# Landscape Features in Italian-style Gardens: Machine Learning and Computer Vision for Symmetry Detection

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*Abstract*

*Between 1304 and 1309, Pietro de'Crescenzi drew up the Opus ruralium commodorum, perhaps the first agronomic treatise of the medieval period, in which it is possible to find a first taxonomic classification of gardens according to the social importance of their owner. The formal rules of the Italian garden remained unchanged until the 18th century, but unfortunately, little 'original' evidence of these jewels remains, because they were destroyed, abandoned or modified over the centuries; very often the proof of their existence can be found in artistic representations or in the treatises of the time. In this research, the landscape features of the Italian garden are investigated, and some automated Machine Learning algorithms are experimented to find symmetries between the plant and decorative elements that characterise it. Although automated symmetry detection has already demonstrated applicability in several disciplines, it is recently showing a new and not yet fully explored potential in art through the development of computer vision; however, some procedural and algorithmic aspects present numerous challenges and problems. For this reason, starting from an examination of the state of the art of current Imaging Detection solutions, we evaluate their applicability in the search for symmetries within artistic representations of Italian gardens.*

*Keywords: Renaissance gardens, symmetry, machine learning, computer vision, Python.*

## Introduction: the characters of the Italian-style garden

The Italian Renaissance garden or even more simply the Italian garden, can be considered as the climax phase of the conceptual evolution and also of the agronomic technique of garden art. The emblematic image of the Italian garden is composed of formal geometries and symmetries created in the search for balance between paths and lowerbeds traced on the ground by realizing hedge borders, artistic trees, espaliers, and vine arbors, often accompanied by fishponds or scenic mechanisms such as games, fountains, or water clocks. Although the formal codification of the Italian garden is chronologically identified with Renaissance Italy, many of its precursor features can be found in the courtly period of 12th-century France, and almost simultaneously in the rediscovery of the Italian country villa. A number of alternative, secular and courtly visions of the garden, or verger in the French language, can be traced in medieval France, which are significantly detached from the culture of monastic gardens [Tosco 2018] deeply oriented more to productive-botanical purposes, developed on large landed estates and mostly resembling from farms [Germano 2022]. In literature, too, it is possible to trace an obvious reference to images and spaces, among which is the garden invested with new functions and values. The French term verger derives from the Latin word *viridarium*, meaning a place of love, pleasure, adventure, rich in symbols even of a magical character, in which amazing events can take place, enriched with fragrant flowers, gushes, trees rich in fruit [Kibler 1992]. For example, in the 12th-century tale Conte

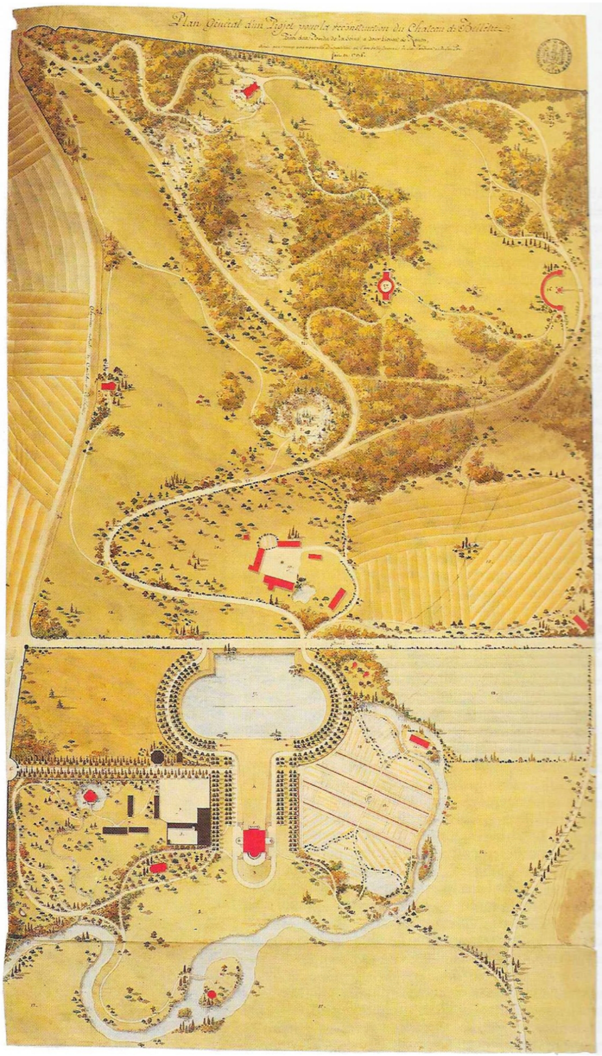



*Fig. 1. Master plan for the park of the Château de Belletre in Normandy [Mosser, Teyssot 1999, p. 12].*

de Floire et Blanchefleur, attributed to Robert d'Orbigny, a significant account of this period and of the garden is depicted, in which the vicissitudes of two lovers are narrated, occurring within the setting of three vergers: the first related to the first and youthful falling in love, the second the site of the deception and alleged death of the beloved, and the third with explicit references to the garden of the Emir of Babylon, exotic and distant, where the happy ending of the story finally takes place [Tosco 2018]. The functional and spatial organization of the late medieval aristocratic garden is enriched by the presence of the park, understood as an enclosed, forested area (from foris, a term originated precisely in this period to indicate areas outside the inhabited area), of considerable size, reserved for hunting purposes as a social and recreational activity for the wealthy aristocracy, adjacent to the castle or villa (fig. 1).
At the turn of the thirteenth and fourteenth centuries the spread of the neologism 'park' spread with pandemic character throughout Europe, and documentary sources indicate their creation mainly in France and England but also in northern Italy, for example with the 'barcho' of the castle of Pavia created by the Visconti after the conquest of the city in 1359 [Azzi Visentini 2004]. In parallel with the increasingly complex and elegant development of the courtly garden, which continued to have in the French aristocratic milieu its gravitational cultural centre, in Italian cultural circles a fascination for the country 'life', a metaphor of rest, refuge and virtue, flourished again [Sberlati 2004], not only in aristocratic circles, but also among intellectuals and more cultured landowners. In view of the mild Mediterranean climate, the beauty of the places, it cannot be surprising that Italy established itself early on as the epicentre of this phenomenon, which is reflected in the most significant agronomic treatise of the time, the *Opus ruralium commodorum* (fig. 2), consisting of twelve volumes compiled by Pietro de' Crescenzi between 1304 and 1309 [Savastano et al. 1922]. The treatise, which takes the form of a kind of manual of agronomy, in addition to discussing fishing, oenology, animal diseases, techniques of crop rotation and alternation, and generally the management of the farm, examines in Book VIII the study of the garden as a space of recreation and well-being for the owner [Sansovino 1522]. In more detail, a kind of taxonomy of the garden is illustrated according to the economic and social level of the owner: the simplest, belonging to the lower classes of small size, usually square in shape, with a herbarium and an orchard; a more complex garden typical of middle class landowners,

characterized by some architectural ornamental structures, still quite simple such as pergolas and pavilions; finally, a more complex level belonging to the higher classes usually surrounded by a wall, containing diversified spaces and functions, such as orchards, nurseries and fishponds, water features, wooded areas populated with animals, and even a palatium made of wooden materials and plant weavings. The historical geographic region of Etruria or, rather, the whole of Tuscany, long regarded as the most delightful area in the whole of Italy, becomes the setting for several of Boccaccio's works, including the Decameron in which the company of boys gather inside a 'magical verziere', to dance, sing, tell tales and live happily [Kern 1951; Usher 1989]. The Tuscan landscape exalted in Florentine humanism found in the Medici family the instrument for its consecration, beginning with Cosimo the Elder with the arrangement of the Mugello estates, particularly in the villas of Careggi, Cafaggiolo and Trebbio, probably under the direction of Michelozzo. They probably all had horti within which the patron hosted parties and delighted in a serene setting; however, establishing the exact ornamental arrangement of the garden in Cosimo's time remains a problem due to the scarcity of documentary sources [Tosco 2018]. Probably the gardens that have most consistently preserved the original characters are those of the Tenuta del Trebbio

*Fig. 2. Summary and Chapter One of the Opus ruralium commodorum of Crescentino translated for Sansovino [1522].*

SOMMARIO
DI TVTTA L'OPERA.

NEL Primo libro ſi tratta del ſito della uilla, delle fabriche, de contadini, & delle coſe che ui biſognano per habitare. carte. 1
Nel ſecondo, della natura delle piante, & della cultura che richiede ogni qualità di terra o di campo. 23
Nel terzo, della cultura de campi campeſtri, della natura & della vtilità de frutti che ſe ne cauano. 52
Nel quarto, delle viti, & delle vigne, & della lor cultura, & della natura & vtilità loro. 103
Nel quinto, de gli arbori, della natura loro, & della vtilità che ſi caua de gli arbori. 153
Nel ſeſto, de gli horti, & della natura, tanto delle herbe, quãto dell'altre coſe che naſcono per ſe medeſime. 215
Nel ſettimo, de prati, & come ſi fanno, & de boſchi, & delle ſelue coſi naturali come artificiali. 278
Nell'ottauo, de giardini, & delle coſe piaceuoli che vi ſi fanno con artificio, quãto a gli arbori & alle herbe. 284
Nel nono, di tutti gli animali che ſi nutriſcono alla villa così per vtile, come per piacere. 293
Nel decimo, de gli ingegni diuerſi da pigliar le fiere ſaluatiche & gli vccelli d'ogni generatione. 376
Nell'undecimo, le regole delle coſe che ſi debbon far in contado, ridotte in compendio perche ſi tengano a mente. 401
Nel duodecimo, di tutte quelle coſe che ſi fanno di meſe in meſe per ordine, da Gennaio fino a Dicembre. 427

1

DI PIETRO
CRESCENTIO
BOLOGNESE
DELLE COSE APPARTENENTI
A BISOGNI ET A COMMODI
DELLA VILLA
TRADOTTO PER MESSER
FRANCESCO SANSOVINO

LIBRO PRIMO.

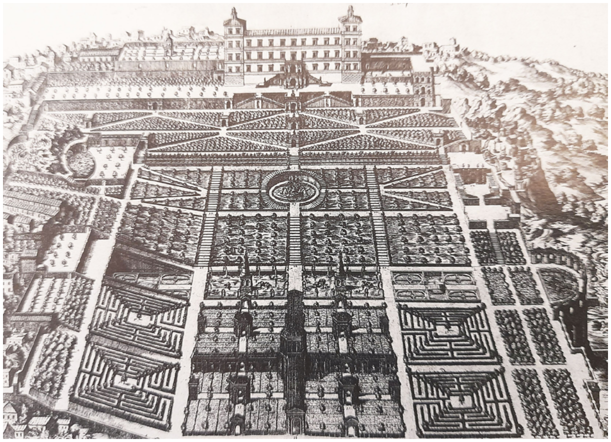
PERCIOCHE IL Cultiuamento della uilla per i ſuoi continui affari, richiede fatiche & ſpetialmente fortezza de gli habitanti, però mi è paruto conueneuole in queſto primo libro inſegnar la dottrina di quelle coſe che s'appartengono alla cognition della ſanità de luoghi habitabili, & di alcun'altre coſe neceſ-

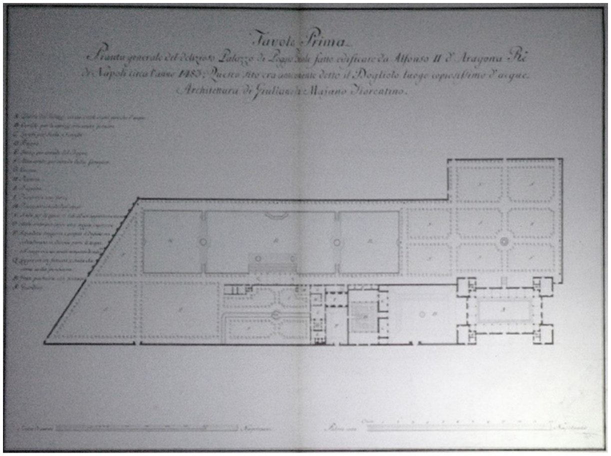
Piet. Cre. A

[Frati 2015], separated from the residence and bordered along the long sides by two parallel pergolas on which rows of vines develop and in the centre a checkerboard of square-shaped flowerbeds. The oldest depiction of this layout dates to the late 16th century and is contained in the lunettes of Flemish painter Giusto Utens [Agostini 2011]. The well-established characters of the garden-model, with the presence of the plant, hydraulic, architectural and landscape elements considered obligatory according to the canons of the period are found in Alberti, who dictates certain rules for their arrangement: the presence of box hedges, perticates that will allow people to enjoy both sun and shade, the arrangement of laurels, cedars, junipers arranged according to geometric figures at equal distances and with mutually corresponding angles (fig. 4) or as it was customary to say at the time *quinunx* [Tosco 2018]. Through the work of Giovanni, Cosimo's second son, the first intervention of a bucolic villa was realized, breaking away from the previous castellan characters. Its location was determined exclusively by landscape needs; the villa and the attached garden were, in fact, built on the side of the hill overlooking the city of Florence, and major terracing works were carried out for this purpose. The Fiesole garden is presented according to Cecil Pinsent's 1915 arrangement, and as with most cases few elements have survived to the present day that can suggest what the exact ornamental architectural arrangement was in the fifteenth century. Perhaps the most innovative character can be traced in the creation of two terraces or, rather, hanging gardens, immediately adjacent to the building, on the hillside designed in such a way as to 'open up' to the valley below [Frommel 2006].The contemplation of the panorama is a theme that becomes more and more intimately linked to the garden, and the arrangement on terraces in hilly areas or otherwise characterized by pronounced morphology turns out to be an increasingly recurrent feature of the period. The explosiveness of the magnificence of the art of the Italian garden grows rapidly, but unfortunately some of these jewels of the Italian landscape are permanently lost, such as the residence of Poggioreale (fig. 4), the realization of which is due to Alfonso d'Aragona, Duke of Calabria and legitimate successor to the throne of Naples. The design by Giuliano da Maiano with the participation of Fra' Giocondo and Francesco di Giorgio included an arrangement of gardens that developed from the sides of the villa on terraces. In front of the villa there was a cruciform garden in axis with the entrance that was populated by orange trees and other fruit plants

*Fig. 3. Etching/pulley of the Gardens of Tivoli by Etienne Du Pérac in 1573 [Mosser, Teyssot 1999, p. 49].*

*Fig. 4. Planimetry of the Villa di Poggioreale attributed to Carlo Vanvitielli, Collection of Lord Bute, Victoria & Albert Museum, London.*






with a fountain in the center. Moving towards the valley, one encountered an enclosure wall in which wind windows opened, allowing contemplation of the view. On the side ran a large fishpond divided into rectangular pools and traversed by walkways with platforms and water features, all around wide green spaces were designated for ball games. The expertly manicured green layout was enriched by inflorescences with artistic pruning, statues and precious marbles [Frommel 1994]. Thus, the garden of Poggioreale was no longer configured as an appendage of the villa, but rather constituted its environment of equal dignity coordinated and consistent with the architectural structure where each element, its arrangement in space was taken care of to ensure the well-being of the Aragonese court [Tosco 2018].Thus, it can be concluded that in sixteenth-century Italy, art in garden composition reached such a high level that French supremacy in this field, recognized until the late Middle Ages, is countered by the emergence even internationally of what is still codified as the Italian garden.

## Symmetries and architecture

The relationship between mathematics and architecture has probably always existed, at least for as long as architecture has existed [Salvadori 2015]; from the simplest applications to define lengths, surfaces or volumes to the most complex ones in structural calculations, the mathematical tool has always provided help to improve the latu sensu quality of the man-made landscape [Mehaffy 2020], with an intense production of theories, proportions, scales, models [Padovan 2002]. Although the possibility of 'enhancing' aesthetics is sometimes regarded by many architects and designers as a mere matter of purely subjective taste, related to one's individual aesthetic goals [Taylor 1994], several researches have already demonstrated the relationship between environmental quality and psycho-physical benefits for people [Cold 1998; Van den Berg 2003]; however, what is still not well understood concerns the form these environments must take to produce well-being [Mehaffy 2020]. For example, Dosen et al. [2013], evaluating the hypothesis that certain geometric forms of architecture may influence well-being but also the individual's preference system seek to develop an appropriate mathematical conceptual apparatus to evaluate human perceptual responses to space. Again Hagerhall et al. [2004], investigate the fractal geometry of natural environments discovering that there is a relationship between preference and fractal geometry and its size, suggesting that the latter may provide an explanation for the preference-nature connection. Interestingly, it can also be noted that the historical theme of symmetries, which in architecture has always been a defining element in any period and geographical context: from prehistory [Hodgson 2011], to the complex structures of ancient Egypt [Rossi 2004], to the architecture of ancient Rome's theaters and Vitruvius' De Architectura [Amadei 2015], via Asian, Indian architecture, and pre-Columbian art [Salvadori 2015], has not received adequate attention regarding its implications on the sphere of psychological well-being and individual preferences [Mehaffy 2020]. In landscape and garden architecture, the principles of order, symmetry, and balance find in axis control (fig. 5) a widely used technique [Eckbo 1964], which has evolved over time to become a kind of "toolbox" and which has allowed the Italian garden to evolve in its maturation process from a simple structure to a complex one, from an

*Fig. 5. Master Plan of Villa Lante (left) and Villa d'Este (right) reproduced by Nieuwlandt, W. and the New York Botanical Garden with garden axes highlighted [Hu 2004, p. 82].*

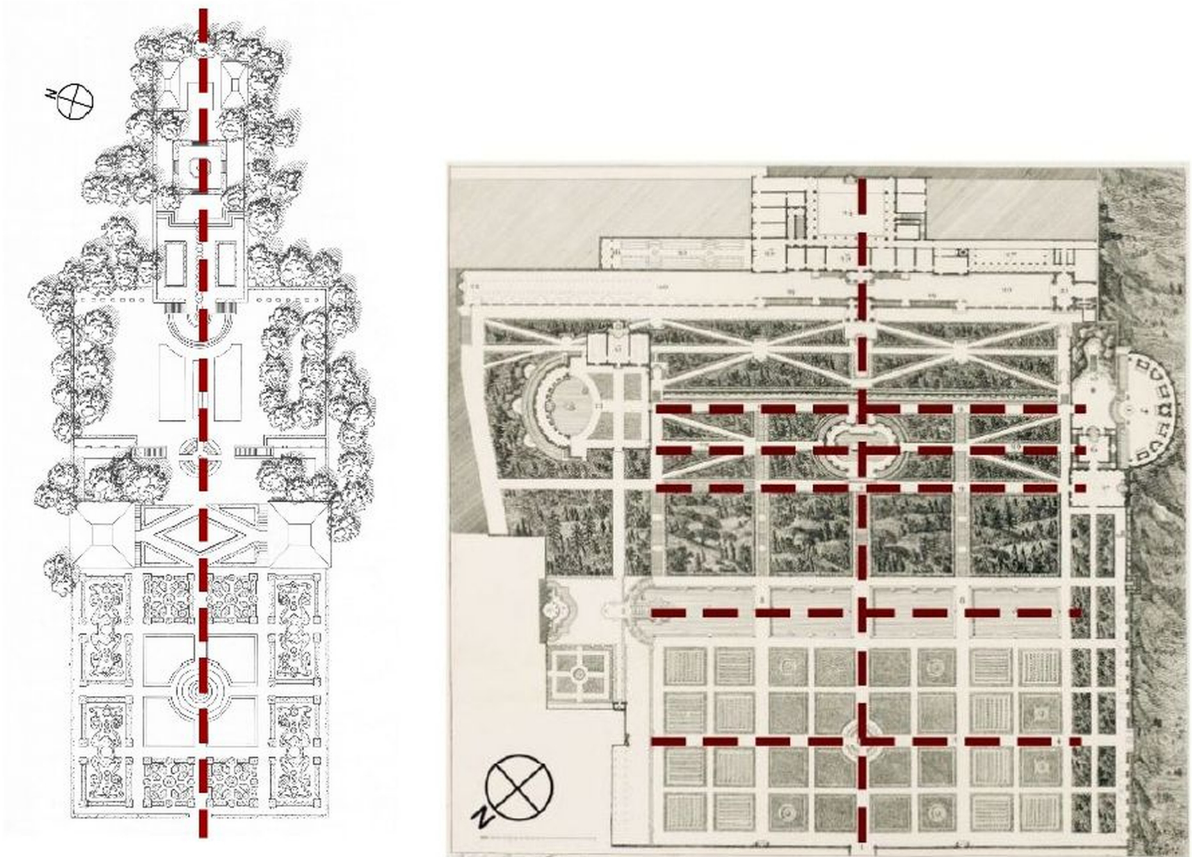
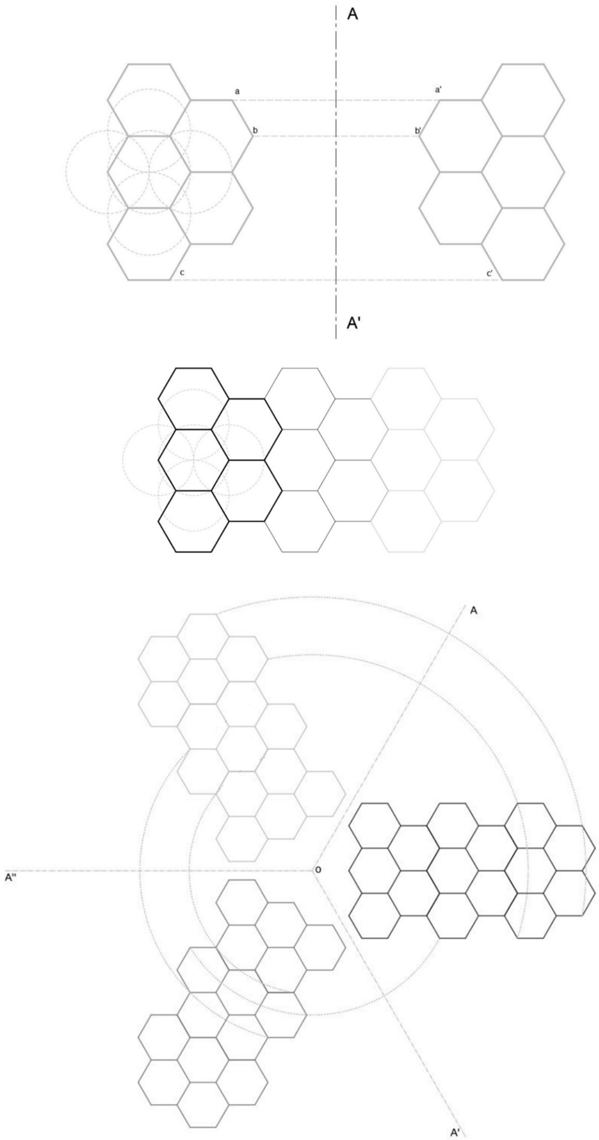


Fig. 6. Schematic examples of reflexive symmetry (top), translative (middle), rotational (bottom) (author's elaboration).




isolated element to a system integrated with the land and its surroundings [Hu 2024]. In mathematics, the term symmetry is used to refer to the invariance of an object with respect to one of its geometric transformations such as reflection, rotation, translation or scale variance [Weyl 2015]. Formally, two objects *A* and *B* that belong to the same space *S* can be said to be symmetric if there exists a transformation *T* such that *B=T(A)* [Mitra, Pauly 2008]. Reflected or 'mirror' symmetry turns out to be the most intuitive type in which any geometric configuration is reflected with respect to an axis. In this case it is used to say that *A* is the mirror image of *B*. In rotation symmetry, the geometric configuration is said to be such if it rotates about a point; there are numerous examples of this class, such as the iris of the eye, or a balloon in which its final shape is the result of the symmetry of the pressures of the gas it contains [Mehaffy 2020]. A translation exists if there is a correspondence between shapes that are not otherwise symmetrical with respect to an axis, such as may be the repetition of a pattern or frieze (fig. 6). Symmetries by variance of scale are similar to translational but occur when the transformation *T* occurs with respect to the size of the object, a very common example being fractal shapes. One should add an additional type, which actually cannot be considered a class in the strict sense as in the previous cases, represented by 'breaks' in symmetries, which occur when there is a perturbation of the symmetric rule. The latter is currently attracting a great deal of research attention to explain the formation of certain biological structures [Isaeva 2012], just as it has already been shown to have several applications in physics and cosmology [Weinberg 1979; Albrecht, Steinhardt 1982].

## *Machine learning* per la *simmetry detection*

The environment in which we live or, rather, the events that unfold in it and that we perceive are often the result of multiple combinations between classes of symmetry [Park et al. 2008]. Usually, the human being's brain is extremely adept at detecting them almost instinctively [Conners 1989; Tyler 2003], conversely, the application of the computer for this purpose still presents some adaptability challenges especially considering that geometric objects or at any rate the elements of this universe can be considered to be practically infinite continuous variables, whereas computers process finite arithmetic elements. In addition, one must consider that geometries with respect to which one wants to determine the existence of one or more symmetrical rules between their parts may be stored in different formats such as raster images, vector images, scanner acquisition result, which inevitably require different procedures and algorithms [Mehaffy 2020].The environment in which we live or, rather, the events that unfold in it and that we perceive are often the result of multiple combinations between classes of symmetry [Park et al. 2008]. Usually, the human being's brain is extremely adept at detecting them almost instinctively [Conners 1989; Tyler 2003], conversely, the application of the computer for this purpose still presents some adaptability challenges especially considering that geometric objects or at any rate the elements of this universe can be considered to be practically infinite continuous variables, whereas computers process finite arithmetic elements. In addition, one must consider that geometries with respect to which one wants to determine the existence of one or more symmetrical rules between their parts may be stored in different formats such as raster images, vector images, scanner acquisition result, which inevitably require different procedures and algorithms [Mehaffy 2020]. The development of symmetry detection algorithms has a long history in computer vision, even it is possible to date the first attempt to detect bilateral reflection symmetry before computer vision itself [Park et al. 2008]. Although, as seen, the classes of symmetry are diverse, the detection of bilateral reflection symmetry or more simply mirror symmetry and its oblique version has dominated the attention of researchers for several decades [Davis 1977; Kanade 1981; Gauch, Pizer 1993; Lei, Wong 1999], while it is only in recent years that there has been an intensification of research into the detection of other classes of symmetry [Hays et al. 2006; Podolak 2006; Prasad, Davis 2005]. Obviously, each research tries to demonstrate the potential of a specific strategy with some experimental results [Park et al. 2008], however, systematic comparison among all of them, perhaps through a standard set of objects or images, remains at present still limited [Žalik et al. 2022]. Some examples, however, can be found in Xiao and Wu [2007]; who develop an overview of symmetry detection algorithms for raster images; or, in Mitra et al. [2013]; they show the results of a comparison between algorithms for evaluating reflected symmetry in 3D objects; finally, Bartolucci et al. [2018]; they compared different methods applicable to biomedical spatial data. The algorithms just described, are certainly effective but still present two types of problems: the first is the specificity of their application, as indicated some are designed to evaluate a particular class of symmetry, the second concerns the inherent computer complexity of the tool or the need to resort to a specific software for its execution, often proprietary, and not opensource with inevitable limitations with respect to a possible customization of the tool. In addition, referring to the object of this research, it should be considered that very often the documentary source of the garden is very often a pictorial representation or woodcut that has inevitably undergone degradation over time, or the morphology of the land may have been altered over the centuries, and therefore one cannot proceed by applying current technologies for high-precision surveying. In view of these issues and especially the special nature of the research object, it was decided to adopt a method with greater adaptability, which is free and opensource and also more intuitive than many of the previously stated solutions. Summarizing it in a few lines before going into the details of the code: having acquired a raster source, and denoting it as set $Z$, one identifies or selects within it a portion that one can denote as object A, obviously such that $A \subseteq Z$, and of which you want to search for a transform of it $T(A)$. The portion may, for example, be a hedge maze, a peach orchard, an arbor, or more; of this one calculates the transform by rotating or reflecting it and obtaining $B=T(A)$. Finally, you query the computer by asking it to look for the transform $B=T(A)$ within the set $Z$. If successful, then:

$$\exists\, B \subseteq Z \leftrightarrow B=T(A)$$

that is, if the portion $B$ exists and is contained within the set Z, then $B$ is the transform of $A$ within $Z$; therefore, $A$ and $B$ are mutually symmetrical portions of $Z$; otherwise:

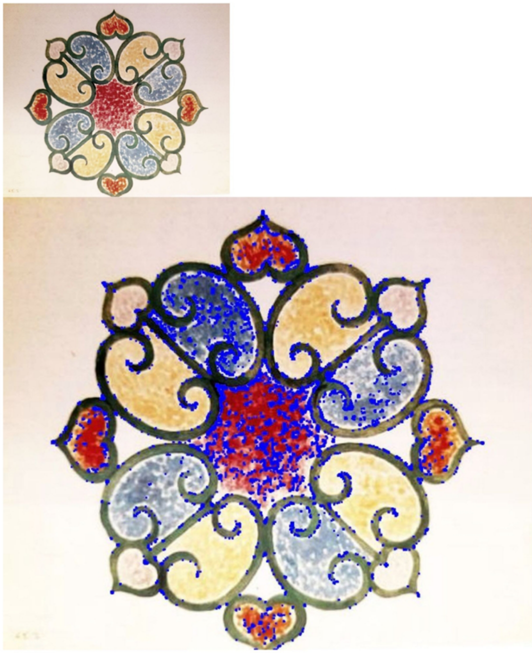
$$\nexists\, B \subseteq Z \leftrightarrow B=T(A) \nsubseteq Z$$

that is, the impossibility of determining the existence of the $B$ transform within $Z$ will imply that A and B are not mutually symmetric portions of $Z$. The problem inevitably arises of how to teach the computer to find the object $B=T(A)$ inside $Z$. The solution proposed in this research is to teach the computer to play a kind of jigsaw puzzle, teaching it the basic cognitive processes used by people to identify one piece of an image among a hundred, a thousand, and more elements and figure out its correct position within the overall image. To illustrate the solution, take as a purely explanatory example the image that follows (fig. 7) and from which four tiles were obtained, stacked on the right. Trying to play around, we can assume that the totally gray piece, the first one starting from the bottom definitely belongs to the sky, however being able to locate its exact position in the image is impossible, any part of the sky would do, the second dowel from the bottom we can attribute to the top of the building, neither lower nor higher, thus there is a vertical constraint but no horizontal one, in fact any translation along this direction would be considered valid (limited to the building), instead for the third and fourth dowels it is possible to identify precisely their origin position in the image. In this case, therefore, it can be guessed that the discriminator for determining the position of the tessellation within the image is the presence of a strategic information graphic element: the corners. So to answer the initial question, one must teach the computer to recognize the angles within an image and in the portions, between which one wants to ascertain the existence of one or more symmetrical rules. In 1988, Chris Harris and Mike Stephens [1988], developed an algorithm that from a gray-scale transformed image electronically acquired in matrix form (in fact, a raster is already a matrix) allows angles to be determined by maximizing the function that evaluates the difference in pixel intensities in all

*Fig. 7. Illustrative example of the methodology depicting Palazzo Venezia in Rome (source: https://turismoroma.it/it/luoghi/palazzo-di-venezia).*

*Fig. 8. Application of the Harris and Stephens algorithm, in the small image above a 16th century watercolour depicting a flower bed [Mosser, Teyssot 1999, p. 79], in the enlarged image the result of korner detection with blue pixels (author's elaboration).*

```
#Planimetria di aiuole del XVI sec.  Test per determinare i korner
#Test con algoritmo di Chris Harris & Mike Stephens, 1989

import numpy as np
import cv2 as cv

filename = 'Acquerello planimetria aiuole pag 79.jpg'
img = cv.imread(filename)
gray = cv.cvtColor(img,cv.COLOR_BGR2GRAY)

gray = np.float32(gray)
dst = cv.cornerHarris(gray,2,3,0.005)


dst = cv.dilate(dst,None)

# Soglia per un valore ottimale, che può variare a seconda dell'immagine.
img[dst>0.005*dst.max()]=[0,0,255]

cv.imshow('dst',img)
if cv.waitKey(0) & 0xff == 50:
    cv.destroyAllWindows()
cv.imwrite('Risultato_KD.jpg',img)
```

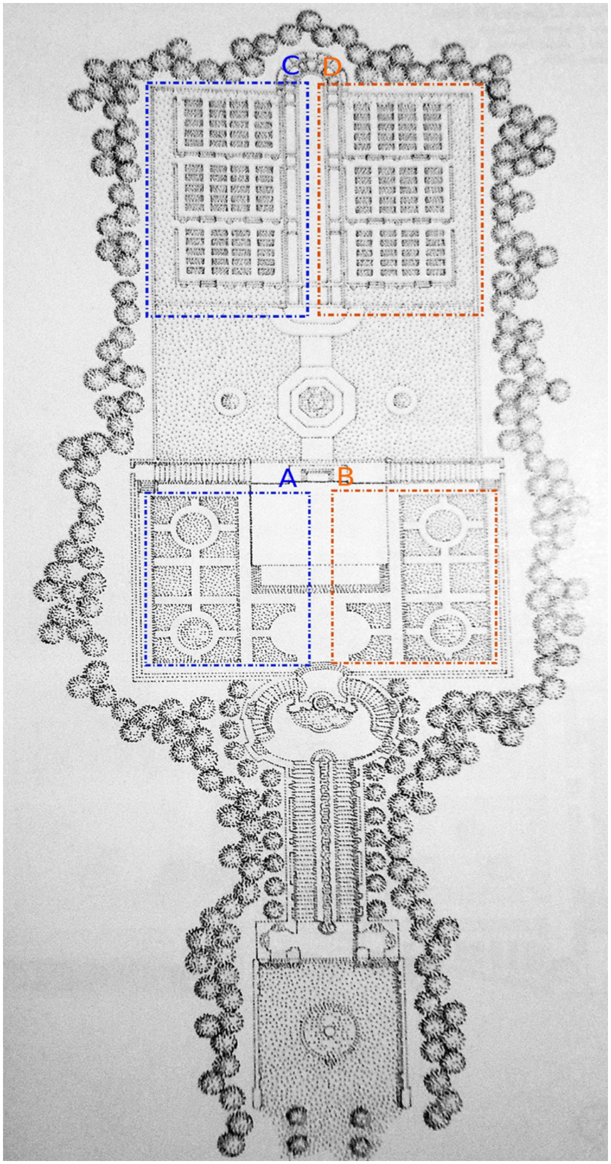


directions. The algorithm can be implemented within the Python programming environment, using the Open Source Computer Vision Library [1] (fig. 8). Harris's algorithm has problems and is ineffective when changing the image display scale. Continuing developments in computer vision and particularly in korner detection have allowed these difficulties to be overcome by providing a more performance-oriented toolset. For example, Lowe [2004] developed an applicable Scale Invariant Feature Transform (SIFT) algorithm, which was further improved by Bay et al. [2006], in terms of computational speed with the Speeded-Up Robust Features (SURF). Finally, the latest evolution is due to Rublee et al. [2011] with ORB (Oriented FAST and Rotated BRIEF), which is a more 'open' solution in terms of code accessibility than the previous two solutions. Subsequent to corner recognition, algorithms belonging to the family of enhanced 'Brute-Force Matchers' can be used to verify the existence of the *B=T(A)* transform within the *Z*-set, enabling them to operate on large datasets, such as FLANN (Fast Library for Approximate Nearest Neighbors), which conceptually can be explained as a solution that verifies and identifies the presence of corners in the Z-set, in the A-portion and in the *B=T(A)* transform, examines their distribution, and 'figures out' whether a given corner present in the transform also exists in the *Z*-set [Noble 2016; Muja, Lowe 2009].

## Experimentation and results

The application of the algorithmic procedures illustrated in the previous paragraph was implemented by choosing as a case study a planimetry of the gardens of Palazzo Farnese found in Mosser and Teyssot [1999], at page 27. The image was deliberately acquired with a non-professional camera characterized by low resolution and without the aid of corrective filters or other mechanical zenithal self-leveling devices. The idea behind this particular choice is to test the korner and image detection algorithms on low quality, blurry images acquired from documentary sources that perhaps over time have also undergone physical deterioration of the paper support with consequent deformation of the image itself; in practice, the choice was made to simulate difficult operating conditions that might arise when operating on representations, even pictorial ones such as those described in the preceding paragraphs. Having then defined the case study (fig. 9), two portions

*Fig. 9. The plan of the Palazzo Farnese gardens used as a case study on which two portions 'A' and 'C' are highlighted for the algorithm test [Mosser, Teyssot 1999, p. 27].*



of the image indicated by two blue dashed rectangles and named *A* and *C* respectively were selected within it. Of these two portions, the respective reflexive and rotational symmetries were processed, and in addition, to test the algorithmic recognition capability, they were scaled and intentionally deformed. More specifically, portion *A* was first reflected and scaled to twice the scale ratio of the original and then rotated 90 degrees. Portion *C* was first reflected and then successively deformed by stretching it along the Y-direction by a percentage of 10%, 20%, 30% and finally 40% from the originally acquired dimensions. Following the determination of the respective *A* and *C* transforms, we first applied the SIFT algorithm for determining the kornerpoints and then the FLANN algorithm whose task is to compare them and through this step recognize the existence of one image within another. The complete code is given below (fig. 10), and a more detailed explication of it is given later.

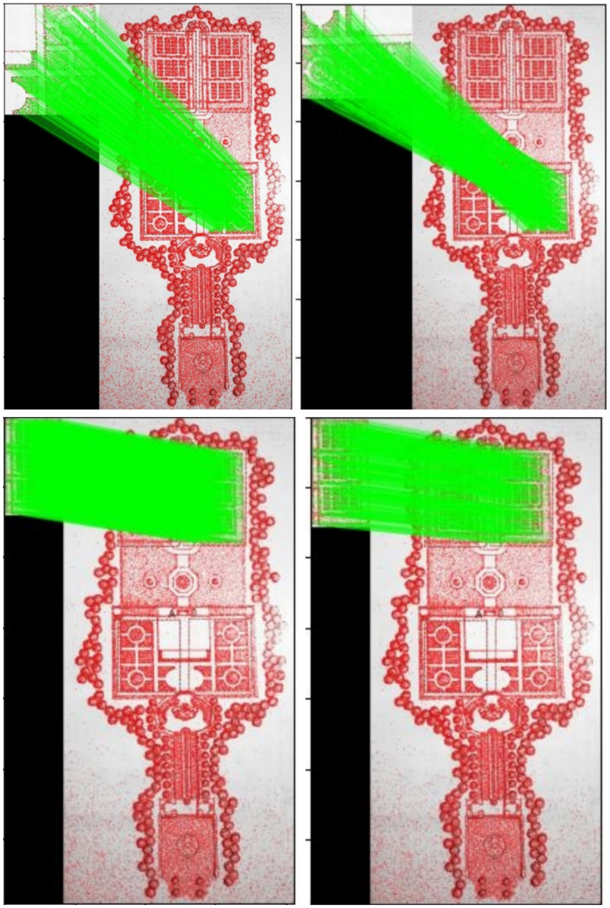
*Fig. 10. The scirpt in Python incorporating the SIFT and FLANN algorithms for kornerpoint detection in the case study (author's code compilation).*

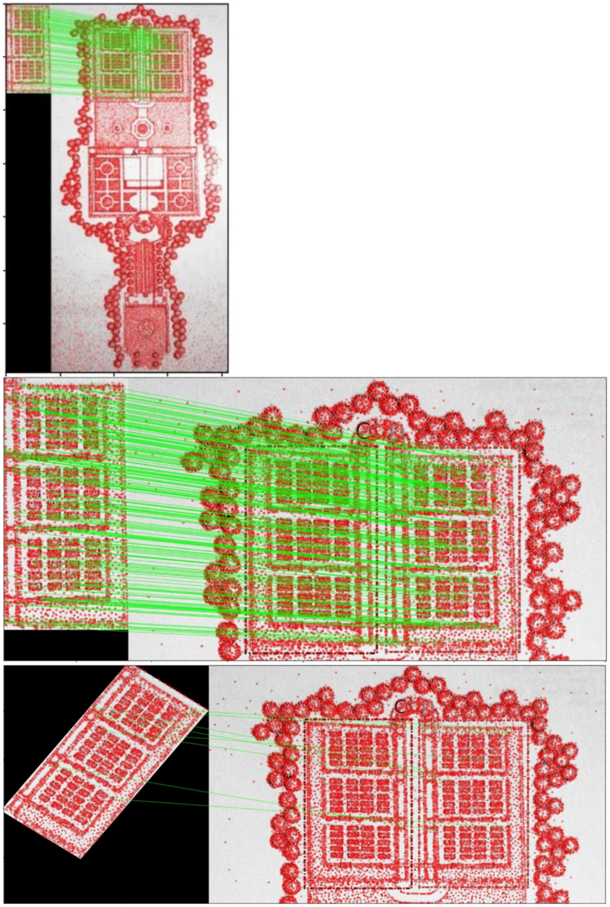
```
import numpy as np
import cv2 as cv
import matplotlib.pyplot as plt

e1 = cv.getTickCount()

img1 = cv.imread('Palazzo_Farnese_D_test_estremo.jpg',cv.IMREAD_GRAYSCALE) #
queryImage
img2 = cv.imread('Palazzo_Farnese.jpg',cv.IMREAD_GRAYSCALE) # trainImage

# Initiate SIFT detector
sift = cv.SIFT_create()

# find the keypoints and descriptors with SIFT
kp1, des1 = sift.detectAndCompute(img1,None)
kp2, des2 = sift.detectAndCompute(img2,None)

# FLANN parameters
FLANN_INDEX_KDTREE = 5
index_params = dict(algorithm = FLANN_INDEX_KDTREE, trees = 5)
search_params = dict(checks=100) # or pass empty dictionary

flann = cv.FlannBasedMatcher(index_params,search_params)

matches = flann.knnMatch(des1,des2,k=2)

# Need to draw only good matches, so create a mask
matchesMask = [[0,0] for i in range(len(matches))]

# ratio test as per Lowe's paper
for i,(m,n) in enumerate(matches):
 if m.distance < 0.4*n.distance:
     matchesMask[i]=[1,0]

draw_params = dict(matchColor = (0,255,0),
 singlePointColor = (255,0,0),
 matchesMask = matchesMask,
 flags = cv.DrawMatchesFlags_DEFAULT)

img3 = cv.drawMatchesKnn(img1,kp1,img2,kp2,matches,None,**draw_params)

e2 = cv.getTickCount()
time = (e2 - e1)/cv.getTickFrequency()
print( time )

plt.imshow(img3,),plt.show()
```

In the first part of the script, through the first three lines of code we import three libraries that are not originally present within Python and they are respectively 'Numpy'' [2] which allows the management and advanced analysis on multidimensional matrices, such as a raster image that is nothing but a two-dimensional matrix; the library 'OpenCV' which as already indicated is the basis of many Computer Vision and Image Detection solutions, and finally the library 'Matplotlib' [3] which allows the creation and visualization of interactive graphs and animations within Python. Immediately afterwards and almost at the close of the code, two counters *e1* and *e2* were inserted, respectively, which allow to calculate the time (*e2-e1*), expressed in seconds, taken to execute the script between the two counters; this expedient was made in anticipation of a development of the research also directed to the quantitative evaluation of the performance of the different solutions that can be developed and implemented. Thus, analyzing the operation of the script between the two counters, we immediately find two lines that create the variables *img1* and *img2* within which will be stored, respectively, the portions of the image to be searched (#queryImage) and the base within which to search (#trainImage). In the following lines, we first start the SIFT algorithm for determining the kornerpoints in the two previously stored images and then save the result of this analysis within two new variables called *kp1* and *kp2* respectively. This result is then passed to FLANN's algorithm, which, as mentioned earlier, allows the comparison and merging of similar kornerpoints between the images being compared. The result is conditional on reaching a certain quantitative standard, which can be modified through subsequent lines of code. For a precise and detailed explication of how both SIFT and FLANN work, see the extensive OpenCV library documentation that is available online. The overall result is stored within a new variable *img3* that will contain the two images, the kornerpoints and the union of those determined to be coincident via line segments (flags). The final result is immediately available and displayed within a window created with the last line of code ('plt.imshow'). In the results obtained, the kornerpoints are represented by small red-colored circles in both comparison images, while the flags joining them are green-colored line segments. The execution of the code was repeated for all the previously indicated case histories, allowing us to observe and evaluate the script's ability for image recognition under different conditions, from simple reflection, to reflection-rotation,

Fig. 11. Top: recognition of the A portion reflected (left) and reflected-rotated (right); bottom: recognition of the C portion reflected and deformed by 10% (left), and 20% (right) (author's elaboration).

Fig. 12. At the top of the small figure is the recognition of the C portion reflected and deformed by 30%, and of which an enlargement can be appreciated; at the bottom is the recognition, again of the C portion reflected, by 30% and rotated by an angle of 45° (author's elaboration).





to intentional image distortion. The outcomes are shown in the following two images and then commented on. In the first two computations (fig. 11), one can immediately see the high effectiveness of the script if the portions to be recognized are only reflected but also reflected and rotated; in fact, one can observe the dense and thick cloud of green links that connect the image to be searched within the overall floor plan. Applying the script in the search for the *C* portion that was first reflected and then deformed along the *Y*-axis from 10% to 40%, it can be observed (fig. 11) how its recognition ability tends to be reduced, but still with a high numerosity of green flags. By raising the degree of deformation to the threshold of 30 percent, the reduction in the script's recognition ability is evident; the density of green flags is significantly lower (fig. 12), and then is further reduced if the portion is reflected, deformed and also rotated. The script's recognition capability is totally nullified by reaching the 40% deformation threshold, in other words, the code being unable to recognize any coincidence between the korners of the compared images, suggests that in this particular situation the *C* portion and its transform *C=T(D)* (reflected and deformed along a single direction by as much as 40%) would not be symmetrical.

## Conclusions

In the research, the possibility of using some automated algorithms belonging to the branch of Image Detection and Computer Vision was tested and evaluated to instruct the computer in recognizing the symmetries existing in the typical spatial distribution of an Italian garden. Starting from an evolutionary historical analysis of the characters and events that stimulated and influenced the birth of the Italian garden, a number of documentary sources were identified that indicate how a large representation of these gardens has unfortunately been lost over the centuries, and the remainder have often been affected by revisiting, in some cases quite recently. Thus it has been ascertained that very often the only evidence of these treasures now lost or altered from the initial plans is found in artistic illustrations or very old documents that may have deteriorated, discolored, distorted or otherwise over time. For this reason, it was chosen to develop an algorithm and test its effectiveness under different conditions to understand its applicability but more importantly its limitations. The chosen case study shows the typical characters of the Italian garden and therefore, by definition symmetrical in the elements that compose it, but this was not the objective of the research, i.e. to demonstrate the existence of symmetries, as much as, if anything, to understand whether the proposed methodology was effective in teaching the computer to recognize these relationships allowing the experiment to be replicated in the future in more complex and less certain situations. As the results show, the outcome of the experiment can be considered successful within certain operational limits. Subsequent developments are already pointing in several directions: understanding and comparing the effectiveness and efficiency of the different algorithms with respect to a casuistry of potential case studies that is much broader than could be discussed in a single research study, census and categorize precisely the variety of case studies that could be presented and that would be used as a standard library for performance comparisons between algorithms, instruct the computer to recognize and distinguish within the representations the typical elements of the Italian garden, (hedges, tree-lined avenues, peach orchards, etc.) through other computer vision algorithms that are actually more frequently used in medical diagnostics through imaging. Initial results, which will be published in future research, seem to suggest the research direction can be considered as fertile and promising in results.

### Notes

[1] https://numpy.org/.
[2] https://matplotlib.org/.
[3] http://opencv.org/.

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### Reference List


Agostini, I. (2011). *Il Trebbio in Mugello. Terre, storia, architettura: tremila anni di un trivium*. Firenze: Aión.

Albrecht, A., Steinhardt, P.J. (1982). Cosmology for grand unified theories with radiatively induced symmetry breaking. In *Physical Review Letters*, vol. 48, No. 17, p. 1220.

Amadei, D. (2015). *L'ordine e la geometria nel teatro antico. Diffusione e fortuna del De Architectura di Vitruvio. Caso studio: il teatro romano di Fanum Fortunae* [dissertazione di dottorato, Università Politecnica delle Marche]. Centro Studi Vitruviani.

Azzi Visentini, M. (2012). La chasse dans le duché de Milano à l'époque des Visconti et des Sforza: les parcs de Pavie et de Milan. In M. Chatenet et al. (eds.). *Chasses princières dans l'Europe de la Renaissance*, pp. 179-215. Arles: Actes sud.

Bartalucci, C., Furferi, R., Governi, L., Volpe, Y. (2018). A survey of methods for symmetry detection on 3d high point density models in biomedicine. In *Symmetry*, vol. 10, No. 7, p. 263.

Bay, H., Tuytelaars, T., Van Gool, L. (2006). Surf: Speeded up robust features. In *Computer Vision–ECCV 2006: 9th European Conference on Computer Vision*, Graz, Austria. Proceedings, Part I 9, pp. 404-417. Berlino: Springer Berlin Heidelberg.

Cold, B., Kolstad, A., Larssæther, S. (1998). *Aesthetics, well-being and health: Abstracts on theoretical and empirical research within environmental aesthetics*. Oslo: Norsk form.

Conners, R.W., Ng, C.T. (1989). Developing a quantitative model of human preattentive vision. In *IEEE transactions on systems, man, and cybernetics*, vol. 19, No. 6, pp. 1384-1407.

Davis, L.S. (1977). Understanding shape: Angles and sides. In *IEEE Trans. Computers*, vol. 26, No. 3, pp. 236-242.

Dosen, A.S., Ostwald, M.J., Dawes, M.J. (2013). Environmental preference and spatio-visual geometry: A method for combining isovists and psychological testing. In M.A. Schnabel (ed.). *Cutting Edge: 47th International Conference of the Architectural Association*, pp. 653-662.

Eckbo, G. (1964). *Urban Landscape Design*. New York: McGraw-Hill Book Company.

Frati, M. (2015). *Alle soglie della villa fiorentina: l'architettura delle dimore rurali nel Trecento*. Roma: Opus Incertum.

Frommel, C.L. (1994). Poggioreale: problemi di ricostruzione e tipologi. In D. Lamberini, (a cura di). *Giuliano e la bottega dei da Maiano. Atti del convegno internazionale di studi*, pp. 104-111. Heidelberg: Heidelberg University.

Frommel, C.L. (2006). Villa Medici a Fiesole e la nascita della villa rinascimentale. In A. Roca De Amicis (a cura di). *Colloqui d'architettura*, pp. 30-59. Heidelberg: Heidelberg University.

Gauch, J.M., Pizer, S.M. (1993). The intensity axis of symmetry and its application to image segmentation. In *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 15, No. 8, pp. 753-770.

Germano, G. (2022). *Uno sguardo nel Medioevo: il monastero, l'orto, la società*. Convegno *La salute nella visione profetica di Ildegarda di Bingen. Un pensiero globale e moderno*, Pavia, Italia.

Hagerhall, C.M., Purcell, T., Taylor, R. (2004). Fractal dimension of landscape silhouette outlines as a predictor of landscape preference. In *Journal of environmental psychology*, vol. 24, No. 2, pp. 247-255.

Hays, J., Leordeanu, M., Efros, A.A., Liu, Y. (2006). Discovering texture regularity as a higher-order correspondence problem. In *Computer Vision–ECCV 2006: 9th European Conference on Computer Vision*, pp. 522-535. Berlino: Springer Berlin Heidelberg.

Harris, C., Stephens, M. (1988). A combined corner and edge detector. In *Alvey vision conference*, vol. 15, No. 50, pp. 147-152.

Hodgson, D. (2011). The first appearance of symmetry in the human lineage: Where perception meets art. In *Symmetry*, vol. 3, No. 1, pp. 37-53.

Hu, H. (2024). Analysis of the Spatial Axis of Italian Renaissance Garden. Taking Villa Lante and Villa d'Este as Examples. In *Frontiers in Art Research*, vol. 6, No. 2, pp. 80-87.

Isaeva, V.V., Kasyanov, N.V., Presnov, E.V. (2012). Topological singularities and symmetry breaking in development. In *Biosystems*, vol. 109, No. 3, pp. 280-298.

Lei, Y., Wong, K.C. (1999). Detection and localisation of reflectional and rotational symmetry under weak perspective projection. In *Pattern Recognition*, vol. 32, No, 2, pp. 167-180.

Lowe, D.G. (2004). Distinctive image features from scale-invariant keypoints. In *International journal of computer vision*, No. 60, pp. 91-110.

Kanade, T. (1981). Recovery of the three-dimensional shape of an object from a single view. In *Artificial intelligence*, 17, pp. 409-460.

Kern, E.G. (1951). The Gardens in the Decameron Cornice. In *PMLA*, vol. 66, No. 4, pp. 505-523.

Kibler, W.W. (1992). Labbé's 'L'architecture des palais et des jardins dans les chansons de geste'. In *Olifant*, vol. 17, No. 3/4,pp. 212-216.

Mehaffy, M.W. (2020). The Impacts of symmetry in architecture and urbanism: Toward a new research agenda. In *Buildings*, vol. 10, No. 12, p. 249.

Mitra, N., Pauly, M. (2008). Symmetry for architectural design. In *Advances in Architectural Geometry, conference Proceedings*, p. 13-16.

Mitra, N.J., Pauly, M., Wand, M., Ceylan, D. (2013). Symmetry in 3d geometry: Extraction and applications. In *Computer graphics forum*, vol. 32, No. 6, pp. 1-23.

Mosser, M., Teyssot, G. (1999). *L'architettura dei giardini d'Occidente. Dal Rinascimento al Novecento*. Milano: Electa.

Muja, M., Lowe, D. (2009). *Flann-fast library for approximate nearest neighbors user manual*. <https://www.cs.ubc.ca/research/flann/uploads/FLANN/flann_manual-1.8.4.pdf> (accessed July 30, 2024).

Noble, F.K. (2016). Comparison of OpenCV's feature detectors and feature matchers. In *23rd International Conference on Mechatronics and Machine Vision in Practice*, pp. 1-6. New Jersey: Institute of Electrical and Electronics Engineers.

Padovan, R. (2002). *Proportion: science, philosophy, architecture*. Routledge: Taylor & Francis.

Park, M., Lee, S., Chen, P.C., Kashyap, S., Butt, A.A., Liu, Y. (2008). Performance evaluation of state-of-the-art discrete symmetry detection algorithms. In *IEEE Conference on Computer Vision and Pattern Recognition*, pp. 1-8. New Jersey: Institute of Electrical and Electronics Engineers.

Podolak, J., Shilane, P., Golovinskiy, A., Rusinkiewicz, S., Funkhouser, T. (2006). A planar-reflective symmetry transform for 3D shapes. In *ACM SIGGRAPH*, pp. 549-559.

Prasad, V.S.N., Davis, L.S. (2005). Detecting rotational symmetries. In Tenth I*EEE International Conference on Computer Vision*, vol. 2, pp. 954-961. New Jersey: Institute of Electrical and Electronics Engineers.

Rossi, C. (2004). *Architecture and mathematics in ancient Egypt*. Cambridge: Cambridge University Press.

Rublee, E., Rabaud, V., Konolige, K., Bradski, G. (2011). ORB: An efficient alternative to SIFT or SURF. In *International conference on computer vision*, pp. 2564-2571. New Jersey: Institute of Electrical and Electronics Engineers.

Salvadori, M. (2015). Can there be any relationships between mathematics and architecture?. In K. Williams, M.J. Ostwald (Eds.). *Architecture and Mathematics from Antiquity to the Future. Volume I: Antiquity to the 1500s*, pp. 25-29. Basilea: Birkhäuser.

Sansovino, F. (1522). *Pietro Crescentio tradotto novamente per M. Francesco Sansovino nel quale si trattano le cose della villa con le figure delle herbe poste nel fine. Con vocabolario delle voci difficili che sono in questa opera & con i disegni degli strumenti co quali si coltiva & si lavora la terra*. <https://archive.org/details/bub_gb_UYXh3J_fijkC/page/n5/mode/2up> (accessed July 30, 2024).

Savastano, L., Chiovenda, E., Mieli, A. (1922). Pietro de' Crescenzi ed il suo trattato d'agricoltura. In *Archeion*, No. 3, pp. 311-318.

Sberlati, F. (2004). Villania e cortesia. L'opposizione tra città e campagna dal Medioevo al Rinascimento. In *La letteratura di villa* e *di villeggiatura: atti del convegno di Parma*, pp. 1000-1050. Roma: Salerno editrice.

Taylor, N. (1994). Aesthetic judgement and environmental design: is it entirely subjective?. In *Town Planning Review*, vol. 65, No. 1, p. 21.

Tyler, C.W. (2003). *Human symmetry perception and its computational analysis*. Routledge: Psychology Press.

Tosco, C. (2018). *Storia dei giardini. Dalla Bibbia al giardino all'italiana*. Bologna: Il Mulino.

Usher, J. (1989). Frame and novella gardens in the "Decameron". In *Medium aevum*, vol. 58, No. 2, pp. 274-285.

Van den Berg, A.E., Koole, S.L., Van Der Wulp, N.Y. (2003). Environmental preference and restoration:(How) are they related?. In *Journal of environmental psychology*, vol. 23, No. 2, pp. 135-146.

Xiao, Z., Wu, J. (2007). Analysis on Image Symmetry Detection Algorithms. In *Fourth international conference on fuzzy systems and knowledge discovery*, vol. 4, pp. 745-750. New Jersey: Institute of Electrical and Electronics Engineers.

Weinberg, S. (1979). Implications of dynamical symmetry breaking: an addendum. In *Physical Review*, vol. 19, No. 4, p. 1277.

Weyl, H. (2015). *Symmetry*. Princeton: Princeton University Press.

Žalik, B., Strnad, D., Kohek, Š., Kolingerová, I., Nerat, A., Lukač, N., Podgorelec, D. (2022). A Hierarchical Universal Algorithm for Geometric Objects' Reflection Symmetry Detection. In *Symmetry*, vol. 14, No. 5, p. 1060.