Oblique Analog Models

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Introduction

Every architectural drawing is by its nature the result of a combined operation of projection lines and intersection with a plane, which is usually a sheet of paper. Even a sketch, in its rarefied figuration, evokes such operations for a representation of a design idea or an executive work, in an unequivocal icasticity.
To the physical model of architecture different attention is given, involving a concrete materialization in reduced, real or dimensionally enhanced form of an equivalent morphology. Nothing to do with projection or intersection, but it's an extension into real space. Those who study representation theory cannot exclude to question the possibility of associating the two afore mentioned operations with the stereometry of a physical model.
Since the advent of the digital, among the few adjectives that can uniquely connote the three-dimensional, plastic, material model, undoubtedly is to be ascribed the term 'analog', which qualifies an architectural object for a peculiarity of its own: it clutters our tables and gives us –in addition to the visual sensation– the haptic pleasure, the smell of the material of which it is composed or the sonority at the simple touch; in clear contrast to a processing all contained exclusively within a computer.
So many can be the declinations that describe the model: from architectural micro-models to full-scale mock-ups, in the ideal path linking the two cities of Mildendo and Brobdingnag in Jonathan Swift's novel

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[Swift 1997], in which Lemuel Gulliver first finds himself disproportionately overdone by a multiplication factor of twelve units, and then underdone by the same multiplier. But also models made with new materials and innovative technical devices, at any scale of reduction, from casts to thermal supports, constructive stratagems that have received special attention especially with the advent of new technologies. Or anamorphic and deformed models that simulate graphical representations –which we will focus on in this essay– whose theoretical value becomes indispensable both for making the object and for fully understanding its figurative stratagem, as in the case of early Eisenman's houses or ephemeral models used in the field of visual perception and in exhibition, stage, and film. And finally, functional models, such as structural solutions –from Antoni Gaudi's catenaries to Pier Luigi Nervi's prototypes– or for sound and visual verifications, to which the contents present in some treatises, from Vitruvian machines to Leon Battista Alberti's "bare and simple" models versus "embellished" ones [Alberti 1546, p. 27v].
Unlike the drawing, however, the physical model can distract attention from its natural peculiarity, namely, being –as Massimo Scolari recalled– "an instrument of initiation for generations of architects who in the realization of objects in the form of small architectures were preparing themselves to build in a big way" [Scolari 1988, p. 16] and considered by Scolari himself "ultimately the best representation of architecture" [Scolari 1988, p. 16]. If we want to question the not immediately noticeable potential of maquettes, we must associate it with a strong theoretical value in the course of its realization. That is, we refer to the generation of deformed models of architecture that evoke specific geometric projections. It is not a coincidence that such experimentation is reserved for a small part of work in the history of figuration, to which only a few scholars have decided to devote themselves, with the aim of linking theoretical contents of the discipline of drawing to artifacts of a purely practical nature.
We will focus our attention on the oblique deformation of architectural models, which can only be done downstream of a rigorously theoretical investigation of the axonometric projections that hold the key to understanding such artifacts.

Axonometric deformations of digital models

Oblique axonometry is one of the representation systems that has had the greatest impact in the field of architecture. Its main advantage lies in the possibility of observing in true size some faces of the model, those parallel to the projection plane, while the edges perpendicular to it are affected by a reduction coefficient that depends on the direction of projection. This way of evoking three-dimensionality on paper is simple and intuitive even without knowing that it is the result of an oblique projection and, therefore, has mathematical concreteness and graphic operability. In fact, very ancient representations have survived that intuitively depict a drawing that could be equated to an oblique axonometry, such as, for example, some first-century frescoes found in Pompeii or Leonardo Da Vinci's drawings of the famous war machines.
Nevertheless, oblique axonometries have not always been well regarded by geometry purists, such as Gaspard Monge, as drawings without mathematical rigor.
According to Joel Sakarovitch [Sakarovitch 1997, p. 133], Monge did not want his students to see illustrations of treatises such as Jean-Baptiste de La Rue's stone-cutting one [de La Rue 1728], which contained some sort of oblique axonometries.
However, when Pohlke's theorem [Pohlke 1860] was proved by his disciple, the German mathematician Hermann Schwarz, in 1864, such representations were mathematically legitimized. From then on, oblique axonometry reached the status of a rigorous representation system, especially in its two particular cases such as military and cavalier axonometry.
A military axonometry is obtained when a model is projected obliquely, onto a projection plane that is parallel to its horizontal faces. Its name comes from the fact that it was a type of axonometry widely used in treatises on military fortifications, to see the ground plan in true magnitude. When the model is projected obliquely onto a projection plane that is parallel to any of its vertical faces, a cavalier axonometry is obtained. Its name derives from the certain similarity of this type of views to the way a rider mounted on horseback would appreciate constructions when observed frontally, saving the differences in what the horseman would see, more similar a frontal perspective.

Undoubtedly, human vision is closer to a perspective representation. Spatial perception can be evoked by two perspective images forming a stereoscopic pair, which is the basis of virtual reality devices. The visual system perceives depth by means of certain pictorial-perspective cues. These monocular cues play a fundamental role in the theory of visual perception. For this reason, perspective is one of the most widely used representational systems to convey the spatiality of the scene to people who are not accustomed to the graphic reading of other types of projections such as axonometric ones.
It is difficult to assess the similarity of the mental image evoked in our mind by an axonometric representation, whether oblique or orthogonal, with respect to what we perceive when we visualize the real model with our eyes. Our brain is able to interpret the pseudo-perspective cues offered by axonometries because of their resemblance to perspective. An axonometry is quite similar to perspective when the point of view is far from the observed object, since, in this case, the projective rays are almost parallel and the convergence effect of the parallel lines, typical of perspective, is less evident. However, the lack of convergence of axonometries tends to induce certain perceptual distortions that the viewer has to get used to and learn to interpret, in the same way as a newborn child has to learn to see, relating the visual cues he perceives to the world around him.
Axonometric drawings are more abstract than perspectives because they lack pictorial depth cues that help establish scale and distances, such as height relative to the horizon and relative size between near and far objects. This grade of abstraction is even more noticeable in oblique axonometries, since they can be distorted to a greater or lesser degree depending on the projecting direction angle.
For istance, when projecting an object with an oblique direction of 45° with respect to the projection plane, the reduction coefficient in the axis perpendicular to the projection plane would be equal to one. This means that it could be measured in real magnitude on any of the coordinate axes of the resulting oblique axonometry, but this representation would be highly distorted and its three-dimensional evocation would be far from the real perception of the model. For this reason, especially in Anglo-Saxon countries, the so-called 'cabinet projection' has been widely used. It is basically a cavalier projection in which the reduction factor is 1/2 This reduction makes that the represented model is perceived more natural and similar to the real model. So, what would be the ideal reduction factor to evoke the shapes and volumetric features of an object with an optimal fidelity?
In order to determine this value, we can perform a perceptual experiment like the one shown in figure 1. There are shown several oblique projections of a cube using different reduction coefficients in military and cavalier projection. The aim is to determine the option in which the cube is perceived as closest to our mental image of an ideal cube. This perceptual test is carried out year after year with a new group of architecture students and very similar results are always obtained. In the case of military axonometry, the majority of students choose the cube represented with a reduction coefficient of 0.7 as the most proportionate option, while in the case of cavalier projection they choose the one corresponding to a reduction coefficient of 0.6. It is curious how our perception works so the value obtained for the military is slightly different from that of the cavalier projection, given that both representations of the cube are identical images that have simply been rotated one with respect to the other.
Therefore, it is up to the designer to choose between a reduction coefficient that offers a closer and more proportionate image in relation to the real object, or to opt for a simpler and more abstract option in which, using a coefficient of 1, the graphic model can be measured in true magnitude in all axes.
Certainly, this degree of abstraction is what arouses the interest of many architects of the modern movement and Bauhaus artists in these kinds of representations, such as the well-known axonometries of Theo van Doesburg or some of Piet Mondrian's drawings (figs. 2a, 2b). With these drawings, the authors delve into art movements such as neoplasticism and elevate architectural representations to the status of artworks.
The maximum abstraction level was reached with the drawings by John Hejduk (fig. 2c), who creates his masterly style by projecting the object in such a way that one of its coordinate planes is parallel to the projection direction, that forms 45 degrees with the rest of the

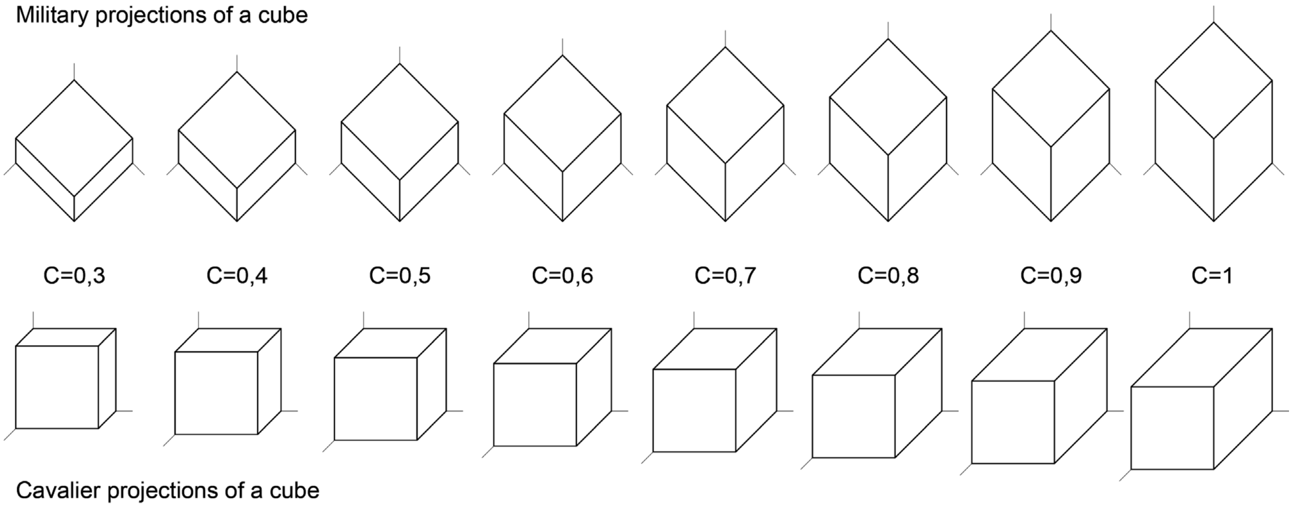


Fig. 1. Different oblique projections of a cube to determine the proper reduction coefficient C (P.M. Cabezos-Bernal).

coordinate planes. The work acquires a certain cubist character, as two of the coordinate planes are projected in true maagnitude, which remind some of Le Corbusier's paintings (fig. 2d).
The interest in this type of representations remained alive in renowned architects, such as Peter Eisenman or Arata Isozaki but, in recent years, oblique axonometries suffer a worrying abandonment, mainly due to the change of paradigm that supposed the arrival of CAD software. Most programmers must consider more convenient or simpler to allow only obtaining orthogonal projections and perspectives from a 3D model and not oblique projections.
In the current scenario, in which modeling in three dimensiona is mandatory, this limitations imposed by the most widespread CAD programs lead most users to use only orthogonal axonometries and perspectives in their representations, so it is the easy way. To overcome this drawback and obtain military or cavalier models from a three-dimensional model, it is possible to perform a projective or affinity transformation, which consists in transforming an orthogonal axonometry into an oblique projection. An affinity relationship can be established between an orthogonal projection and an oblique one, as shown in figure 3. It shows how a cube is projected onto two planes by means of a cylindrical projection. One of the planes is orthogonal to the projection direction, therefore, an orthogonal projection of the cube is obtained on it. The other plane is oblique with respect to the projecting direction and also parallel to the horizontal faces of the cube, so an oblique projection is obtained on this plane, specifically a military axonometry of the cube. The affinity relationship between both projections is defined by three elements. Firstly, by the affinity axis, which is the intersection between the two planes of projection. Secondly, by the affinity direction, which is perpendicular to the affinity axis, and thirdly by the affinity ratio R, which can be determined by means the relation $R = A_2S/A_1S$.
It can be observed that this affinity relationship is equivalent to perform a non-uniform scale change in the direction of affinity. Therefore, an orthogonal axonometry can be easily obtained from a 3D model with any CAD software and then transformed into an oblique projection, which can result in a military or cavalier projection, by being scaled in the direction of one of the coordinate axes of the axonometry (fig. 4).
This operation can be carried out by means any graphics software that allows non-uniform scaling.

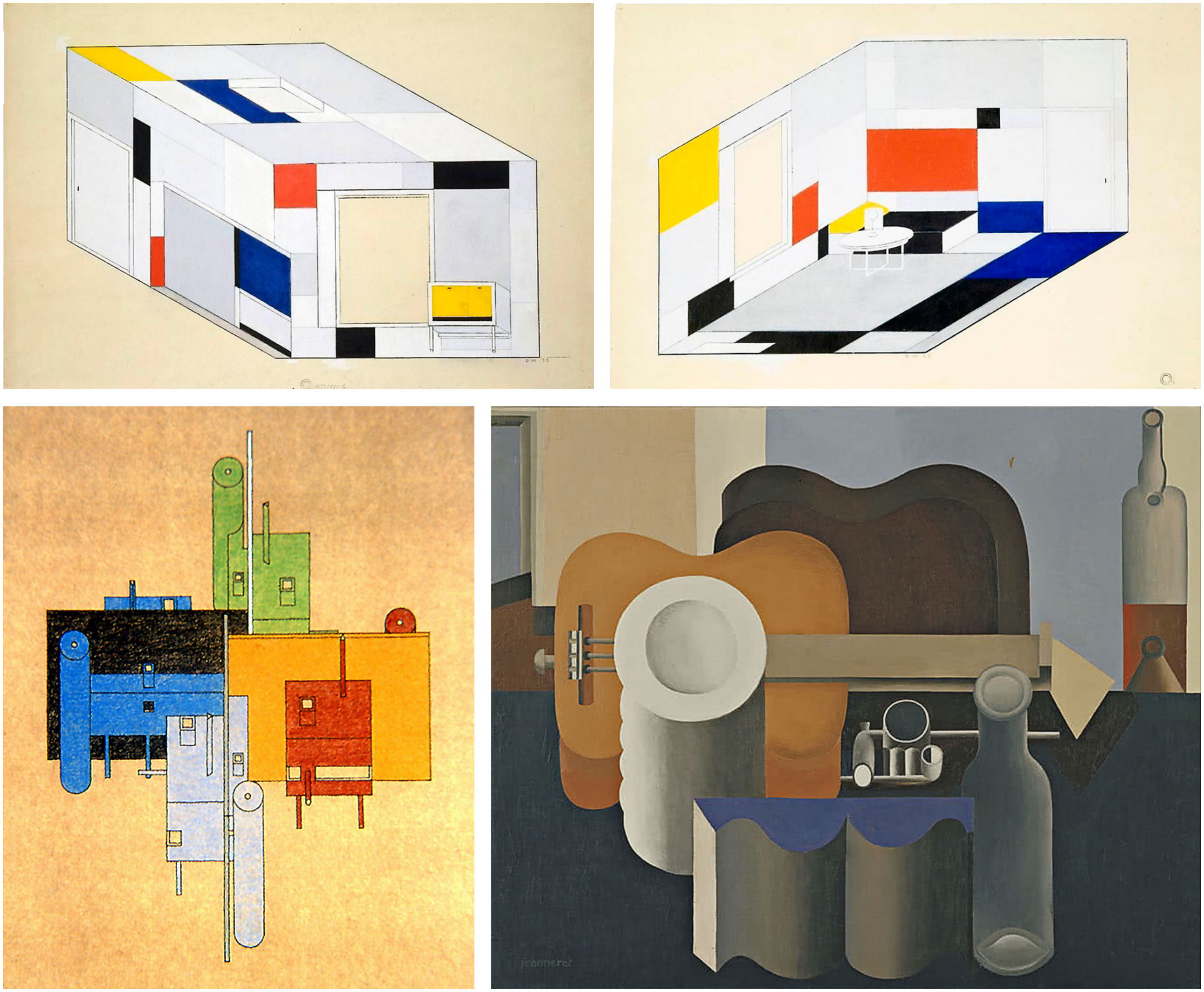
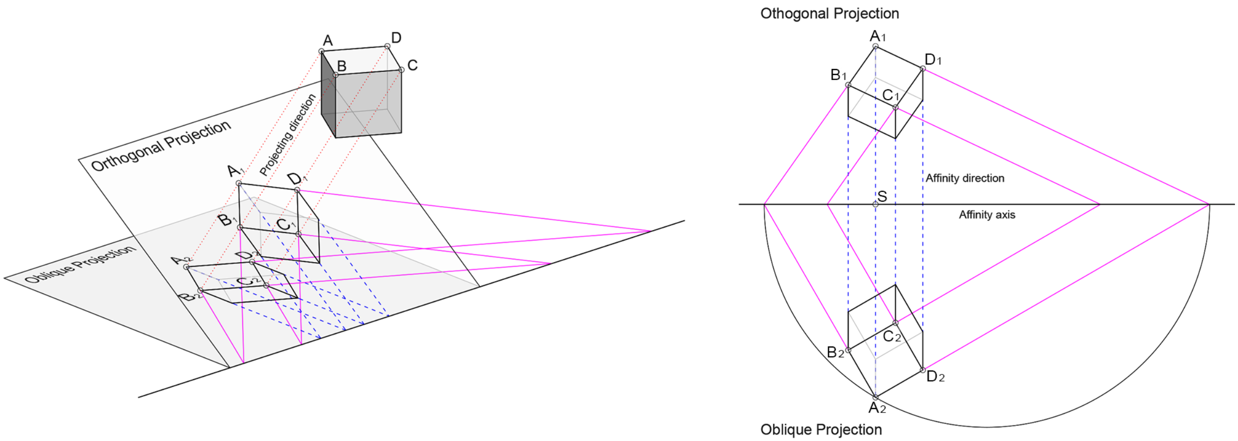
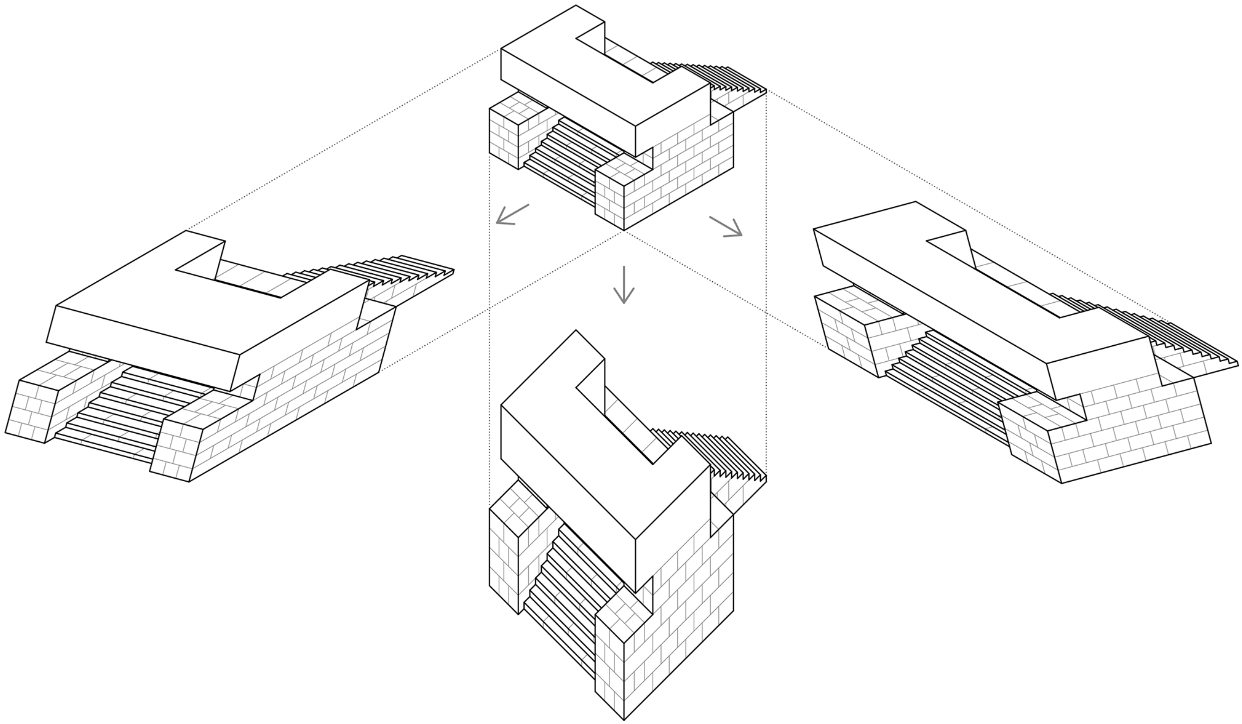


Fig. 2. a, b (top): Piet Mondrian, Color drawings for the salon of Ida Bienert, 1926 (Dresden, Staatliche Kunstsammlungen Dresden); c (bottom left): John Hejduk, Oblique axonometric drawing of the North East South West House, 1977; d (bottom right): Le Corbusier, Still Life, 1920, MOMA (Museum Of Modern Art).





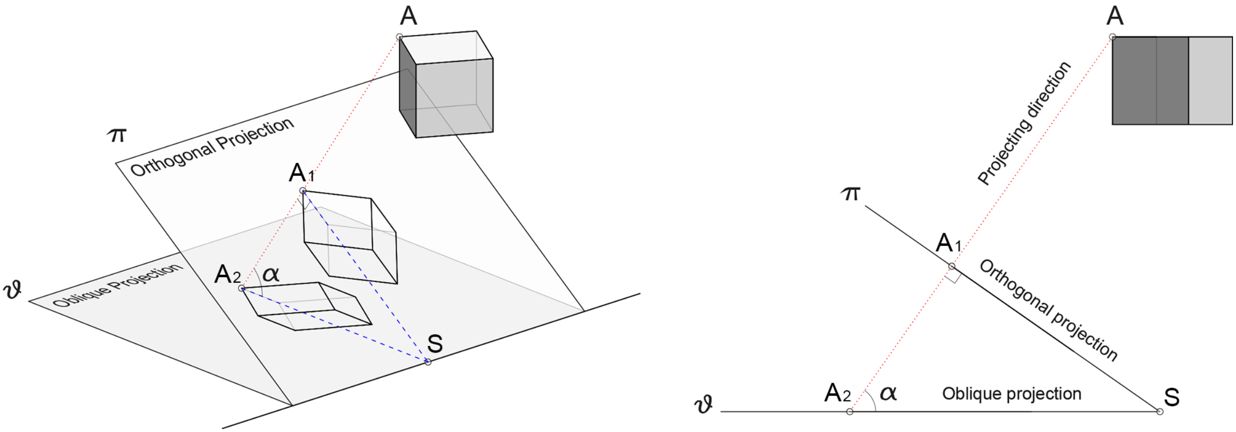
FFig. 3. Affinity relationship between orthogonal and oblique projections (P.M. Cabezos-Bernal).

Fig. 4. Transforming an orthogonal projection into oblique by applying a non-uniform scaling (P.M. Cabezos-Bernal).

For example, in *AutoCAD* this option is possible when working with blocks, since their *X*, *Y*, *Z* scale coefficients can be changed independently.
The scale factor will be equal to the affinity ratio *R*, which can be calculated graphically with the relationship $R = A_2S/A_1S$, seen above in figure 3. The position of point A_2 is determined with the help of the half circle shown in figure 3. The affinity ratio or scale coefficient can also be determined analytically as a function of the angle α formed by the projection direction with respect to the oblique projection plane (fig. 5).
Analyzing the right triangle A_1SA_2 and applying the laws of trigonometry, it can be deduced that the affinity ratio or scale coefficient $R = A_2S/A_1S = (A_1S/\sin \alpha)/A_1S = 1/\sin \alpha = \csc \alpha$ [Cabezos-Bernal, Cisneros-Vivò, 2003b; 2010; 2016].
The choice of projection angle is not a trivial matter, since it would affect the distortion of the represented figure. As can be noticed in figure 6, the oblique axonometries obtained from the orthogonal projection show excessive distortions resulting in inadequate reduction coefficients. As previously discussed, an appropriate reduction coefficient for a military projection would be $C = 0.7$, while for a cavalier projection it would be $C = 0.6$. The projecting direction corresponding to these coefficients would be 55° and 59°, respectively. The scaling coefficient to be applied for these angles would be $R = \csc 55° \approx 1.22$ and $R = \csc 59° \approx 1.1666$. Figure 6 shows the transformation of two orthogonal projections to obtain a military projection (left) and a cavalier projection (right). In the case of the military projection, the orthogonal axonometry has been obtained with a view that forms 55° with respect to the horizontal planes of the model, which are what we will see in true magnitude after the transformation. In the case of the cavalier, the visual forms 59° with the vertical planes of the model, so they will be seen in true magnitude after the conversion. By using these angles, more proportionate oblique projections are obtained after the scaling operation.

Designing oblique analog models of architecture

There is no doubt that anyone who is in the presence of an axonometric model of an architecture feels a sense of discomfort and disorientation not immediately understanding from which point the work should be viewed or why the author decided to make the physical deformation of the object. This sense of visual detachment is similar to the concept of estrangement that Viktor Šklovsky described in addressing his studies of the work of art: "I have already examined estrangement in Tolstoy. A variant of this artifice consists in fixing and emphasizing only one detail of an image, thus changing the usual proportions. Thus, in the illustration of a battle, Tolstoy develops the detail of a moist chewing mouth. This detail placed in the foreground causes a particular shift" [Šklovsky 1974, pp. 100, 101]. As is the case of an anamorphic projection, the axonometric model predicts that it is only possible to understand the underlying visual logic by viewing from a privileged view point that, unlike anamorphosis, simulates a 'parallel' projection and not perspective. We have already recalled some theoretical contributions elaborated by authors such as Ginés Martinez de Aranda, Alonso de Valdevira and especially Juan Caramuel de Lobkowitz between the sixteenth and seventeenth centuries in another essay to which for brevity we refer [Sdegno 2019b], along with others on the subject [Cabezos-Bernal, Cisneros-Vivò 2003a; Sdegno 2003].
The reference to specific case studies such as some early houses by Peter Eisenman and Massimo Scolari's installation at the I Venice Biennale of Architecture is functional to present the outcomes of the experimentation conducted by the authors on the theme of oblique generation of models.
As it is well known, Eisenman has from the beginning favored axonometry in his cognitive inquiry. He did so in his doctoral dissertation –recently published [Eisenman 2009]– in which we find architectures reproduced in parallel projection: from Le Corbusier's villas to Giuseppe Terragni's Casa del Fascio, just to mention the most significant examples. The purpose is to analyze their morphological characteristics, especially in the rigorous relationship between mass and surface.
Later, he continued to use axonometry in his early designs of single-family houses, for example from House I to House IV, reflecting specifically on the type of projection adopted. Precisely in the case of House IV, in fact, the author writes that if "frontality is the preferred point of view of modernism in



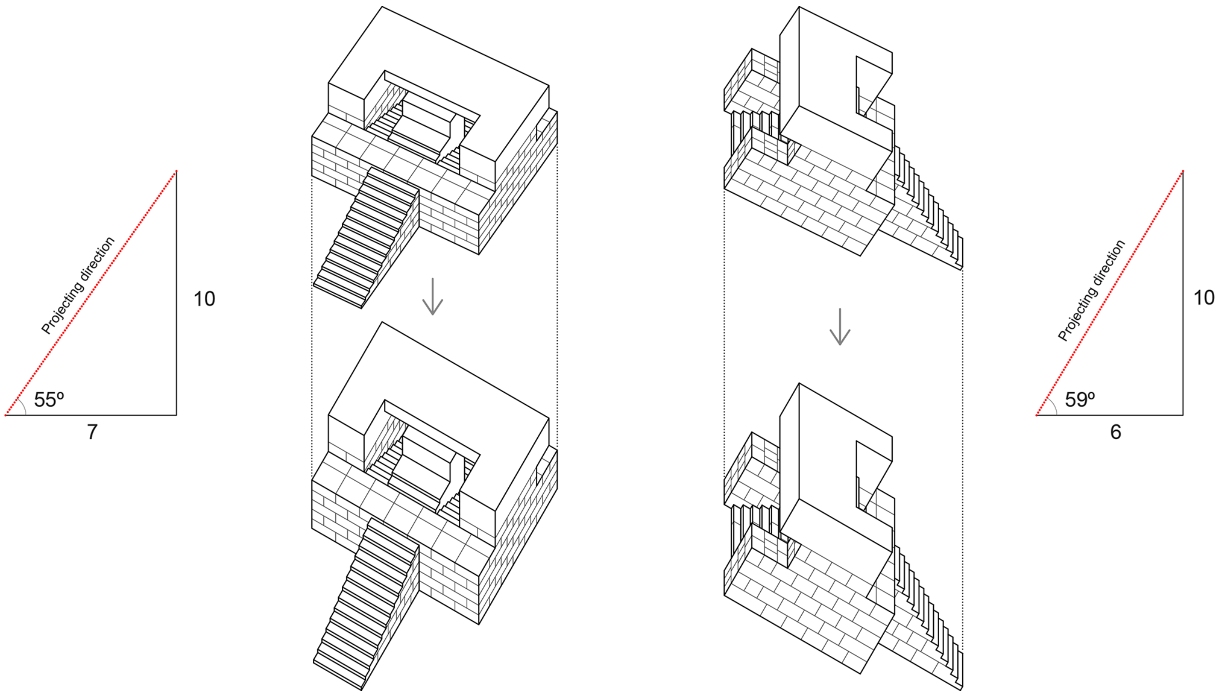


Fig. 5. Projection direction angle and geometric relationships that allow determining the scaling factor in an analytical way (P.M. Cabezos-Bernal).

Fig. 6. Transforming orthogonal projection into obliques. When using appropriate projection directions to obtain the initial orthogonal axonometric views, the resulting oblique projections have the proper reduction factor (P.M. Cabezos-Bernal).

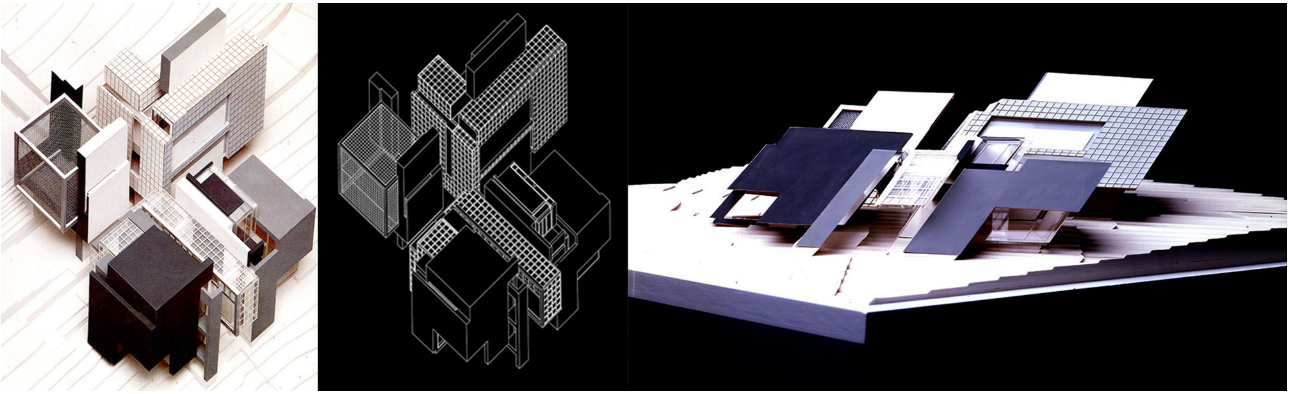


Fig. 7. P. Eisenman, House X, Bloomfield Hills, Michigan, 1975; a (left): top view of oblique model; b (center): military axonometry drawing; c (right): side view of oblique model (from <https://eisenmanarchitects.com/House-X-1975Z>, accessed 10 May 2024).

House IV the oblique view has been equated in importance with the frontal one" [Aureli, Biraghi, Purini 2007, p. 72] [1]. Also from such considerations Eisenman will come in 1975 to propose for House X [Eisenman 1982] an architectural solution that he will comment as follows: "Houses are, in general, conceptually vertebrate: in addition to possessing a necessary structural dimension, that is, they are metaphorically 'vertebrate'. They have a center, usually a hearth or a staircase: the roof pitches from the middle and an overall centrality emerges from their configuration. […] House X is non-vertebrate" [Aureli, Biraghi, Purini 2007, p. 88]. It is not a coincidence that the conceptually 'invertebrate' house will also take on such a configuration visually, when it materializes physically in the form of an oblique model (fig. 7a), deformed so that it can be superimposed on an oblique military axonometry (fig. 7b) when viewed from above, but unequivocally invertebrate when the eye subtracts from that zenithal vantage point and rotates around the object (fig. 7c). If, in such a case, architecture interrupts its constitutive semantic relationship to reduce itself to pure syntax, the axonometric model can be properly semantic –when it declares its affinity with military oblique projection– while it abandons both values –semantic and syntactic altogether– when observed from any point of view, becoming pure abstraction.

A second project by Eisenman, the House El Even Odd, from 1980, presents itself as an oblique model, at once concrete in its material description, but abstract in its theoretical conceptualization: "the House El Even Odd," –as the author observes– "is an axonometric object that explores the criteria for reading representation in architecture and thus addresses the issue of disciplinary limits. […] An axonometric model, in antithesis to an axonometric drawing, is the transformation of the three-dimensional representation of a three-dimensional reality: process and real thing at the same time" [Aureli, Biraghi, Purini 2007, p. 100]. A detailed and articulate description on the theoretical level shows how the figurative stratagem has now become a real working hypothesis, in which representation is not a final outcome of thought but accompanies the design process.

Along with oblique models on a small scale we should mention the work of Massimo Scolari, who has devoted an entire volume to axonometry [Scolari 2005], collecting some theoretical contributions already published since 1984, to which he has added further essays on the subject. His *Elementi per una storia dell'axonometria* [Scolari 1984] has undoubtedly provided rigorous keys for all those wishing to study this particular form of representation, both historically and theoretically. The "anti-perspective", as axonometric projection is called by the author

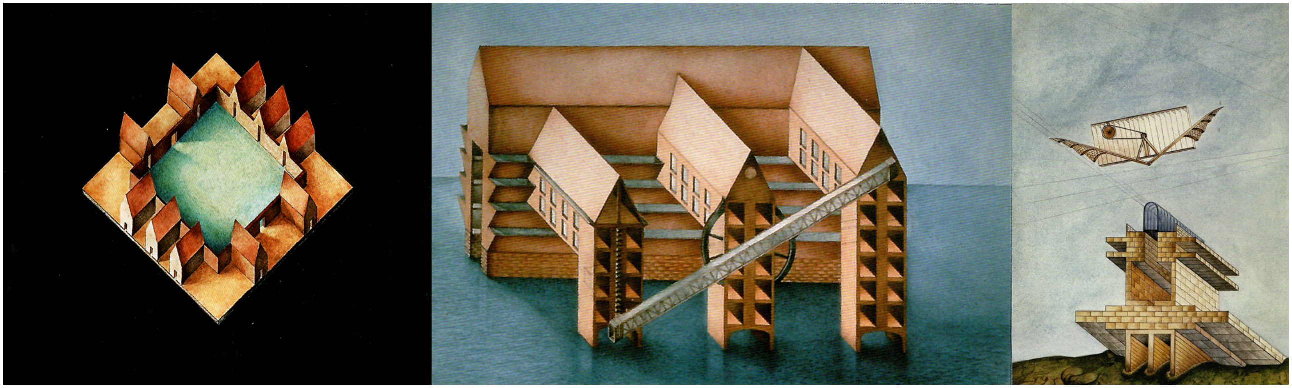


Fig. 8. a (left): M. Scolari, Urban enclosure, watercolor on paper, 1979; b (center): M. Scolari, Architettura lagunare, watercolor on cardboard, 1980; c (right): M. Scolari, Gas Station Inn, watercolor on paper, 1975 (from Marzari 2007, pp. 70, 77, 79).

since the volume's subtitle [Scolari 2005], now requires the same attention that has been devoted to perspective studies so far. Indeed, it is not only a functional expedient for a technical figuration of a scaled object, but can reserve other qualities, both on the level of pictorial representation and architecture. It is not a coincidence that architectures in parallel projection are among the prevailing subjects of many of his pictorial works, such as the watercolor *Recinto urbano* of 1979 (fig. 8a), *Architettura lagunare* of 1980 (fig. 8b) or the work *Gas Station Inn* of 1975 (fig. 8c) whose symmetrical pattern will be recognizable in the oil painting *Porta per città di mare* of 1979 (fig. 9a), which we have already discussed in a previous issue of this journal [Sdegno 2019a], which will see its physical transformation within the *Strada Novissima* created for the 1980 I Venice Biennal of Architecture, curated by Paolo Portoghesi entitled *La presenza del passato* (The presence of the past) [Portoghesi 1980]. The *Porta*, in fact, will materialize at the natural scale in a mock-up (fig. 9b), rigorously traced in technical form and accompanied by the dimensions (fig. 9c) that will ensure its realization along with the other nineteen installations of the *Strada*. As the author himself will describe the project, "the door was constructed as a footprint-calculus of the pictorial image so that the geometry of the two converging parallel projections would be maintained in the actual construction" [Scolari 1987, p. 54]. In further clarification Scolari clarifies the mechanism adopted in figuration: "Usually the pictorial image of an architecture is an oblique parallel projection or perspective. To become an architectural drawing and project this image must be rendered in plan and elevation. An oblique projection 'alla cavaliera' (axonometry) usually shows the receding side in true measure, but with a distortion in the corners. We 'read' by custom and indoctrination those angles as right angles even if they are drawn acute or obtuse" [Scolari 1987, p. 54]. As the author would reveal in a 1991 interview with Léa-Catherine Szacka, "I made a manifesto about the representation of architecture instead of a manifesto about my architecture. I started with a painting and built it in 3D. The idea was to start with representation and make a construction" [Szacka 2016, p. 168]. Having crossed the threshold of the oblique model, in fact, the visitor was confronted with the pictorial work on the opposite wall: "to remove any design ambiguity" –Scolari comments– "I placed, immediately after the entrance, the painting '*Porta per città di mare*' (1979): so that upon entering the representation one could find nothing but a representation" [Scolari 1987, p. 54].

Eisenman's small-scale military oblique model and Scolari's full-scale "cavalier" oblique model show two distinct lines of research, although in –if you will pardon

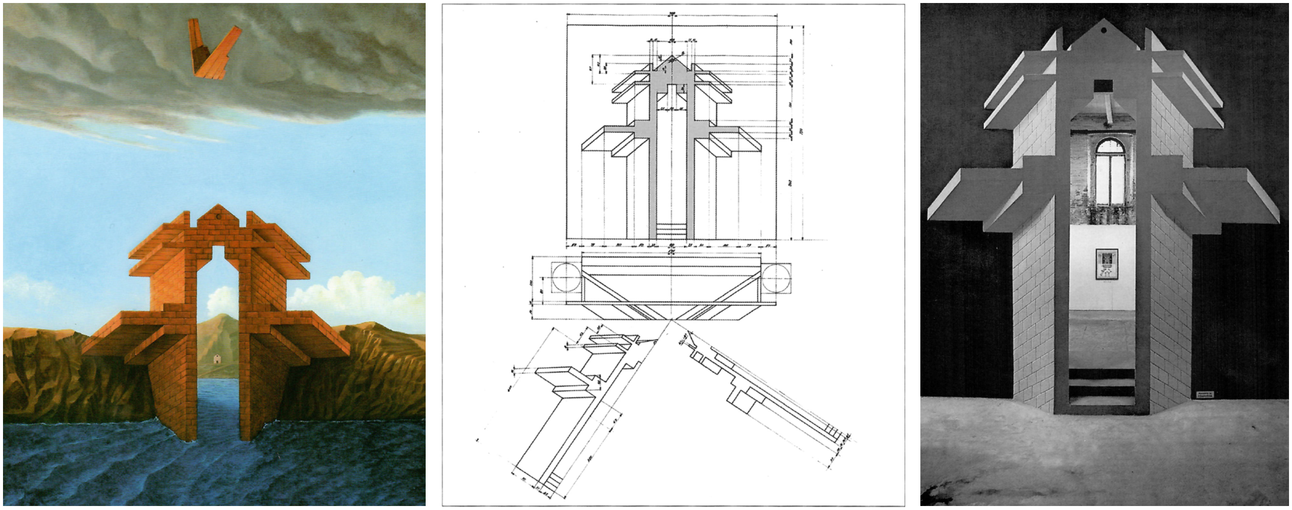


Fig. 9. a (left): M. Scolari, Porta per città di mare, oil on paper pasted on board (1979-1980); b (center): M. Scolari, Executive project of the installation Porta per città di mare for the I Venice Biennale of Architecture (1980); c (right): M. Scolari, Installation Porta per città di mare at the I Venice Biennale of Architecture, 1980 (from Marzari 2007, pp. 86, 87, 89).

the term– parallel progression: areas of research of definite interest although the peculiarity of such an experimental approach does not seem to have been grasped in those concerned with the discipline of figuration.

Downstream of these investigations on the subject, some experiments were initiated that could combine the study of oblique axonometry, the potential of computational modeling and physical prototyping of digital models. The activity-conducted by one of the authors of this paper [2] at the European Ceramic Workcentre (EKWC), has seen further development with application to other case studies. The objective of the research was to investigate a method of representation that could complement the specific contents of physical representation by maquette with those related to the disciplinary nature of parallel axis drawing.

Therefore, it was decided to focus on some of Andrea Palladio's works with the specific investigation of some significant details, such as architectural orders.

The first case study is Villa Emo [Palladio 1570, *Book II*, p. 55], whose linear configuration with the main body of the building in a central position and the two lateral *barchesse*, allows us to propose an oblique model in parallel projection such that the elevation can be visualized in true form and the axonometric restitution of the plan and side elevation. By preserving the measurement of the *x* and *z* axes, and tilting the y axis of the digital model by 45° in the negative direction, it was possible to highlight the previously prepared plan layout through a reduced extrusion of the wall structure to transform it into bas-relief. In this way, plan and elevations immediately declare the contents of the villa, although, in this case, the mirror symmetry of the entire layout is negated by the necessity required by the oblique projection. The deformed model was then subjected to rapid prototyping procedure with the process of Selective Laser Sintering (SLS) in nylon powder at the scale of 1:200, and then translated into silicone mold and reproduced in plaster (fig. 10). Making the mold in silicone negative has the potential of being able to replicate the object while avoiding further production with SLS systems, which are far more expensive than manual reproduction.

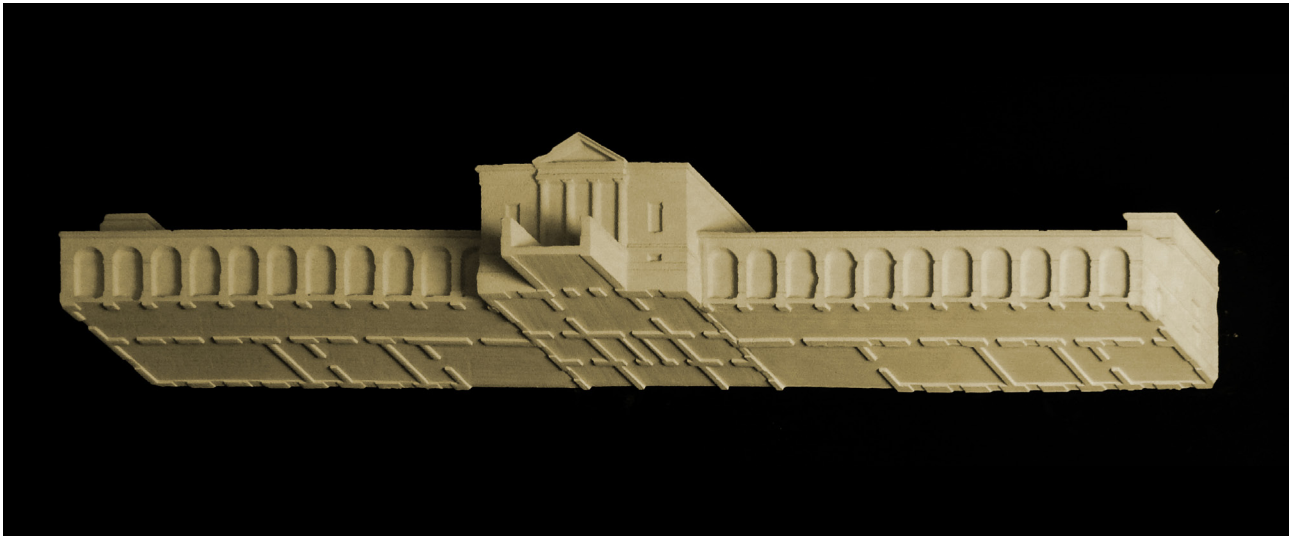


Fig. 10. Oblique analog model of Andrea Palladio's Villa Emo, plaster (A. Sdegno with B. Gernand, Protoservice realization, 2007).

A second case study dealt with one of the most iconic pieces of architecture: the Villa Capra known as "*La Rotonda*", designed by Andrea Palladio and made in a different way –as far as the roofing is concerned– than the drawings in his treatise [Palladio 1570, *Book II*, p. 19]. The double figurative register, related to the roofing, urged research in the field of representation that, while taking into account the initial objectives dictated by the theme of oblique deformation by computational means, would allow some significant aspects of the work to become evident. We therefore proceeded in synchronous work on two distinct models: that published by Palladio in *I Quattro Libri dell'Architettura* and that published by Ottavio Bertotti Scamozzi in his treatise [Bertotti Scamozzi 1778, pp. 8-13]. The main elevation was then analyzed, dividing it into two quarters of the entire work, which, as is known, has a configuration with double mirror symmetry in the pronaos, although the internal distribution does not present the same logic. Wanting then to make the section evident, we went to a computational deformation in the opposite direction of the digital models related to the two solutions, such that –placed in appropriate positions– they could preserve the symmetrical layout of the villa, although altered in the two different morphological configurations. Keeping the *x* and *z* directions in true form, a negative value was given to y in the slope coefficient equal to half a right angle (-45°) for the original solution and an equivalent positive one (+45°) for the model derived from Bertotti Scamozzi's treatise, so that the on-axis section would also be visible, albeit in its non-straight configuration. These two obliquely deformed models, too, following the rules of cavalier axonometry, were then reproduced at a scale of 1:200 with a selective sintering rapid prototyping system (SLS), resulting in an opaque nylon maquette with a quality of 1/10th of a millimeter (fig. 11).
A final experimentation involved the realization of a Doric order –again modeled from information in Palladio's treatise [Palladio 1570, *Book I*, p. 27]– of which several solutions were made. On the one hand, an oblique model that repeated the computational work done previously, although this time applied to a single architectural detail. On the other, the multiple restitution of the same subject, to which a recursive

Fig. 11. Sections of the oblique analog model of Andrea Palladio's Villa Capra, plaster (A. Sdegno, Protoservice realization, 2008).

Fig. 12. Composition of one straight and eight oblique capitals, lactic acid polymer (A. Sdegno, 2019).

deformation was applied starting from the centrally located straight model. While in both situations the decision was made to opt for the deformation of a semi-model capital, in the first case a ceramic solution was landed with a thousand-degree firing –again starting from a physical SLS prototyping then treated with a negative mold on silicone rubber– while in the second a reduced size was used, using a lactic acid polymer filament for FDM additive printer, which could allow rapid identification of objects in the determined tilt, at an angle of 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315° (fig. 12).

The rigorous method of geometric construction of the deformation –both in the works of Eisenman and Scolari described above, and in those we have proposed here for the Palladian villas and the Doric order– does not preclude that to the sense of figurative precision of the oblique objects, observed from the privileged point of view –albeit not axonometric– is counterbalanced by a sense of estrangement toward the same when examined from different views, such as to raise in the observer a perceptual sensation similar to the estrangement described by Šklovsky that we have mentioned before, although declined now in the field of figuration of architecture.

Credits

Although sharing the general layout of the essay, the paragraphs *Introduction* and *Designing oblique analog models of architecture* are by Alberto Sdegno and the paragraph *Axonometric deformations of digital models* is by Pedro Manuel Cabezos-Bernal.

Notes

[1] All descriptive texts of works in the volume Aureli, Biraghi, Purini 2007 are by Peter Eisenman and are taken from <https://eisenmanarchitects.com/Projects> (accessed 10 May 2024).

[2] The activity was carried out by Alberto Sdegno in early 2007 at the European Ceramic Workcentre (EKWC) in 'S-Hertogenbosch in the Netherlands, in collaboration with London artist Bruce Gernand.

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