

Lunette Vaults in Guarini's Work. Digital Models between *Architettura Civile* and *Modo di Misurare le Fabriche*

Roberta Spallone

Abstract

In Guarini's systematization of vaulted systems based on geometric reasoning, the lunette vaults created by cutting main vaults of different geometric shape and inserting the groins take on a primary role. This operation increases the natural lighting of the rooms and ensures airiness and dynamism to atria and halls of Baroque palaces. In the *Architettura Civile* (published posthumous in 1737) and the *Modo di misurare le fabbriche* (1674), he describes the geometrical origin and the measurement of the vaults, respectively. In the first treatise, the lunette vaults emerge as a typology rich of shape variations and implementations in civil buildings, in the second the surfaces of the vaults are attributed to the geometric matrices of the single elements and present some novelties with respect to the *Architettura Civile*, having semi-ellipses, and arcs of circumference and ellipse, as directrices. In this work the significant relationships between these two Guarini's treatises, whose complementarity emerges in the continuous reference to the Geometry-Architecture binomial, are developed and deepened. The graphical analysis and digital modeling of surfaces allow not only the recognition of the geometric primitives underlying the structure of the vaulted systems and the verification of the measurement rules that Guarini largely attributes to himself, but also the comparison with the architectures realized in the period of the second expansion of Turin.

Keywords: graphical analysis, digital modeling, lunettes, lunette vaults, treatises, Guarini.

Introduction

Guarino Guarini (1624-1683), was an abbot of the Theatines order and a scholar with wide-ranging interests, mainly architecture, mathematics and philosophy. He worked in the century permeated by the *esprit de géométrie*, fueled by the discoveries and studies in the mathematical field and in particular of pure geometry. In this period a leading position of discipline within the philosophical method established, through the theories of Galileo, Bacon, Descartes, Malebranche, Desargues.

Guarini, after a series of journeys in Italy and France, during which he devoted himself to theoretical studies and architectural design, in 1666 settled in Turin. Here he published some of his most important treatises and created a series of religious and civil architectures, emblematic of the Baroque period.

Guarini was the first to develop a rigorous and systematic discussion of vaulted systems which is divided into three complementary treatises: *Architettura Civile* (published posthumously in 1737), *Euclides adauctus* (1671), and *Modo di misurare le fabbriche* (1674), in which he illustrated the geometric origin, the stereotomy, and the measurement of the vaults, respectively.

Among them, the *Euclides adauctus* takes the role of theoretical reference based on the geometric discipline, with respect to the approach of the other two treatises constantly referred to the architectural works.

In this systematization, started in the *Architettura Civile* with the construction of a real vocabulary of shapes geometrically based, the lunette vaults, generated by cutting

the main vaults of different geometric shapes and inserting the groins, take on a primary role. This operation increases the natural lighting of the rooms and ensures airiness and dynamism to the brickwork vaults that feature atria and halls in Baroque palaces. Indeed, from the time of the second expansion, begun in 1673, Turin is endowed with numerous palaces and noble houses of new construction and reshaping, in which the experimentation of unitary vaulted brick systems of considerable complexity is particularly lively.

The geometric nature of the vaults described in the *Architettura Civile* has been the subject of previous studies by the author [Spallone 2016; Spallone 2017, pp. 91-120]. Among these, the lunette vaults emerge as a typology rich of shape variations and implementations in civil buildings. Indeed, starting from the late Renaissance models having axial development, Guarini creates a new radio-centric system, on an oval plan, applied to the designs of the atria of the palaces Carignano and Provana di Collegno in Turin.

In the *Modo di misurare le fabbriche*, the surfaces of the vaults are attributed to the geometric matrices of the single components. Among these, the lunettes, considered both as parts of cross vaults and as completion elements of main vaults, present some formal novelties with respect to the *Architettura Civile*. These shapes are recognizable in the lunette vaults both in the noble halls and in the service rooms in the palaces built in Piedmont between 17th and 18th centuries.

The brick lunette vaults, of Renaissance tradition, have an important precedent in the vault that Tibaldi had designed for the Sacristy of the church of the Santi Martiri in Turin (1592), while the great composed vault of the main hall of Reggia di Venaria (1667-68) seems to usher in a season of widespread attention for the vaults with cuts and lunettes in civil architecture [Piccoli 1999, p. 87; Scotti Tosini 2006, p. 95].

Working methodology

In this paper, the significant relationships between *Architettura Civile* and *Modo di misurare le fabbriche* about the development of the theme are examined in depth, comparing theories and drawings, whose complementarity emerges in the continuous reference to the binomial Geometry-Architecture.

The graphical analysis and the digital modeling of the surfaces are realized starting from the drawings in the plates of the *Architettura Civile*, and the graphic diagrams printed with xilographic technique and inserted in the text of the *Modo di misurare le fabbriche*. They allow not only the recognition of the geometric primitives underlying the shape of the lunette vaulted systems, and the verification of the measurement rules that Guarini largely ascribes to himself, but also the comparison with some built architectures.

The pseudo-axonometric diagrams, traced with some uncertainty, that describe the different models of lunettes, and the double orthographic projections (plan and section), that specify the spatial features and the proportions of the lunette vaults allow the control of the rigorous classification and ideation process followed by Guarini. This involves retracing the steps, applying the methods of graphical analysis and reconstructive digital modeling. For the purposes of this research all the lunettes and lunette vaults mentioned in the two treatises have been modeled, including their variants.

As regards to the *Architettura Civile*, the aims of modeling are the identification of Guarini's vocabulary of shapes, the control of consistence between the orthographic projections, and the reconstruction of the compositional logics underlying the proposal of *exempla* applicable to the construction of palaces. As regards to the *Modo di misurare le fabbriche*, the aim is to complete the analysis, documented by the figures, up to the verification of the Guarinian calculation methods, which are extremely useful in estimating the quantities of materials needed for the construction and renewal of the buildings of the city during the Baroque age.

Lunettes and lunette vaults, terminological issues and convergences between the two treatises

In the *Modo di misurare le fabbriche* the theme of the lunettes is developed in Chapter 3 of Part Two, devoted to the calculation of the surfaces of 'cloister and lunette vaults' on square or polygonal plans. From the chapter's title emerges that Guarini is dealing with vaults composed by portions of the first 'round body' classified in the *Architettura Civile* –the cylinder– which, appropriately cut, gives rise to those that today are called coves and groins.

While in the current architectural lexicon, by lunette is meant the portion of vertical wall between the arc of intersection of the wall with a groin and the impost plane, in

Fig. 1. Geometric shaping of lunette vaults. Image: Guarini 1737, Plate XIX; digital modelling: Roberta Spallone.

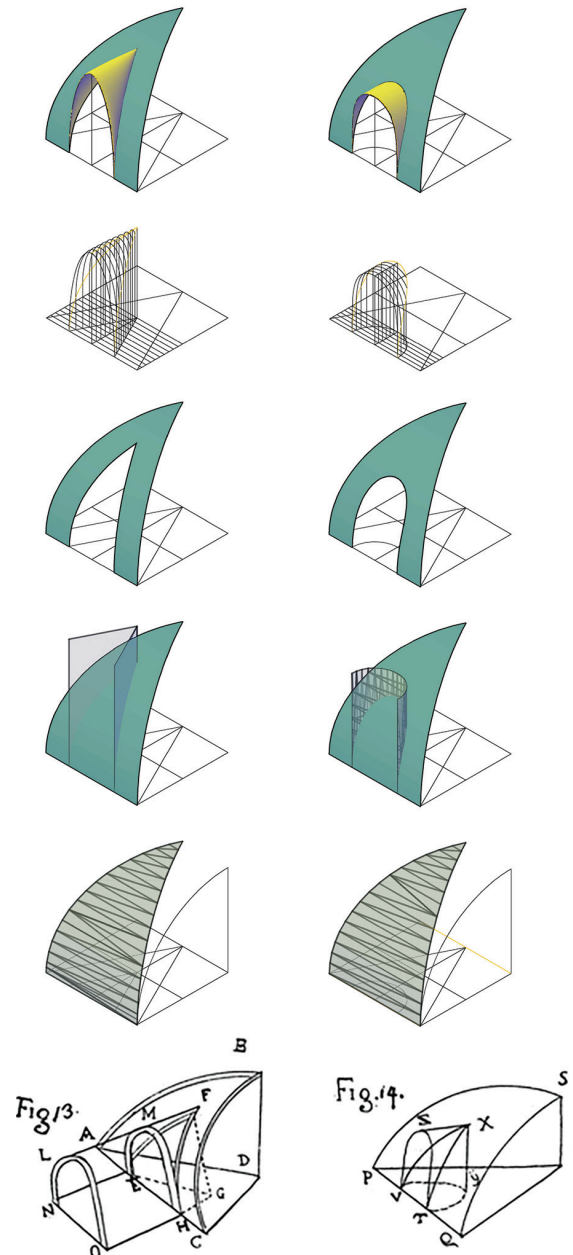
the *Modo di misurare le fabbriche* the term 'lunette', or 'Luna', is used both to indicate one of the four groins of a cross vault (propositions 14 and 23), and as a synonym of groin inserted in a main vault (proposition 16), and as a separate element (proposition 24).

In the *Architettura Civile* the lunette vaults are described in Treatise III, Chapter XXVI, entitled 'About the vaults, and various ways of making them', within Seventh Observation, which defines the geometric model of such vaults, and Octave Observation, which shows three examples applied to architecture. In this treatise the meaning of lunette vault is different: "if the vault of any kind will be intersected by many lunettes, it will be called lunette vault" [Guarini 1737, p. 187]. Thus, here is valid the definition of lunette vault as a cover composed of a main vault (barrel, cross, cloister, dome) cut and completed by groins, as the drawings in the XIX and XX plates confirm.

Beyond the terminological issues, the first part of the discourse concerning lunettes and lunette vaults has significant convergences between the two texts, also from the graphic point of view.

In the *Architettura Civile*, the deepening of lunette vaults follows the description of the elements shaping the vaults born from the 'six round bodies' – cylinder, cone, conoid, sphere, rotational ellipsoid (or oval), and scalene ellipsoid/oval. These primitives are considered as geometric matrices of the main simple vaults with single and double curvature (barrel, conical, conoidal, dome, sail) and composite (cross-shaped, cloister, 'gothic', lunette), after operations of cuts and compositions of parts.

The explanation of the lunette vaults concludes this typological repertoire. Indeed, they are composed vaults that can take as a main surface one of those previously described. Guarini specifies: "In all the aforementioned bodies, two kinds of cut can be made, one is triangular [...] but does not arrive in the middle [...] and then, is made another triangular cut on the other side, which reached up to the aforementioned [...] and then that void is filled with a piece of cylinder cut triangularly with the same angle of the cut, and as high as the same cut" [Guarini 1737, p. 187]. In the drawing of such vaults, in figures 13 and 14 of plate XIX, groins with a triangular and circular plans are inserted into a cylindrical cove, while the groin having a square plan mentioned in the text does not appear.



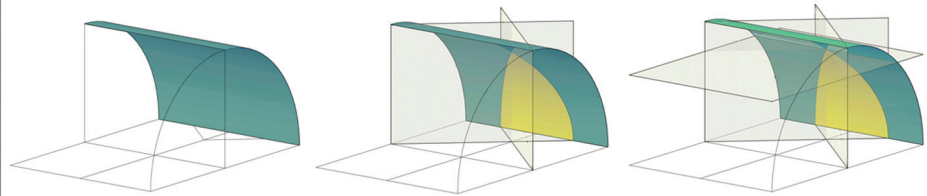
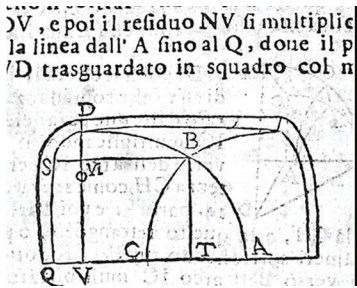


Fig. 2. Cut of a main vault aimed to inserting a groin. Image: Guarini 1674, Prop. 16; digital modelling: Roberta Spallone.

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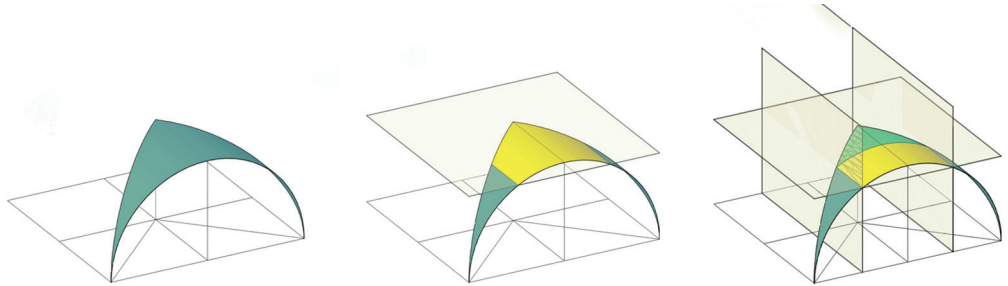
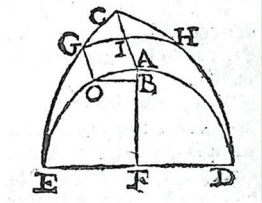


Fig. 3. Round-shaped lunette and its portions. Image: Guarini 1674, Prop. 14; digital modelling: Roberta Spallone.

As seen above, Guarini defines the surface of the groin as a portion of cylinder cut from the same planes that intersect the main surface but, as is well known, to guarantee the joint with the latter, this groin cannot be geometrically described as cylindrical, but it must be generated by parallel and variable sections (like arcs of circumference or ellipse) (fig. 1).

In the *Modo di misurare le fabbriche*, the issue of the creation of lunette vaults can be read by relating two propositions. In Proposition 16, Guarini presents the problem of calculating the surface to be subtracted to the main surface consisting of a quarter of a cylinder, after cutting with vertical planes. The author explains the necessity of such a calculation to achieve a correct measurement of the lunette vaults: "As it is reasonable to measure the lunettes, that certainly make greater the surface of a vault, so it is convenient to remove that surface from the same

vault, which occupies the space of the lunette" [Guarini 1674, p. 109].

This case establishes an undeniable relationship with the built architecture. Indeed, Guarini explains how to survey the position of some particular points and the horizontal plane tangent to the vertex of the groin, finalizing these operations to the calculation. The graphic diagram shows a doubling of the lines of the main vault to signify the wall thickness (fig. 2).

In Proposition 14 there is the ideal complement to the previous problem, namely "to find the surface of a lunette, or of a groin of a cross vault, whose directrix is a semicircle" [Guarini 1674, p. 107]. Indeed, here is illustrated the method for finding the surface of a groin with circular directrix, meant as part of a cross vault on a square plan. Following the beginning of Chapter 3, the reconstructive model hypothesizes a square plan for the whole vault,

therefore the horizontal projection of the groin results to be an isosceles triangle, rectangle at the vertex, with base equal to a and sides to $\sqrt{2}a/2$.

Finally, in the groin are made sections with two vertical planes parallel to the key line and with a plane parallel to the directrix. Therefore, Guarini starts from the case previously developed in the *Architettura Civile* and seen above, and extend it to extremely lowered lunettes whose section is an arc of circumference, very widespread to give light in low-pitched vaulted rooms, such as mezzanines and basements (fig. 3).

Diverging developments between architectural designs and surface calculation

In the two treatises, the Guarinian discourse continues in a divergent way: in the *Architettura Civile* developing three examples of lunette vaults, in the *Modo di misurare le fabbriche* widening the casuistry of lunettes' shapes.

The first two examples in the *Architettura Civile* are based on late-Renaissance lunette vault models.

In the first case, a cloister vault cut with planes parallel to the intersections between the coves is hypothesized. The rampant groins, inserted for completing the cuts, are in equal numbers for each side and have different spans of the perimeter arches on each walls, which are, therefore, semicircular along two sides of the room and semi-elliptical along the others. The main vault on a rectangular plan has a ratio of 3:4 between the sides, and semi-elliptical directrices of the two semi-cylinders that generate the coves of the cloister vault. The rampant groins, having a triangular plan of different base along the two sides of the room are generated by sections (fig. 4). This model could refer to the drawing of the honor hall in the Guarinian design of Castello di Govone, present in another plate of the *Architettura Civile* (fig. 5).

In the second example, a rectangular room with a ratio of about 1:2 between the sides, is covered by a barrel vault with cloister heads. The lunettes, defined by cuts parallel to the intersections between the barrel and the coves, have the same shape and are in a different number on the sides. The compensation, necessary for the regularity of the system, is realized by setting a different distance between the lunettes along the sides. It should be noted that the Guarinian drawing of the section of the main vault is not geometrically consistent with the plan: the profile of the section is elliptical rather than plane in the central part and circular at the ends (fig. 6).

The vault described above could refer to a drawing of the section of the design for the central body of Castello di Racconigi, attributed to Francesco Lanfranchi and dating back around 1665 [Dardanello 2006, p. 435]. Guarini will succeed to him in the task of transforming the castle from 1677 (fig. 7). The third case witnesses the passage from the late-Renaissance to the Baroque style, through Guarini's invention of a lunette vault set on the ovate plan that recalls, for its geometric layout, the vault of the atrium of Palazzo Carignano, even if the author does not explicitly refer to it. In the drawing shown in the plate XX the ratio between the axes of the oval plan is 4:3, while the section reveals that the main surface is generated by rotation of the oval around the major axis: the resulting rise is equal to the minor semi-axis. The horizontal projections of the groins, according to the indications of the text, have the sides of the same length and the vertex tangent to an internal oval equidistant from the main one. Guarini also shows a variant with different sides in plan for the diagonal groins.

In the reconstructive model, the rotational oval that shapes the main surface is cut with vertical planes drawn on the projection of the groins. In the section, these cuts generate vertices of the groins higher than in Guarini's drawing. The rampant groins, which originate from round arches, are modeled by parallel arcs with variable curvature along an oblique trajectory to the horizontal plane (fig. 8).

As said, the model in the treatise seems to be inspired by the ideation of the vault in the atrium of Palazzo Carignano, whose first drawings date back 1678-1679. The survey of the atrium of the palace showed remarkable measures of the central space to be covered with a single vault: in fact, the major axis measures about 14.65 m, the minor 10. The vault, consistently with the construction requirements, is rather lowered: the rise is about 2.20 m.

The comparison between the geometric diagram of the treatise and the realization in Palazzo Carignano reveals some morphological differences: in the atrium the main surface can be assimilated to a scalene ovaloid and the groins have triangular projections with different bases in plan. There are also some proportional differences: the ratio between the axes is between $\sqrt{2}:1$ and 3:2, while the rise is between a quarter and a fifth of the span as suggested by Guarini in a passage from the treatise [Guarini 1737, p. 188] (fig. 9).

In the *Modo di misurare le fabbriche*, the link with architecture is less stringent: while acting as an aid to the architect's operational activity, the reasoning "is maintained on a the-

Fig. 4. Cloister vault with lunettes. Image: Guarini 1737, Plate XIX; digital modelling: Roberta Spallone.

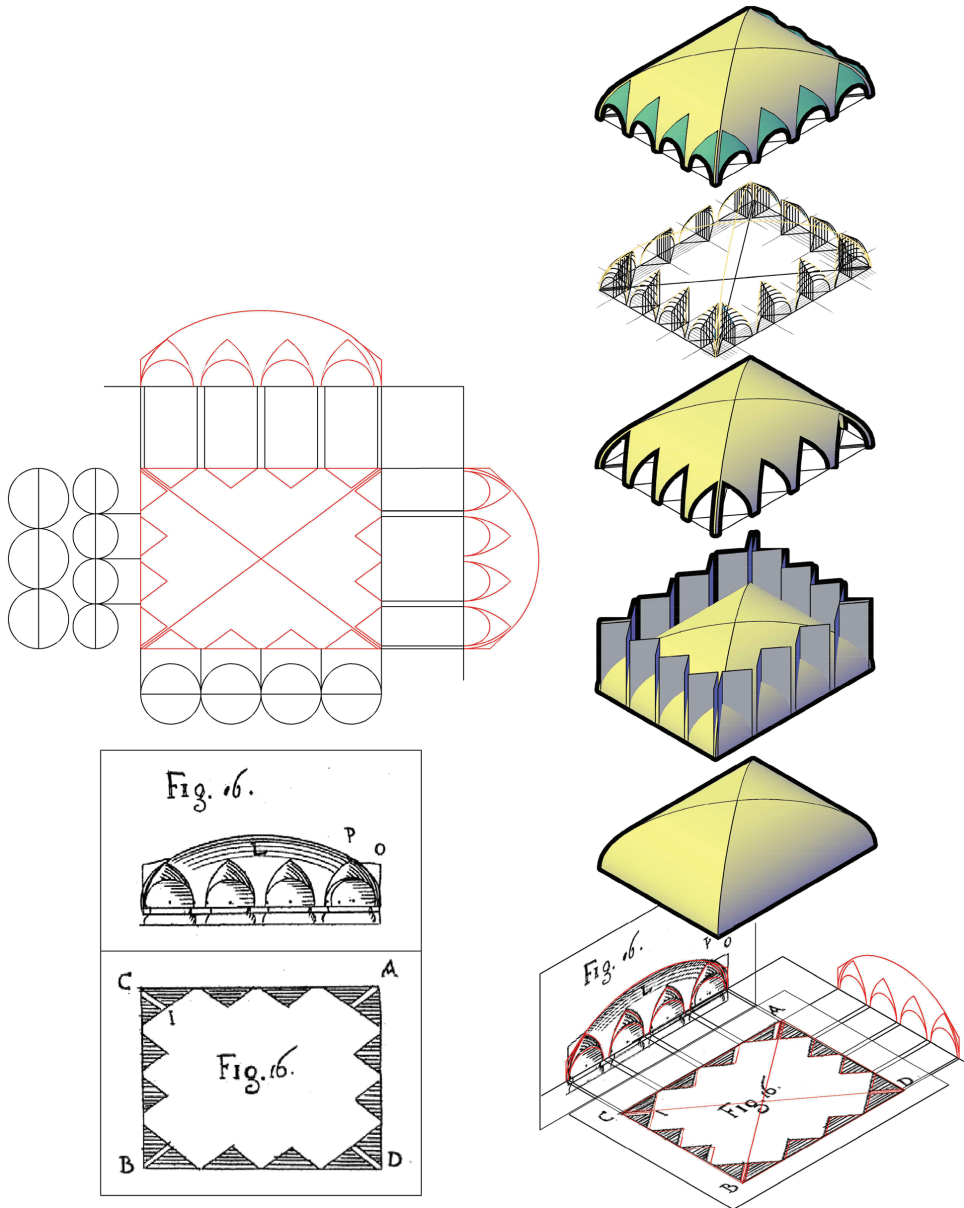
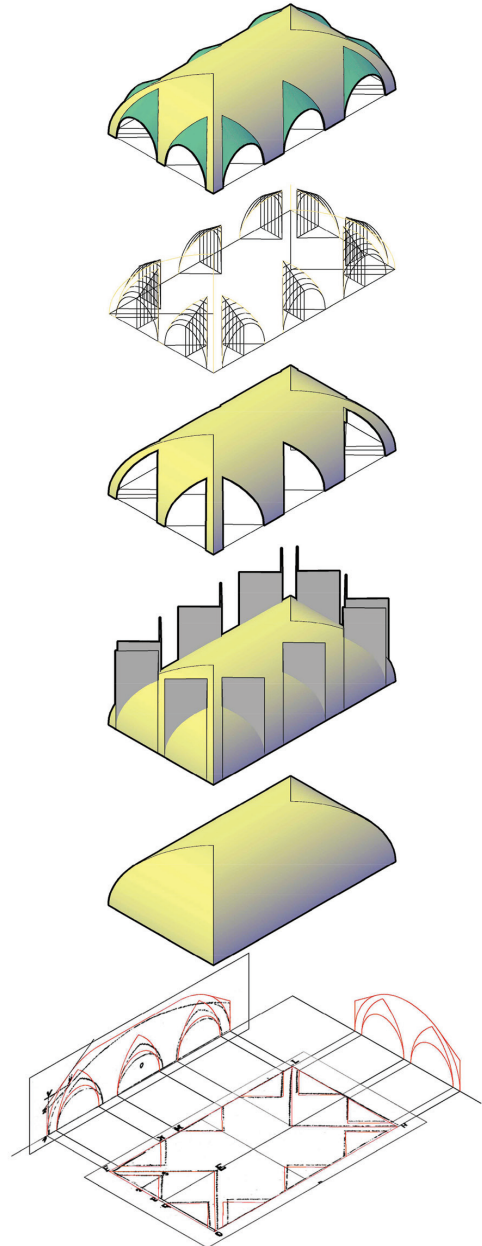
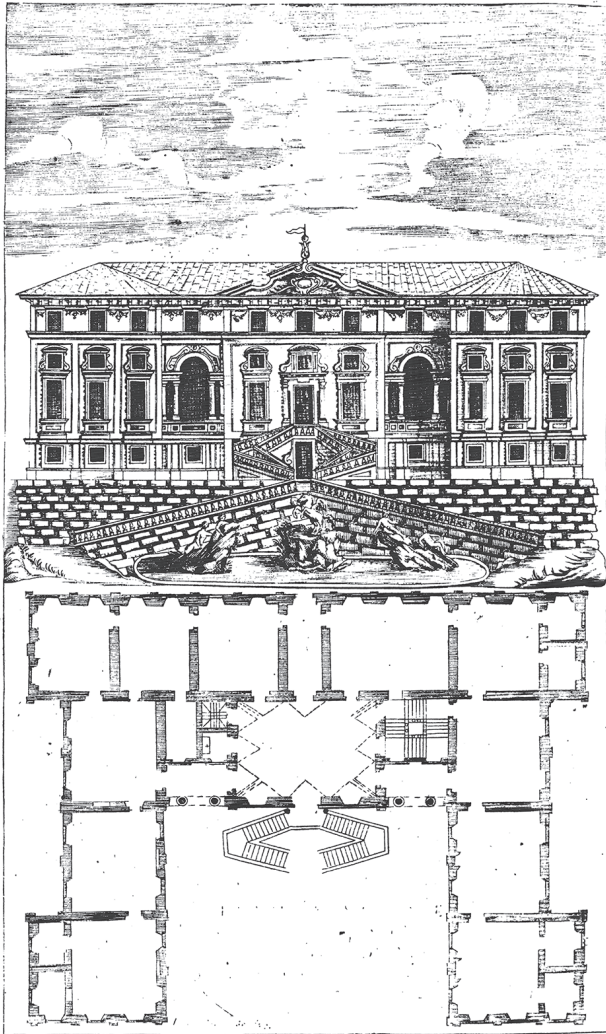


Fig. 5 Guarini, plan and elevation of Castello di Govone. Image: Guarini 1737.

Fig. 6. Barrel vault with cloister heads and lunettes. Image: Guarini 1737, Plate XX; digital modelling: Roberta Spallone.



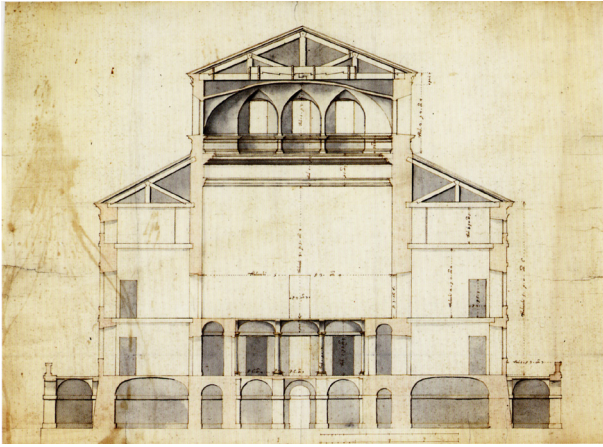


Fig. 7. Lanfranchi(?), cross section of Castello di Racconigi, c. 1655. Image: AST, Archivio Savoia-Carignano, cat. 95, mazzo 2, fasc. 124.

oretical level of absolute abstraction, in which the scientific and speculative mentality of the author decisively prevails” [Vagnetti 1970, p. 508].

The theme of the lunettes is carried out, in addition to the already commented propositions 14 and 16, in the propositions 23 and 24.

The description of the geometrical constructions of the lunettes is developed through the identification of different directrices, which give rise to lunettes round-shaped (semicircle), lowered arch (circumference arc), elliptic (semi-ellipse), lowered elliptic (arc of ellipse), archiacute (arcs of circumference), for a total of nine different shapes. The reference surfaces are semi-cylinders with different directrices or portions of them in the first four cases, and a semi-face of a pyramid, defined as ‘concave’ by the author, in the last. All the lunettes have triangular projections in plan.

In the book it seems evident Guarini’s auspice, on the one hand, to assimilate the lunettes with geometric figures of which he can offer formulas for the calculation - exact or approximate -, on the other hand, to widen the spectrum of shapes. Compared to the diagrams in the plate XIX of the *Architettura Civile* it is possible to note analogies in regard to the type of projection, and the use of dashed lines and letters but, in the plate, the lines are doubled in correspondence with arches, sections and intersections, aiming to simulate the future construction.

The modeling works are based on the recognition of the geometric primitive, the directrix, and the horizontal projection of each lunette. This is followed by: the setting of horizontal and vertical cut planes that generate the variants, the automatic interrogation for the calculation of areas, and the comparison of the result with that obtained by applying Guarini’s calculation methods. The latter are based on the four arithmetic operations and on the proportions with an unknown quantity, which he calls ‘rule of thirds’, while π is approximated with the fraction $22/7$, according to the teachings of Archimedes.

Proposition 23 expands the series of lunettes to elliptical ones and their portions. Also these are designed as elements of a cross vault on a square plan, so their horizontal projection is identical to that of Proposition 14. In the graphic diagram the height of the lunette is given by the semi-major axis but the method is also valid for the minor one (fig. 10). Both are widespread in the built vaults, just think of the lunettes in the atrium and the vestibule of Palazzo Provana di Collegno designed by Guarini himself, in which elliptic lunettes with different impost spans alternate (fig. 11). Proposition 24 concludes the discourse on the theme through an original construction, generated by the lateral half-faces of a ‘concave’ pyramid with a square base, or portions of them. The half-face, rotated 180° , can represent half of a pointed lunette whose plan is an isosceles rectangle triangle (fig. 12). This example is consistent with the reference to the Gothic vaults in Proposition 15 and in the *Architettura Civile* where a paragraph is devoted to them, and illustrated in plate XIX.

Conclusions

The parallel reading of the *Architettura Civile* and the *Modo di misurare le fabbriche* about the theme of the lunette vaults is testimony of a production cycle of vaulted systems that goes from the ideation, to the construction, the calculation, and the verification of the quantities of materials during construction.

In the two treatises there is a further confirmation of Guarini’s vision according to which architecture is dependent to mathematics and therefore must be traced back to its geometric matrices [Tavassi La Greca 1968, pp. 452-453; Vagnetti 1970, p. 509].

The lunette vaults appear in the *Architettura Civile* as the last element of the vocabulary of shapes established by

Fig. 8. Lunette vault on a ovate plan. Image: Guarini 1737, Plate XX; digital modelling: Roberta Spallone.

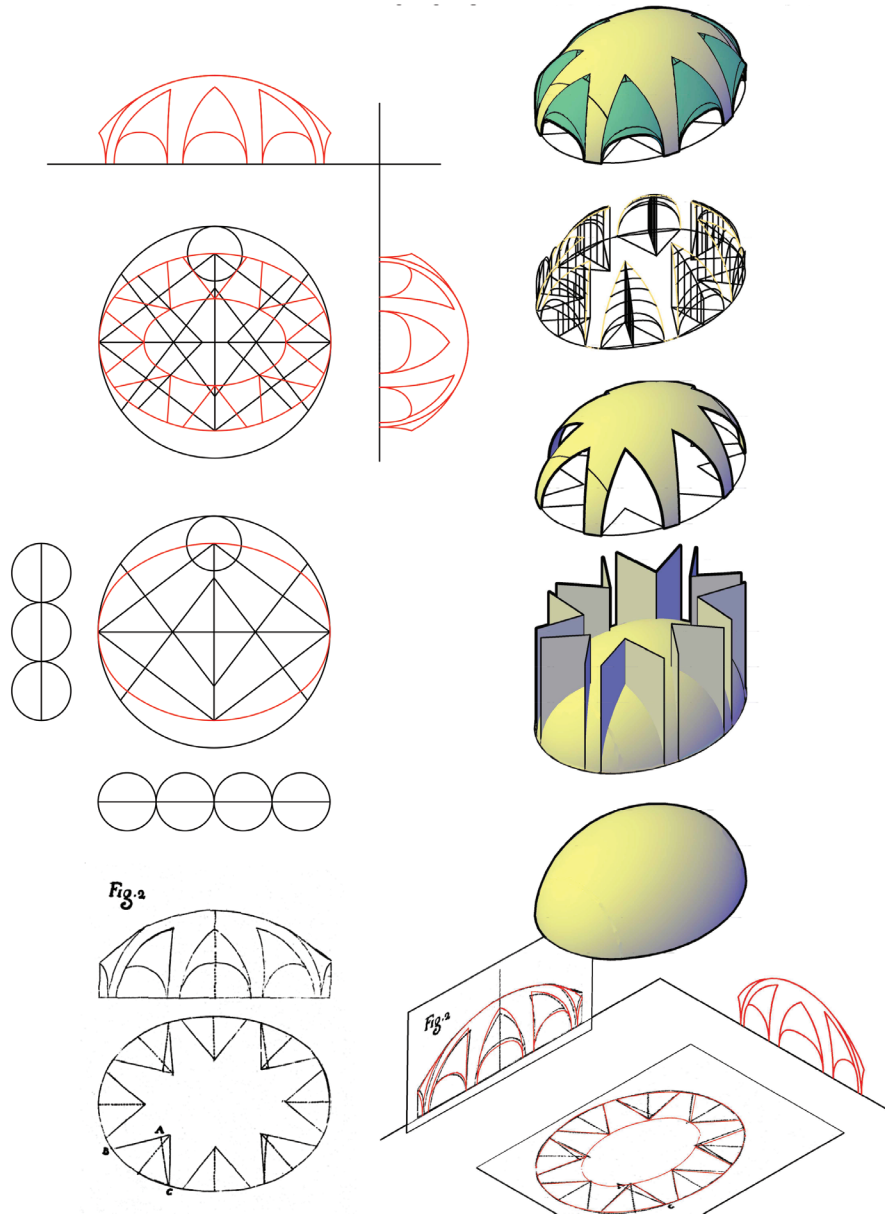
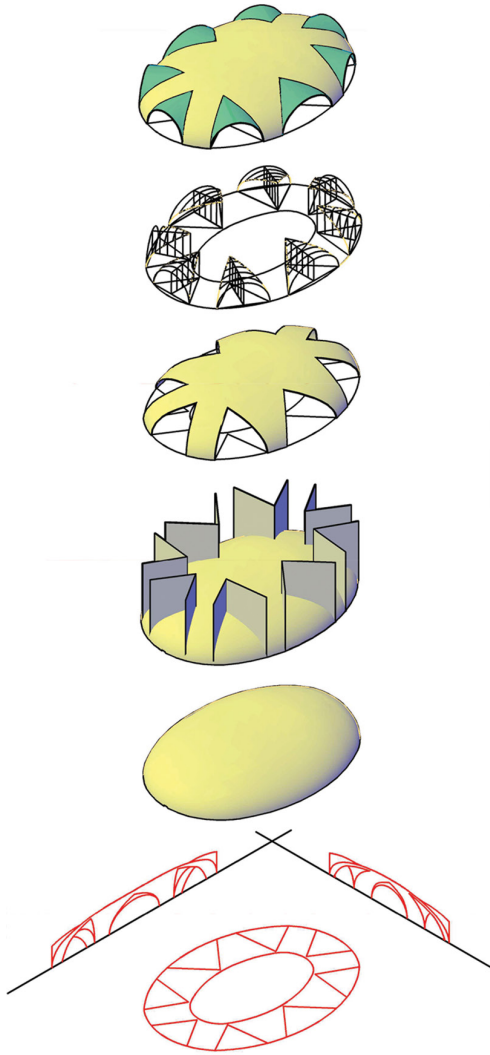


Fig. 9. Vault in the atrium of Palazzo Carignano. Digital modelling: Gabriele Piazza and Lorenzo Valenzisi. Photo: Marco Vitali.



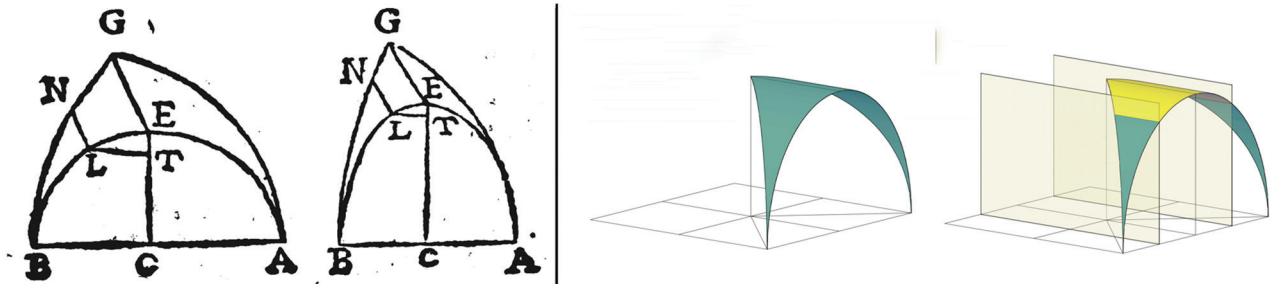


Fig. 10. Elliptic lunette and its portions. Image: Guarini 1674, Prop. 23; digital modelling: Roberta Spallone.



Fig. 11. Elliptic lunettes in the vaults of the atrium and vestibule of Palazzo Provana di Collegno. Photo: Marco Vitali.

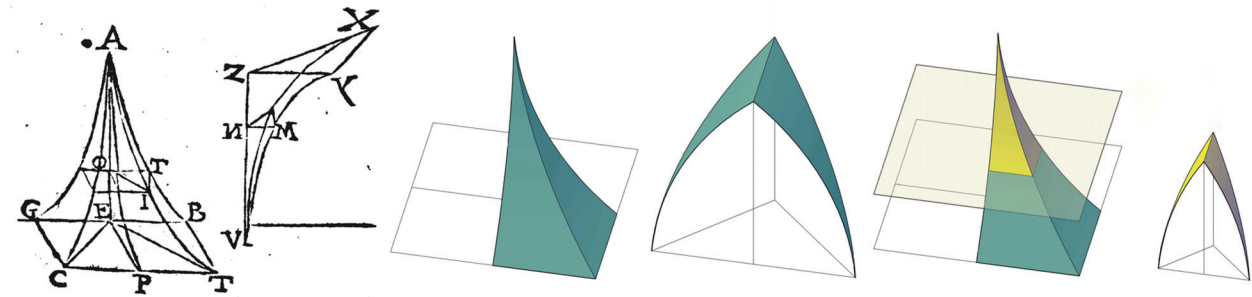


Fig. 12. Pointed lunette obtained from a 'concave' pyramid and its portions. Image: Guarini 1674, Prop. 24; digital modelling: Roberta Spallone.

Guarini for the solution of the issue of the vaulted coverage for the rooms in civil buildings. The combinability of the lunettes with simple and composed vaults of which he first treated, makes them a versatile element for the composition of multiple shapes, as Guarini demonstrates in the immediately following passage.

In the *Modo di misurare le fabbriche*, the described shapes present some novelties with respect to the *Architettura Civile*: here the lunettes, all having a triangular projection, have directrices that extend to the arc of circle, ellipse and arc of ellipse, pointed arc, and its portion. The semi-elliptical lunettes are frequent in the vaults of large rooms between 17th and 18th centuries, allowing the creation of radiocen-

tric or axial systems permeable to light even in presence of reduced rises. The lunettes having as a directrix round or elliptic arcs are widely used in the service rooms of reduced height.

The three-dimensional models allow not only to verify the geometrical and spatial consistence of Guarini's diagrams and drawings and the degree of precision of his calculation methods, but above all, to compare design tools and processes based on geometry, and enrich the discourse on lunette vaults with new examples. Therefore, the present research can constitute a new knowledge basis for the investigation on the Guarinian vaults and, in general, on the vaults in the Baroque civil architecture.

Author

Roberta Spallone, Department of Architecture and Design, Politecnico di Torino, roberta.spallone@polito.it

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