The Palatine Chapel Revealed: Methods and Tools for the Geometric Interpretation of Aachen Cathedral

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Abstract

This study presents an integrated workflow combining digital surveying and 3D modeling for the geometric analysis of architectural surfaces, with application to the vaulted system of the Palatine Chapel, the Carolingian nucleus of Aachen Cathedral. Since 2022, a collaborative research initiative has conducted an extensive 3D survey of the entire complex. Based on these data, the investigation addressed the morphology of the vaulted structures, focusing on their geometric genesis. The chapel is organized around a central octagon with three levels, surrounded by two ambulatories that mediate the transition to a sixteen-sided perimeter. The lower ambulatory, covered by groin vaults alternating quadrangular and triangular bays, produces a highly permeable space. The upper ambulatory is more articulated, defined by semicircular arches that structure spatial rhythm in plan and elevation, with radial chapels and triangular transitional units. The analysis distinguished the geometric principles of triangular and quadrangular vaults by comparing ideal mathematical models with point cloud data. The proposed workflow proved effective in linking constructive knowledge with morphological analysis, clarifying the relation between design intent and built form. The results provide new evidence on medieval construction processes, reference models, and the technical expertise required to achieve complex vaulted systems.

Keywords: 3D surveying and modeling, Early medieval vaults, Palatine Chapel, Aachen Cathedral, stereotomy

Introduction

Within the broader framework of the *Aachen Cathedral Project* [1], the study focuses on the original Carolingian core of the monument, the Palatine Chapel, in particular on the vaults of the annular ambulatory that surrounds the large central octagon, with the aim of investigating their geometry and constructive logic.
As illustrated by Bruno Schindler [Pieper, Schindler 2017], the chapel presents a vertical articulation in multiple levels for both the external ambulatory belt and the central octagon. To this vertical segmentation is added a horizontal sequence, which, level by level, distinguishes progressively less 'public' areas from the entrance toward spaces reserved for the officiants (fig. 1).
On the ground floor it is possible to identify a radial ambulatory composed of a succession of polygonal rooms, each one story high, likely intended for the faithful, as in a parish church (shown in blue and light blue), contrasted with a triple-height space within the central octagon reserved for the clerics (in green). On the upper floor, given the central void, the layout assigned the sector adjacent to the *Westwerk* to the emperor, while the remainder of the ambulatory likely accommodated members of the court. Opposite the *Westwerk*, on both levels, were the altars (in red), both visible from the imperial throne.
As previously discussed [Bianchini 2024], this spatial configuration of the Palatine Chapel proves highly dissonant

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precisely as a result of the opposing tensions between the ambulatories: centripetal on the ground floor, centrifugal on the first floor, seemingly resolved only within the central triple-height void. Alongside other evidence, this sophisticated arrangement suggests that the Palatine Chapel is the product of an extremely clear program in the mind of the designer, who meticulously calibrates the role, form, and function of each architectural component in pursuit of its realization.
The unusual geometry of the complex vaulted systems covering the ambulatories exhibit not only reveals knowledge and skills apparently inconsistent with the conventional know-how of the 8th century but also reinforces the idea that the Palatine Chapel was conceived as a unified project, essentially autographic and rigorously controlled from design to execution. This points to the presence of a single designer endowed with considerable autonomy, currently identified as Odo of Metz.

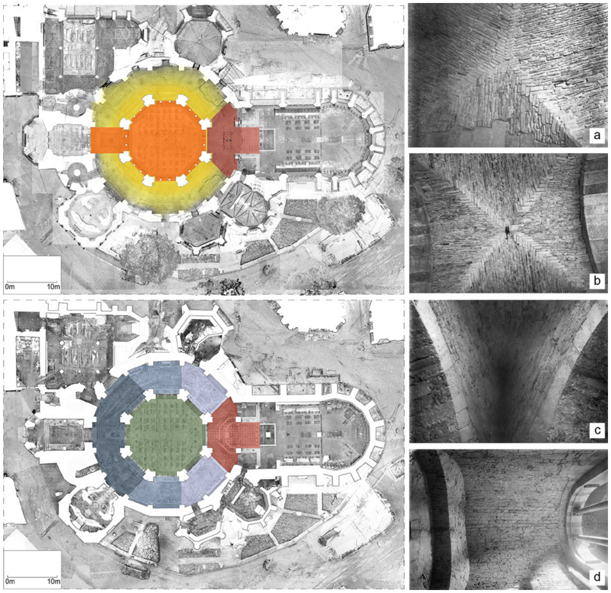

Fig. 1. Spatial organization of the Palatine Chapel according to Bruno Schindler: ground floor for the faithful (blue) and clerics (green); imperial octagonal oratory (orange); first floor for the court (yellow); sanctuaries on both levels (red). Right: vault typologies (© Photo Archive of Aachen Cathedral).

Palatine Chapel

The Palatine Chapel is most eloquent manifestations of Charlemagne's political and religious program, recalling Late Antique and Byzantine models known to the Emperor prior to the construction of the chapel [Ulrike, Beckmann 2012]. It introduces original solutions, particularly in the conception and construction of the vaulted systems, which makes it unique within the architectural panorama of the 8th-9th centuries.
Comparison with coeval monuments confirms this distinctiveness [Pieper, Schindler 2017; Attenni et al. 2023]. For instance, unlike the Basilica of San Vitale in Ravenna where the galleries are recessed and the dome was lightened by the use of fictile tubes, at Aachen the masonry is entirely executed in stone and brick, while the uninterrupted spatial continuity of the ambulatories up to the octagonal core provides the ensemble with a pronounced distributive and visual unity. Other examples of centrally planned architecture, such as the Mausoleum of Santa Costanza in Rome or St. Gereon in Köln, share with the Carolingian Chapel the pursuit of compositional unity, yet they do not exhibit a comparable degree of experimentation in constructive and geometric solutions. In this respect, the Palatine Chapel may be regarded not only as a re-elaboration of existing models but as a moment of advanced geometric and constructive experimentation.
The differentiation between the two levels of the ambulatory renders this experimentation particularly evident. On the ground floor, a system of groin vaults (figs. 1a, 1b), distributed across quadrangular and triangular bays, ensures visual and distributive continuity consistent with the processional function of the space. By contrast, the upper ambulatory displays a more articulated spatial organization: semicircular arches link the inner piers to pilasters set into the outer wall, subdividing the space into clearly defined bays (figs. 1c, 1d). Here, radial chapels provide an alternation between open and more intimate areas, suited to specific liturgical practices, while the triangular transitional sectors act as connective elements between the inclined barrel vaults and the external structural framework.

Research objectives

The survey campaigns conducted from 2022 to 2024 have produced a high-resolution three-dimensional numerical model. Geometric analysis of these data reveals that the

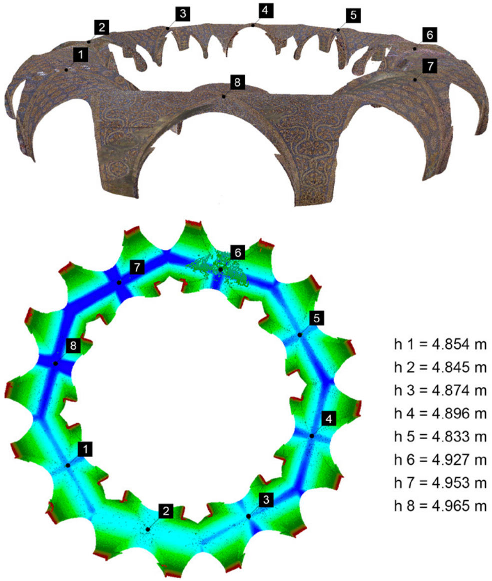

Fig. 2. Top: numerical model of the groin vaults; bottom: elevation map visualization and ridge height analysis of the quadrangular-plan vaults (elaboration by the authors).

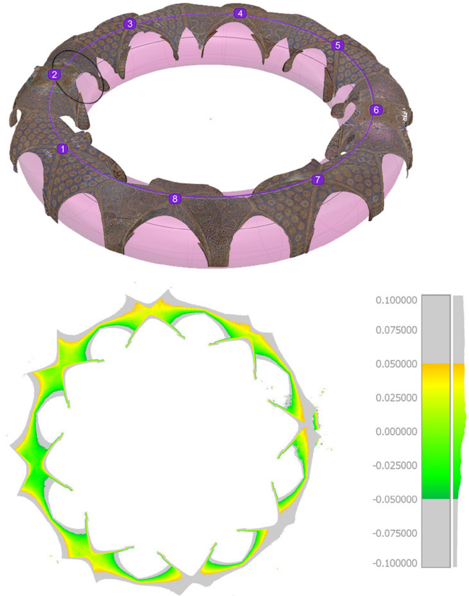

Fig. 3. Top: modeling of the annular vault; bottom: cloud-to-mesh distance within a ± 5 cm range (elaboration by the authors).

various vaulted systems were conceived according to distinct yet coordinated geometric logics, expressing a design articulation that goes beyond mere structural functionality. The research has a twofold objective: on the one hand, to clarify the generative principles of the different types of vaults; on the other, to investigate their meaning within the broader framework of Carolingian architectural culture, assessing whether they stem from conscious knowledge or from pragmatic responses to construction and spatial constraints. The study redefines the role of the Palatine Chapel as a laboratory of geometric experimentation, capable of integrating structural, symbolic, and functional requirements into an innovative architectural language.

Geometric genesis of groin vaults

The lower ambulatory of the Palatine Chapel forms a continuous ring surrounding the central octagon, articulated into quadrangular and triangular bays covered by groin vaults. A first interpretative hypothesis considered these vaults as the outcome of the intersection between an annular vault and radial cylindrical surfaces. To test this hypothesis, the survey data were directly analyzed, visualizing the elevation values of the ridge lines through the elevation map function [2]. The visualization clearly showed that the ridges are not aligned on a single plane nor do they lie on a common circumference, as would be expected from a toroidal construction.
In order to verify the reliability of this observation, the graphical evidence was cross-checked with direct measurements on the point cloud, focusing specifically on the ridge heights of the vault intrados. The recorded elevations range from a minimum of 4.83 m (bay 5) to a maximum of 4.96 m (bay 8), with an overall variation of about 13 cm. For most of the bays (1 to 5 and 6 to 8), deviations remain within a range of ± 2-3 cm, while a sharper discontinuity is observed between bays 5 and

6, which may reflect a differentiated building phase or subsequent structural deformation (fig. 2).
To further verify that the ridges are not actually arranged along a circumference, an ideal torus was generated by translating a circular section –derived as the mean of several circumferences obtained from radial vertical slices of the annular path– along the ring trajectory. The average radius of this circle was 2.32 m, resulting from a range between 2.27 and 2.38 m. The comparison between this theoretical surface and the point cloud, however, revealed significant discrepancies: only a limited portion of the data fell within the ± 5 cm tolerance range (fig. 3). Corroborated by the elevation map analysis, this outcome led to the rejection of the annular generation hypothesis for the groin vaults. Additional confirmation is obtained by interpolating the ridge lines of the groin vaults within a polygon. The resulting figure is an octagon with sides measuring on average 8.85 m, except for a single longer one (8.99 m), corresponding to the octagon that defines the original layout of the Palatine Chapel [Buchkremer 1955].
The investigation was then redirected toward reconstructing the groin vaults as the intersection of cylindrical surfaces. For the quadrangular bays, two cylinders with radii of 2.52 m and 2.32 m were identified, whose intersections define the ridge profiles of the imposts. In the case of triangular bays, a more complex system was required, based on the intersection of three cylinders: two continuing the geometry of the quadrangular bays (with an average radius of 2.33 m) and a third, smaller cylinder (radius 1.99 m), which allowed the triangular bay to be integrated into the overall structural framework. Modeling these surfaces produced ideal reconstructions of both quadrangular and triangular groin vaults, subsequently compared with the point cloud.
The comparison demonstrated a general coherence between the theoretical models and the surveyed geometries, with deviations usually within ± 3 cm. Such values are compatible with medieval construction tolerances and with possible deformations accumulated over centuries [3]. Nevertheless, certain localized areas, especially at the junctions between triangular and quadrangular bays, exhibited greater discrepancies (figs. 4, 5). These may be interpreted as the outcome of executional flexibility, privileging gradual on-site adjustments.

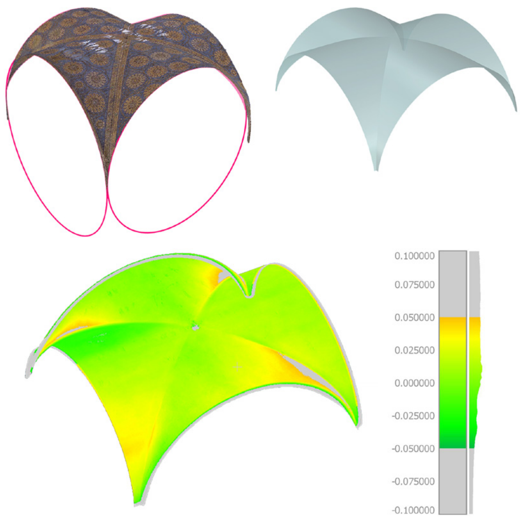

Fig. 4. Top: modeling of the groin vault with quadrangular plan; bottom: cloud-to-mesh distance within a ± 5 cm range (elaboration by the authors).

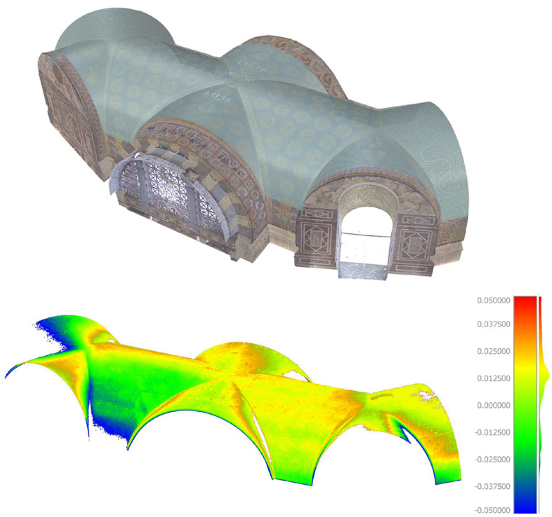

Fig. 5. Top: modeling of the groin vault with quadrangular and triangular plan; bottom: cloud-to-mesh distance within a ± 5 cm range (elaboration by the authors).

From a constructive standpoint, the ground floor groin vaults raise significant questions. The absence of separating arches indicates that the vaulted surfaces were directly sprung from the masonry, likely supported by modular centering elements that could be reused across different bays. The relative homogeneity of the reconstructed cylinder dimensions suggests the use of a limited number of centering models, adapted to the needs of quadrangular or triangular bays.
In terms of generative logic, the groin vaults of the lower ambulatory appear to be based on a rigorous geometric principle –the intersection of cylinders– flexibly adapted to the varying planimetric conditions. This points to a remarkable level of constructional expertise, capable of transforming relatively elementary geometric schemes into an articulated and coherent system that was both structurally stable and spatially effective.
The comparison between ideal models and built reality thus highlights a dual dimension: on the one hand, the design intention of applying a unifying geometric rule based on the groin vault; on the other, the practical necessity of localized adjustments during execution. This dialectic between design and construction provides valuable insight into the organization of the Carolingian building site and the ability of its craftsmen to translate geometric principles into material solutions.

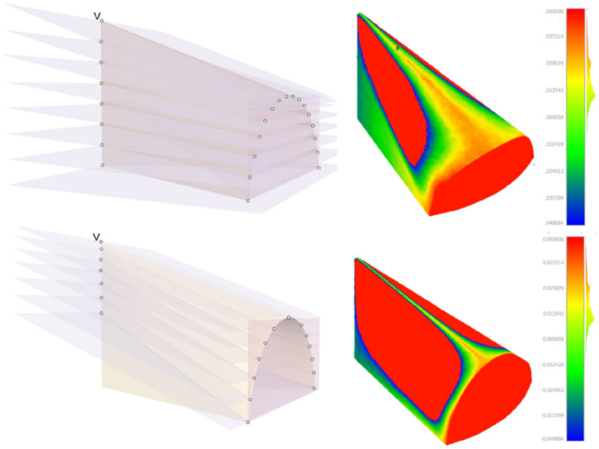

Fig. 6. Top: generic conoidic surface; bottom: conoidic surface with director plane. Right: cloud to mesh distance values in ± 5 cm range (elaboration by the authors).

Geometric genesis of conic vaults

After analyzing the vaulted system of the ground floor, the investigation turned to the conical vaults located in the ambulatory of the first floor [Attenni et al. 2023]. This typology, foreign to the building tradition of the 8th century [Bianchini 2024], immediately proved to be of considerable interest, raising significant questions regarding the level of geometric and technical knowledge attained in that period.
From a compositional perspective, the conical vaults expand towards the vertical outer walls of the chapel, arranged to form a cylinder. As a result, the intersection between the vaulted surface and the cylindrical wall produces a profile described by a skew quartic curve. Beyond the architectural novelty of this solution, such curves are not easily employed to directly describe the geometry of the cone as a ruled surface; for this reason, it was necessary to identify a plane curve to be adopted as the directrix. For this purpose, the curve resulting from the intersection between the vault and a vertical plane passing through the points where the quartic curve meets the springing plane of the vault was chosen. This arc, which closely approximates a circumference, provides an effective representation of the cone's directrix. At the same time, the solution appears consistent with a plausible constructional practice, based on the definition of a semicircular arch with a wooden centering and its subsequent connection to the cylindrical wall.
Although preliminary investigations have already strongly suggested the use of a cone as the geometric matrix underlying the form of the vaults, it was nevertheless deemed necessary to consider two other surface typologies that could be considered equally plausible within the architectural context:
- a conoidal surface with a circular vertical directrix and an oblique director plane parallel to the crown line [4];
- a more general conoidal surface (fig. 6).

The comparison among the three surfaces quickly demonstrated that the conical surface provided the most convincing match with the measured data. Consequently, we proceeded by modeling the vault as a cone, using the two inclined lines at the springing plane as generatrices, taking their intersection point as the

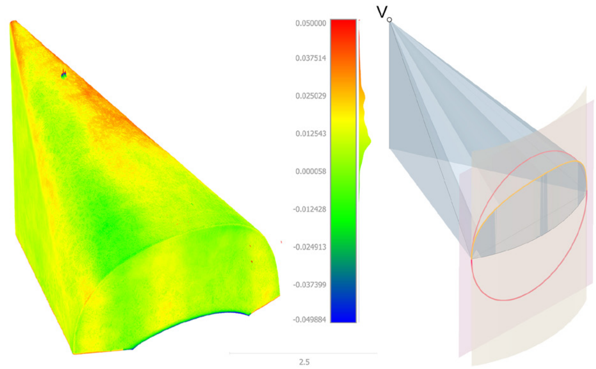

Fig. 7. Left: cloud to mesh distance (± 5 cm) between the point cloud and the cone; right: conic surface intersected by a vertical plane (red) and the cylindrical surface (orange) (elaboration by the authors).

vertex *V*, and adopting the vertical circle previously discussed as the directrix.
We then observed that this conical surface intersected the cylindrical surface representing the outer wall along a quartic curve, which closely approximated the surveyed point cloud.
To further validate the model, the resulting surface was compared with the 3D point cloud using a cloud-to-mesh distance analysis. This comparison revealed a standard deviation of just 1 cm and a maximum deviation of 4 cm at one side of the springing level (fig. 7).
After confirming the strong correspondence between the conical surface and the captured point cloud, we turned to identifying the 'standard' elements of this cone: its axis and its perpendicular base. To construct the axis, we followed a mathematical modelling approach [Salvatore 2012a]. First, we generated a generic sphere with its center at the vertex of the cone. The intersection between this sphere and the conical surface (fig. 8a), together with the intercepted portion of the cone, defined a solid (fig. 8b). The barycenter (*B*) of this solid (fig. 8c), together with the vertex *V*, lies on a straight line representing the internal axis *z* of the cone (fig. 8d). Since the *z* axis is not perpendicular to the circular conic section lying on the vertical plane, the directrix of the cone can be identified as an ellipse. This elliptical curve can be easily constructed by intersecting the cone with a plane perpendicular to the *z* axis. Its geometry is completed by determining the conjugate orthogonal axis (fig. 8e).
To this point, we then reconstructed the circular section of the cone to compare it with the arch used as the conoid's directrix. Indeed, any elliptic quadric cone contains two circular sections. One of these was obtained following the method proposed by Théodore Olivier [Olivier 1852, pp. 199-202], later interpreted through mathematical digital modelling by Marta Salvatore [Salvatore 2012b, pp. 151-154]. The procedure involves constructing a sphere tangent to the sides of the triangle defined by the major axis of the elliptical base and the vertex of the cone. The intersection of this sphere with the cone produces the desired circular section (fig. 8f).
Finally, comparison between this reconstructed circle and the original directrix's curve derived from the point cloud intersection with the vertical plane, revealed a close overlap, differing only by a minor angular rotation.

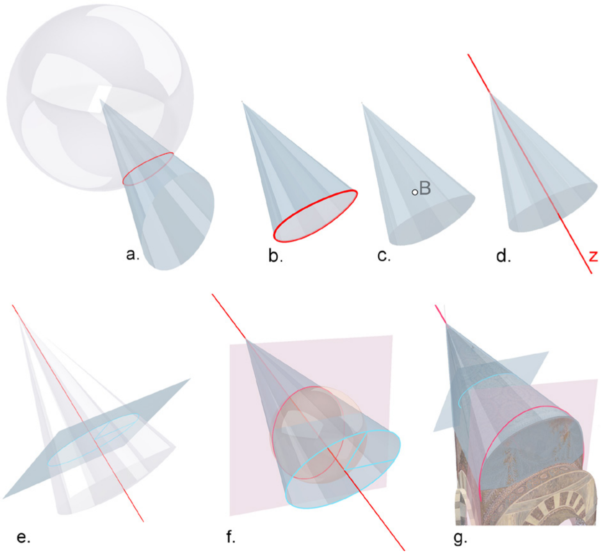

Fig. 8. From a. to d.: construction of the axis of a quadric cone; e.: construction of the ellipse; f.: construction of the two planes that cut the elliptic cone generating circumferences; g.: final reconstruction (elaboration by the authors).

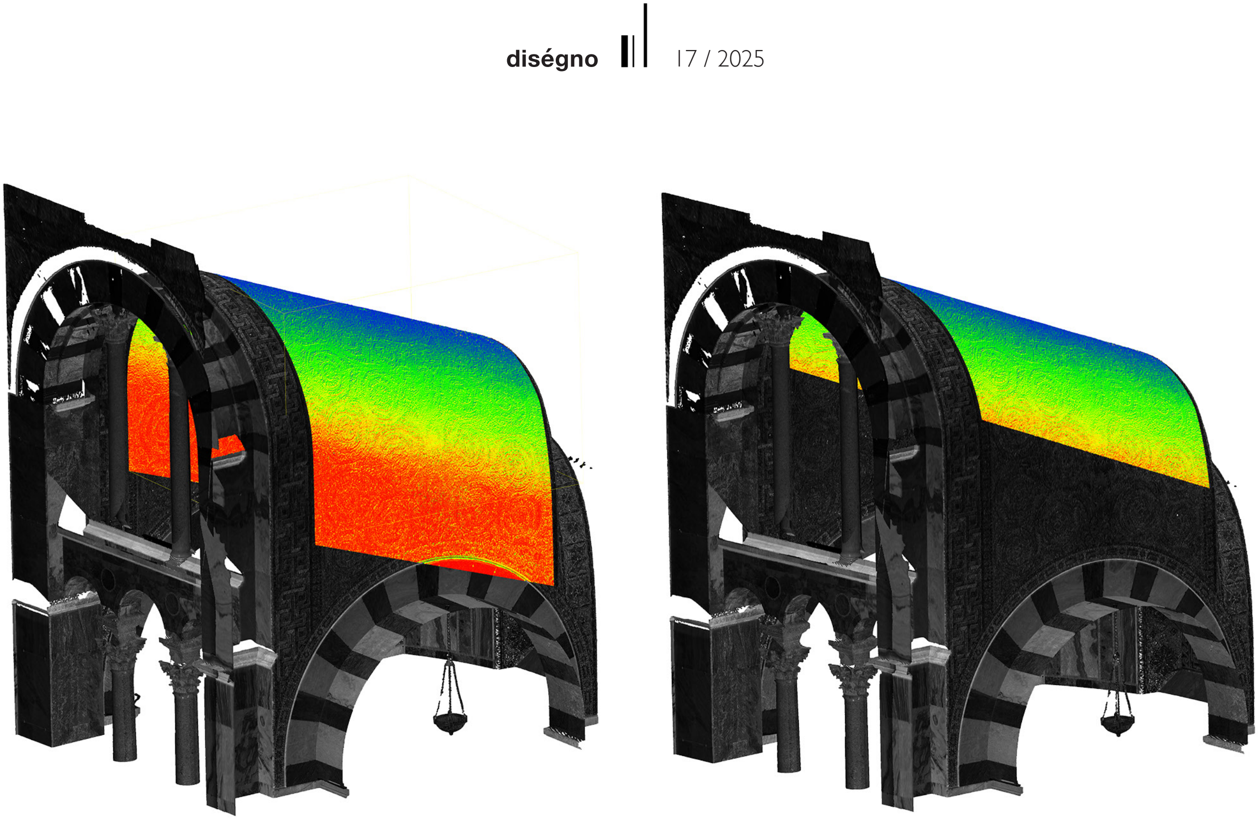

Fig. 9. Left: visualization of the verticality values on the point cloud; right: segmentation of the cylindrical portion of the vault by excluding the points with significant verticality values (elaboration by the authors).

Based on this analysis, we can conclude that the vaults of the Palatine Chapel are indeed conic in shape, and that their on-site construction was plausibly guided by three primary elements: the vertex, the generatrices at the springing plane, and the vertical circular section (fig. 8g).

Geometric genesis of the inclined barrel vaults

The same type of analysis was also carried out on the adjacent inclined barrel vaults, located on the first level of the Palatine Chapel and converging towards the octagonal dome. Although these vaults refer to an architectural and constructional matrix that appears more frequent, the digital reconstruction of this typology has made it possible to highlight peculiar geometric features and to verify the mastery of an architectural practice grounded in a deep knowledge of conic figures and properties.
The first phase of the study focused on identifying the springing plane. To this end, the analysis of the verticality of the points [5] allowed the segmentation of the point cloud of the entire bay, isolating the cylindrical curved surface from the vertical walls (fig. 9).
Then, the construction of the cylinder that best approximated the real surface was initiated, in order to analyze its properties. To understand the type of theoretical surface to interpolate with the point cloud, it was necessary to geometrically examine both the vertical and the transverse sections of the vault. Starting from the identification of the vault crown, the point

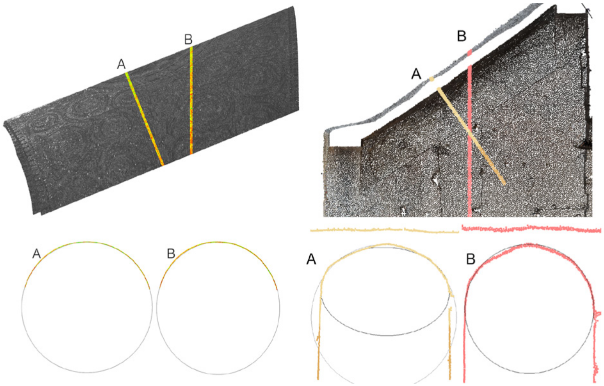

Fig. 10. Inclined vaults of the Palatine Chapel (left) and of the Colosseum (right). Sections perpendicular to the axis (A) and vertical sections (B) with extraction of geometric profiles (elaboration by the authors).

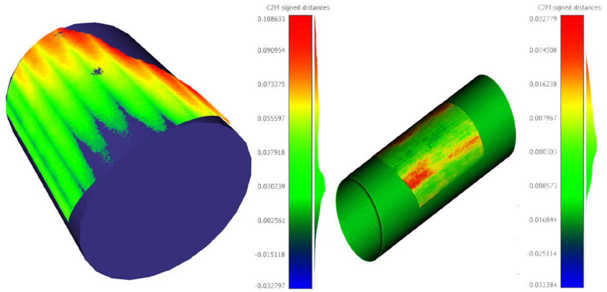

Fig. 11. Visualization of the distance between the cylinder and the point cloud. Left: the cylinder calculated using shape detection algorithm.; right: the one using best fit algorithm (elaboration by the authors).

cloud was first intersected with a vertical plane and then with a plane perpendicular to the crown, in order to analyze the two resulting profiles.
This analysis highlights a particularly interesting element: the section perpendicular to the crown is circular, whereas the vertical section is elliptical, with the minor semi-axis corresponding to the radius of the cylinder (2.61 m) and the major semi-axis, equal to the vault's vertical rise, measuring 2.81 m. This configuration, already noted by Raabe, Trautz and Di Pumpo [2019], deviates from a constructional tradition based on designing the inclined barrel vault as the translation of a semicircle contained in a vertical plane along an inclined axis, thereby generating an inclined cylinder [Adam 2014, pp. 189-211].
As a purely demonstrative comparison, the same analytical method was applied to one of the inclined vaults of the Flavian Amphitheatre [6] to verify the actual peculiarity of the system adopted in Aachen. This investigation clearly showed the use of an oblique cylinder as its geometric genesis (fig. 10).
This peculiarity inevitably raises questions about the organization of the construction site and the preparation of the centering needed for its realization. Leaving aside constructional aspects for the moment, the following phase modeling concerned the reconstruction of the solid with an axis inclined by 21.52° with respect to the horizontal and a circular base, that is, a right cylinder. The reconstruction of the theoretical cylindrical shape from the point cloud can be carried out either semi-automatically –through shape detection and best-fit algorithms– or manually. In this case, the first approach was preferred, since the apparent morphological correspondence between the segmented points and the reference theoretical shape encouraged the adoption of a method capable of producing the statistically most appropriate reconstruction, avoiding additional subsampling steps.
Nevertheless, the shape detection algorithm for the cylinder [7] did not produce satisfactory results. In fact, it further subdivided the already segmented point cloud into smaller portions, calculating a distinct theoretical cylinder for each. This behavior is attributable to constructional deformations of the vault, which are reflected in the point cloud and hinder the identification of a single cylindrical surface. By increasing the tolerance in the deviation between the reconstructed theoretical shape and the corresponding points, the algorithm is able to generate a single solid, approximating the cylindrical surface of the vault while minimizing local errors. However, the theoretical cylinder thus obtained showed significant deviations from the point cloud, incompatible with the constructional irregularities observed.
The second attempt at automated shape reconstruction employed a best-fit algorithm [8] [Bianchini, Carnevale, Griffo 2024]. The algorithm used is iterative in nature, with the objective function defined by the characteristic parameters of the general function of a right

cylinder with axes not parallel to the Cartesian axes (components of the direction vector, axis position, and radius value). This procedure, carried out in the *Matlab* environment, favors a single global solution for the reconstruction of the shape rather than multiple detailed local fittings (fig. 11).
Starting from the theoretical cylinder constructed using the best-fit algorithm, the final phase of the study focused on identifying its characteristic elements, namely the position of the axis, the radius of the transverse section, and the ellipse of intersection with the vertical plane (fig. 12).
Having identified the right cylinder as the most suitable geometric model to approximate the inclined barrel vault, it is now appropriate to examine the practical implications of this choice, particularly with regard to the design of the temporary works. The vault, constructed with greywacke and travertine blocks of heterogeneous shape and size, displays a coursed masonry pattern, in which regular courses of thin, elongated blocks alternate with larger squared elements (fig. 1d). This textural arrangement necessitates the use of a wooden centering, the original configuration of which is not easily reconstructed.
Assuming a fully vertical centering with its base coinciding with the cylinder's diameter would result in a guiding curve with an elliptical profile. By contrast, employing a semicircular arch centering would require it to be inclined perpendicular to the springing plane. Both scenarios imply a notable complication of conventional construction practice, likely driven primarily by the deliberate intention to realize a right cylindrical geometry.
Raabe, Trautz and Di Pumpo [2019, pp. 26-29] propose a construction model based on the use of a movable centering, slightly smaller than the vault diameter and horizontally shifted by approximately 10 cm between the two sides. According to the authors, this procedure would account for the formation of an elliptical arch arranged on the vertical

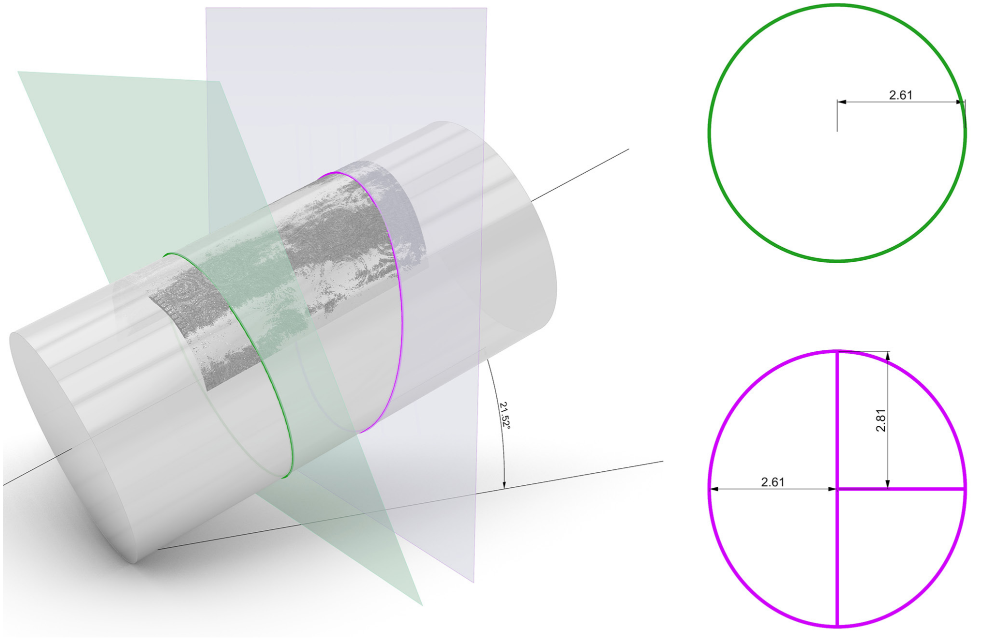

Fig. 12. Left: the theoretical cylinder (in gray), the vertical plane (in purple), and the plane perpendicular to the axis (in green); right: representation of the two intersection curves (elaboration by the authors).

plane. However, this operational principle would produce a flattened elliptical arch, in contrast with the high-rise profile indicated by the analysis of the vault's vertical tracing.
Ultimately, synthesizing the various experimental observations, it becomes evident that the builders possessed a profound understanding of the geometric properties of conic sections, as well as the ability to translate this knowledge into practical construction solutions.

Analysis of results

In recent decades, the evolution of 3D survey methodologies has marked a fundamental milestone in the history of representation, opening new scenarios for the documentation and interpretation of architecture. Today, we are witnessing a further phase of this development: attention is no longer focused solely on data capturing and processing techniques, but increasingly on their analytical potential. Thanks to the high degree of accuracy they can achieve, point clouds do not merely represent a faithful record of reality; they become the raw material for experimenting with tools capable of interpreting them in a manner consistent with their nature.
From this perspective, the case of the Palatine Chapel has provided an opportunity to explore the various strategies currently available, with the aim of fully conveying the informational value of the collected data. From the application of shape detection and best-fit algorithms to the use of more traditional mathematical modeling, the range of possibilities demonstrates that the study of architectural history can no longer disregard advanced analytical approaches. These tools not only allow us to address well-established questions but open the door to new inquiries and perspectives, revealing unexpected aspects that architectural construction inherently preserves.

Conclusions

Within the broader framework of the *Aachen Cathedral Project*, the present study represents a further step in the investigation of the geometries emerging from the analysis and interpretation of the metric data collected on the Palatine Chapel.
The evidence presented, whose reliability is firmly anchored in the accuracy of the original data and the rigor of the applied analytical methodology, reveals an unexpected mastery of conic geometry –an expertise that stands in marked contrast to the state of Western geometric thought as transmitted through historical texts. This characteristic also carries significant architectural implications, reinforcing the hypothesis that the Palatine Chapel was conceived as a unified design and meticulously controlled at every stage. In this context, the presence of a single, talented architect identified in the sources as Odo of Metz, appears both a sound and reasonable hypothesis.
The compositional analysis of the building further highlights several original features, as exemplified by the dynamic relationship established between the ambulatory bands and the void of the central octagon. At the first level in particular, there is an alternation of open quadrangular sectors and blind triangular wedges, which appear to be vaulted with right circular cylinders and conical surfaces, both inclined.
These peculiarities, from both a constructive and formal standpoint, suggest a connection with Eastern building traditions [Raabe, Trautz, Di Pumpo 2019], and specifically with Armenian models [Bianchini 2024]. This comparison, combined with the historiographical hypothesis attributing the same origin to Odo of Metz, seems to outline a coherent picture: an architect of exceptional talent conceiving the Palatine Chapel as a unified project, integrating Roman and Byzantine elements while simultaneously referencing his own cultural background. In doing so, Odo demonstrates not only full mastery of architectural forms but also a level of geometric knowledge that is strikingly advanced in comparison with what is conventionally acknowledged in historiography.
The case of Aachen thus reinforces the idea that, during the so-called 'dark Ages', medieval Practical Geometry served as the primary locus for the preservation and development of Western geometric thought, rather than merely providing basic constructions for key sectors such as building, urban planning, or defense [Bianchini 1995]. The vaults of the Palatine Chapel offer a first significant indication of this, although they have so far received limited attention within the historiographical framework, which has tended to privilege written sources while overlooking the fact that knowledge transmission occurred primarily through oral instruction and apprenticeship. Hence arises the hypothesis of an alternative history of medieval geometry –one not recorded on parchment but inscribed in stone, now ready to be revealed through the analytical, measurement, and simulation tools available to us today [9].

Notes

[1] The *Aachen Cathedral Project* is an international research initiative involving Sapienza Università di Roma (Carlo Bianchini, Carlo Inglese, Guglielmo Villa, Martina Attenni, Marika Griffo, Roberto Barni), Robert Gordon University in Aberdeen (Douglas Pritchard), and RWTH Aachen University (Yannick Ley), in partnership with the Dombauhütte of Aachen cathedral (Jan Richarz). The aim of the project is to provide the most comprehensive possible documentation of the cathedral's buildings and their constituent materials through the integrated use of advanced surveying technologies. The project is co-funded under the framework of the Italian National Recovery and Resilience Plan (PNRR), Extended Partnership 5 'CHANGES', Spoke 8, Thematic Line 1: Production, Organization, and Communication of Knowledge, coordinated by Prof. Carlo Bianchini.

[2] The elevation map in *Autodesk ReCap* allows the assignment of a color value to each point of the point cloud according to its height relative to a reference axis. This tool does not generate an independent metric analysis but provides a color-coded representation that facilitates the visual identification of height variations or local deformations.

[3] A direct examination of the stone was not possible, since all the original structures at both the lower and upper levels are covered by Hermann Schaper's 20th-century mosaic [Radel 2022]. However, it is reasonable to assume that these interventions did not significantly alter the original geometries and proportions of the underlying structures.

[4] The decision to verify the correspondence between the survey data and the surfaces of these conoids, beyond purely geometric considerations, also considers their (much later) probable use for constructive purposes. Reference is made to the shipwright's circular wedge studied by John Wallis –known as Wallis's *cono-cuneus* [Wallis, 1648] – and to the cone terminating in a line described by Guarino Guarini [Guarini 1671, tr. XXV, theor. 1, prop. VIII; Guarini 1737, cap. IV, oss. 6]. In both cases, the surfaces can be classified as conoids with a plane directrix; however, unlike the one examined in the present study, their circular directrix is perpendicular to the horizontal springing plane, which defines the orientation of the director plane.

[5] The analysis was carried out using the geometric features computation tool integrated in *CloudCompare* v2.13.

[6] The point cloud originates from 3D survey and HBIM modeling activities coordinated by the Archaeological Park of the Colosseum (Project Manager: Dr. Federica Rinaldi). The work was carried out by the awarded temporary consortium led by CFR (Consorzio Futuro in Ricerca), Ferrara (scientific coordination: Marcello Balzani, Guido Galvani, Fabiana Raco), in collaboration with GEOGRA' Srl (Sermide), ETS Srl, and JANUS Srl (Rome).

[7] For the experiment, the RANSAC Shape Detection plugin integrated in *CloudCompare v2.13* was used.

[8] The cylindrical best-fit algorithm in the *Matlab* environment was developed and tested within the context of Flavio Carnevale [Carnevale 2026].

[9] In agreement with the arguments presented, Carlo Bianchini was responsible for drafting the *Introduction* and *Conclusions* sections; Martina Attenni was responsible for drafting the sections *The Palatine Chapel*, *Research objectives* and *Geometric genesis of the groin vaults*; Marika Griffo was responsible for drafting the sections *Geometric genesis of the Conical vaults*, *Geometric genesis of the inclined barrel vaults* and *Analysis of the results*.

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