation with the use of brick as a building module for the creation of a Penrose tiling. The synergy between aperiodic geometry applied to a traditional building system demonstrates the innovative potential of computational design (elaboration by the authors).



macro-structural level can lead to brick distributions and spacings that, while complying with local rules, are inadequate to ensure the overall stability conditions required by the construction system. To address these issues, a process initially developed for creating structures from irregular elements [Allner et al. 2020] has been adapted, utilising the physical simulation of the structure through the *Kangaroo* plug-in [Piker 2013]. Each brick is defined as a rigid body connected to adjacent bricks with variable resistance springs. When the physical solver is activated, the springs pull the bricks towards each other, optimising the spaces between them. By controlling the maximum distance at which two bricks are considered connected, it is possible to close spaces of varying sizes, thereby reducing the structural instability.

Further experiments will study new composition schemes (fig. 8) and constructions based on heterogeneous elements, characterised by variations in scale or geometric differences. This also includes irregular or partially deteriorated components, such as recycled materials from existing structures, which can be integrated into the system through appropriate redefinitions of the topological and geometric connection relationships.

Conclusions

The natural world, admired for its efficiency in solving problems, offers complex models that can be translated into algorithms describing form, as natural and computer systems share similar structural patterns, albeit often invisible. In design disciplines, these algorithms can generate not only representation models, but also concrete artefacts. This shows how the conversion of form into computer language does not reduce reality, but rather reveals its complexity. The cases presented here are examples of this, most notably the reinterpretation of the brick as a modular unit within a geometric language, where rules are extrapolated from natural morphologies.

The possibilities are not limited to visual representation: fractal parameterisation in *Grasshopper* allows for light and shadow simulations, ventilation assessments and thermo-visual optimisations, transforming the brick into a multifunctional architectural device. In this way, the design process approaches a synthesis between generative geometry and construction tradition capable of integrating formal complexity and material concreteness.

Acknowledgements

Although this research is a collective work, paragraphs *Introduction* and *Fractal Geometry: References and Digital Models* can be attributed to Michela Rossi and Giorgio Buratti, and *Designing complex factual struc-*

tures: from broccoli to rosettes to Giorgio Buratti and Andrea Rossi. The algorithmic definitions in figures 2-5; 7 are by Buratti Giorgio, while the algorithm that calculates the geometries in figure 6 is by Andrea Rossi.

Notes

[1] In mathematics, the Lebesgue measure is the measure typically used for subsets of a Euclidean space of any dimension. It is a complete positive measure that generalises the elementary concepts of area and volume of subsets of Euclidean space.

[2] The definition is attributed to Charles Hermite, an eminent French mathematician who, in a letter to a colleague, defined continuous but non-differentiable functions as 'monstrous' or 'pathological' because they challenged the intuitive notions of regularity and differentiability on which much of the mathematical analysis of the time was based.

[3] Goethe quotes and uses it in some of his writings, especially in *Maxims and Reflections* (*Maximen und Reflexionen*, posthumous collection of 1833), where similar formulations on the natural limits of growth and perfection can be found.

[4] A species of water lily characterised by large leaves, supported by a complex system of radial ribs and cross veins, which ensure lightness and structural strength.

[5] Munari simplifies Leonardo's observations on the design/shape of trees: Branches tend to curve upwards unless their weight or that of fruit prevents them from doing so. The reason for this is that each branch competes for greater exposure to sunlight; Branches growing in the lower part of the tree are larger than those growing in the upper part; Branches that are more central, and therefore less exposed

to light, tend to wear out more and appear less attractive; The most attractive and vigorous branches are those at the top of the tree due to their exposure to light and air; When the branches of a tree fork, the angle they form is always the same, regardless of which branch is considered; Statement 5 is always true unless the branch is old. The older it gets, the more obtuse the angle becomes; When a branch divides into two branches, the inclination of the two branches will be different, and the thinner one will be more inclined; When a branch divides into two branches, the sum of the sections of the latter is equal to the section of the parent branch; The inclinations of the main branches are as many as the new branches that start from them without colliding; This inclination bends more the thicker the branches are; The point of intersection of the leaf always leaves a scar on the branch to which it was attached until, due to the age of the tree, the bark cracks and bursts.

[6] In fact, Roman broccoli is a medium-early autumn-winter variety of cauliflower.

[7] *Wasp* is a plug-in for *Grasshopper*, developed in *Python*, which offers combinatorial tools for designing with discrete elements. The description of each part includes all the necessary information for the aggregation process (part geometry, position, and orientation of connections), while providing a set of valuable tools for constraining the resulting aggregation. *Wasp* was developed by Andrea Rossi as part of research into digital materials and discrete design at the DDU Digital Design Unit at TU Darmstadt, led by Prof. Oliver Tessimann.

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