A Codification of the Cubic Projection to Generate Immersive Models

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Abstract

The results of this research aim to describe the representation system made with immersive visualizations. The main goal is to transfer techniques that characterizes immersive models from raster images to models generated with vector images. This way is passed over the boundary that supposes the simple substitution of the elementary geometrical entity, point (or pixel), with the line. In order to do it, the transformations from bi-dimensional views to the immersive panoramic will be codified. It is proposed an advancement for the state of the art, connecting the studies with the descriptive geometry field, opening the cone of vision to the whole space that surrounds the observer. The work considers, as a departure point, previous studies about hybrid immersive models using both equirectangular and cubical projections. But, while for the equirectangular projection there is a complete bibliography for both intuitive (trail-error) and mathematical methods, for cubical projection instead, the procedures are just approximated, without a necessary theoretical and complete framework. The used method previews, as a mandatory passage, the development of geometrical formulations that will constitute the base for the mathematical ones. Announcing future steps, the content here presented in graphical terms, will look to be synthesized in an algorithm with digital graphics output, written ad-hoc, looking to propose a new advanced technique of representation.

Keywords: cubical projection, cubemap, equirectangular projection, advanced representation techniques, VR panoramas.

Introduction: immersiveness and immersive model

"*Inmersividad, proviene del lat. immersus, part. pas. de immergĕre, sumergir. Hablar de nuevas tecnologías nos lleva al concepto de inmersivo, de inmersión y de proyectos inmersivos. El criterio utilizado es estar sumergido, o contenido por algo. Lo que nos hace suponer-imaginar que al estar sumergido o inmerso hablamos de un medio ambiente, de un macro ambiente, de una escenografía, de una construcción, o una arquitectura variable*" [Lolas 2014].
An immersive model is a set of communicative elements (for example graphics, texts, sounds, videos), connected to a virtual environment. It will be this environment to allow the interactivity of the user with the content.
Environment and content then can be connected in different ways: for instance, the content overlaps a virtual environment (Virtual Reality) or the content overlaps a part real and part virtual environment (Mixed Reality). There would be a third possibility where the content overlaps just a material (real) environment (Augmented Reality) but this possibility will be excluded from this article as our focus is precisely on the environment creation. As, in this case, the generation of the environment is not object of representation, the problem is reduced in a technical difficulty (the execution) and not to a theoretical question.
In *La geometria descrittiva: evoluzione di una teoria* is commented "*Ci si accorge, allora, che la rappresentazione matematica con le sue caratteristiche di continuità e accurato controllo metrico è affine al metodo di Monge,*

mentre la rappresentazione numerica, discontinua e approssimata, ma agile nella visualizzazione foto realistica, è affine alla prospettiva" [Migliari 2012, p. 26]. Based on this, as we are focused on the representation regarding the environment itself, it is possible to study the sensitive perception produced by shapes in the space (as extension of classic perspective) or the mathematical model for its definition (as extension of Monge system). Thus, two clear objectives are defined: first, how to represent basic shapes in an immersive model, and second, the relation from the perceived image with its constructive definitions. Understand these two concepts results fundamental to comprehend the space when, for example, the immersive model is applied to architecture. This space will be defined with a extension of classical perspective and its methods, enlarging the technique and finding a fluid way to read objects in relationship with the architectural environment and this with the urban context.

As a consequence of a growth use of immersive techniques, must be pointed how important is to regulate the process to warranty a scientific procedure. It is in fact enough to see the exponential increasing of use and applications based on panoramic photography. On another hand, the construction of a generic environment involves both architects and engineers mainly because: first, the conception and materialization of the space constitutes their raw material; and second, because the state of art shows the autonomous existence of this representation system which lacks scientific discipline almost completely, being this slot where the article takes place.

The definition of the term 'virtual' here used, goes further the usual connection with the tools of execution of the drawings. It is proposed widely, in the terms expressed by Vito Cardone in his *Modelli grafici dell'architettura e del territorio*, that is "*con l'aggettivo inteso come ciò che è latente, ma soprattutto come ciò che possiede una potenzialità; ossia – secondo la categoria aristotelica – come ciò che è fondamentalmente potenziale inteso*" [Cardone 2015, p. 342].

We extend the definition to survey models (reconstructions) or models destined to architectural planning. As we will see, this opening will not link us to the used tool but to the communicational coherence of the representation, or, to the thought process for the construction of the scene.

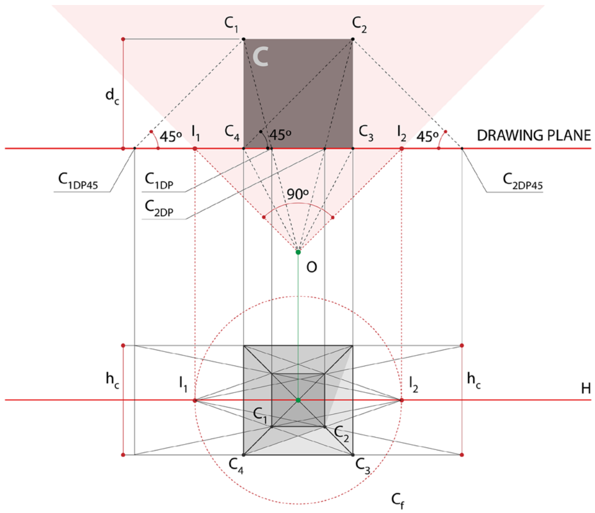

Fig. 1. Conical perspective solved using the 45° diagonals to find C_1 and C_2 (graphic elaboration by Lucas Fabián Olivero).

The context

The first steps of the current immersive models find origin in entire environments that we could refer as 'immersive installations'. Some examples are some *trompe l'œil*, the Sistine Chapel, the compositions of Andrea Pozzo, in St. Ignatius of Loyola at Rome etc.

In these cases, the room is a real space and the representation is based on one specific point of view. Following that point, the spectator can dive into epic scenes and study the symbolic content.

From here, maybe the first systematic attempts to create virtual environments with entertainment purposes were between the 18th and 19nd Century.

Then 360° painted scenes for the panoramic rotunda of Robert Baker and multiple projections of the Cinerama [Rossi 2018, pp. 1389, 1390; Cabezos Bernal, Cisneros Vivó, Soler Sanz 2014, p. 5] brought to the public the access to remote places or historic moments, such as the ruins of Pompeii or the Battle of Waterloo.

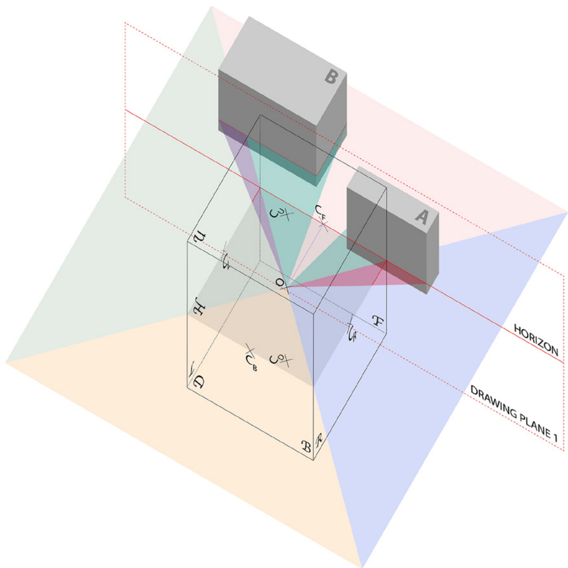

Fig. 2. Generic scene to be represented in cubical projection (graphic elaboration by Lucas Fabián Olivero).

Up to this point, an entire huge mechanism serves to create the illusion of the audience. Sometimes this device even tried to 'confuse' spectators passing them through installations designed for that purpose: "The building was designed to disorient people as they passed from the actual to the virtual world. Spectators had to walk down a long dark hallway and climb shadowy stairs before emerging onto the viewing platform" [Shannon 2016].
Finally, one nearest link is the panoramic photography, evolution of the first panoramic drawings. Nowadays, thanks to the Internet it is very easy to find whole or partial immersive panoramas from elsewhere. In fact, digital architectural navigation to countless places and urban exploration had become a daily habit thanks to billions of people that have a photographic machine in their mobile phones and the use of many applications able to capture single images and then compose the entire panorama through an instant stitching such as Google Street View.

Today, immersive models are used mainly to disseminate information but at the same time, it can be seen a strong growth for technical uses. For example, to visualize architectural and urban projects or as a basis for photogrammetric modelling [Barazzetti, Previtali, Roncoroni 2017].

Anamorphosis

At the base of all these productions and as a common denominator, we find anamorphoses. An anamorphosis is a way to reduce dimensions in the most literal sense of the word. For example, a real object that exists in three dimensions is reduced in one dimension when it is drawn in the plane, however and as is logical, the object and its representation are equivalents in meaning, but not in matter.
In order to appreciate the 'natural form' of an anamorphosis, there are two conditions: first, the observer must watch the scene from the exact point where the rays of the conic projection converge.
If not, the representation will look as 'deformed'. Indeed, Andrea Pozzo marked those points in the above-mentioned St. Ignatius for the best appreciation of the scenes in the main nave and in the false dome. Nevertheless, when the same representation is watched outside of the focus point, is not entirely correct to affirm that is deformed (as a synonym of erroneous, poorly performed). The representation is just correct for a single spatial point and for a specific field of view, such as pointed by the mathematician António Araújo in *Anamorphosis: Optical games with Perspective's Playful Parent* [Araújo 2017, pp. 73, 74].
There is also a condition to produce an anamorphosis: to have the knowledge of the surface where the rays are being projected. The distribution of the spatial points in the final representation will depend on the shape of that surface. As that surface must be flattened, without its knowledge is impossible to determinate the group of possible developments in the plane.
In particular, classical perspective itself is a special case of anamorphosis. Indeed, the surface is known (a plane) and the group of projected rays is reduced to those contained in the cone with a field of view of 90°. The result of following these two conventions, is a type of representation where the deformed outline of the objects can be overlapped to the reality, fooling our perception and giving the

sensation of something truth. In fact, "Perspective had been developed as a method representing space and spatial objects according to our seeing" [Leopold 2016, p. 409]. Thus, it has been popularized by its character by common agreement either for its reading or for its elaboration.
In the case of our immersive models, the used surface is no longer a plane but a spatial container. The surface now surrounds the observer, closes around it and defines a specific projection related to its intrinsic characteristics. The next step is defined as the flattening of the surface and the codification of the previously mentioned possible line developments. So, for example, the equirectangular projection is the flattened map of the projected points and lines in a sphere, while the cubical projection (or cubemap) is the resulting flattening when the used projection surface is a cube.

Digital and hybrid immersive model

During the history, the used tools to do a representation have switched from analogical to digital, giving as a result nowadays a big range of expressive possibilities. The choice can be done according to the technical-communicative needs defined by the object of the representation itself. In particular, the digital technology, that has started with the imitation of analogical techniques, has gradually become an independent language of representation.
The immersive models referred in this article are, in their final version, digital, or better said, assembled and visualized with digital technology. Effectively, in order to have the most complete fruition and interactivity degree, the content additions and the final visualization are carried out with computer support. Maybe the best-known example of a digital immersive model is the virtual tour where the used environments can be from 360° panoramic photographs to environments entirely created with modelling programs.
Passing over the rusty debate about the primacy of digital or analogical techniques, it is proposed a hybridization of digital models with analogical drawings. In this way, it is possible to recover some of the innumerable advantages that both techniques can offer separately strengthening mutually. We will call this Hybrid Immersive Model. The hybrid immersive model is generated with analogical techniques and mounted, navigated, enhanced and visualized with digital ones.

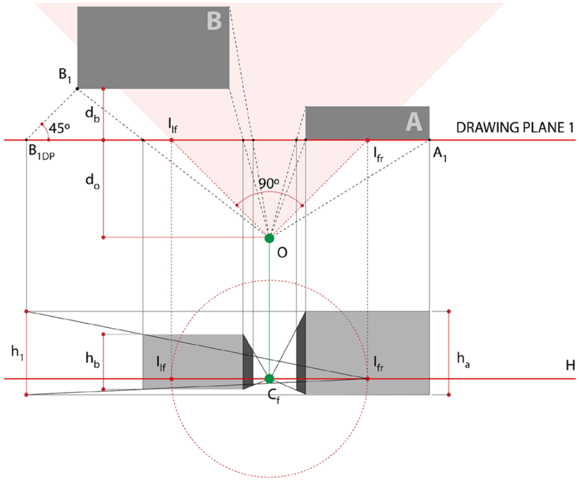

Fig. 3. Construction of the classical conical perspective of the scene (graphic elaboration by Lucas Fabián Olivero).

There are different kind of projections used in computer sciences that allow to synthesize the entire panorama around the observer. Most of them represent at least six vanishing points according with the x, y, z axes of the 3D euclidean space, however, this work will just refer the equirectangular and the cubical projections.

The state of the art

It has been studied a varied casuistry of immersive artworks cases. The presented selection bases on two criteria: first, the chosen surface for the creation of the virtual environment, and second, in the used drawing method.
For the first case, the chosen examples reach the two most used surfaces for VR navigation: the sphere and the cube. In particular, they refer the equirectangular projection (among the various spherical projections) and the cubical projection respectively.
For the second criteria, are referred two main methods used indistinctly for each projection: the intuitive (or trial-and-error) and the geometric / analytical / mathematical. The intuitive method considers the use of preformatted grids. Following the template, the users can 'create' immer-

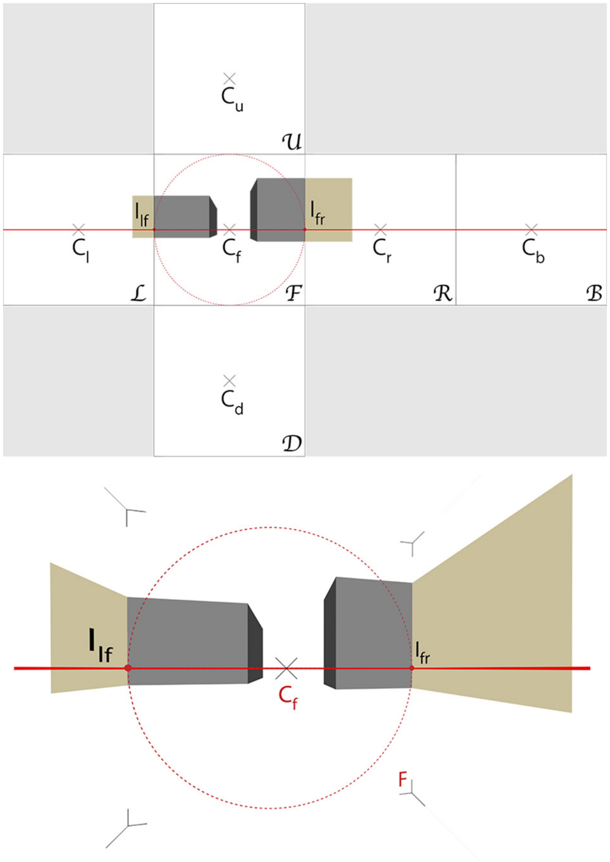

Fig. 4. The composed perspective placed into the cubemap and its immersive visualization (graphic elaboration by Lucas Fabián Olivero).

sive perspectives, although they are not necessarily really conscious of what are they doing. In fact, the depth effect of perspective can be also the result of guessing with approximative trial-and-error proves.
A geometric / analytical / mathematical method instead, aims to spread a deep knowledge: the reasoning to understand the whole projection in the space. This article is part of this second group.
Crossing the criteria, for spherical projections using intuitive methods, it is possible to find an increasing number of architects and artists using grids, such as Arno Hartmann (Germany), Sandnes Frode Eika (Norway) and Bruno Sucurado (Argentina). Here, the grid has been used either to produce imaginary virtual environments (project) or to re-produce synthesizing real environments (survey). Another use given to these templates is as a base for 360° raster drawing software, such as *Sketch 360* [1].
For spherical projections using geometrical-mathematical methods, works such as *Drawing Equirectangular VR Panoramas with Ruler, Compass, and Protractor, Constructing a Total Spherical Perspective* [Araújo 2018a; 2018b] and *La prospettiva e la costruzione dello spazio figurativo* [Masetti 2014], evolve or complement previous works such as *L'œil, au centre de la sphere visuelle* [Michel 2013] and *La perspective curviligne: de l'espace visuel à l'image construite* [Barre, Flocon, Bouligand 1967]. All these studies aim to the analytical development of perspective using spherical projections. The methods cover from partial field of views up to the whole 360x360°. In any case, there is a common *modus* breaking down of the whole system into the simple construction of partial elements like points, lines and planes. Only then is given a method that includes an integral solution. The result is an exhaustive base material in scientific terms. In particular, the intention to solve these systems is pointed to the use of simple instruments such as the ruler and compass or, as Migliari refers with the use of the classic geometry "*la geometria classica che impiega esclusivamente la retta e il cerchio*" [Migliari 2012, p. 27].
For cubical projection using intuitive methods, there are many blog entries as tutorials, such as *4 Steps to Create a 360 VR Illustration/Painting in Photoshop* of the Studio Behind 90 office [2] or *Draw Sketches for Virtual Reality Like a Pro* [Kurbatov 2017]. These publications try to solve the 'how-to' without mention the projection itself and its characteristics. In some cases, the problem is partially solved converting the equirectangular grid into the cubemap and then reverting to the equirectangular for-

mat again. There are more automated procedures also such as the Oniride plugin [3] created for Photoshop 2015.5. Furthermore, as described in the *How to create 360 Virtual Reality (VR) Illustration with Adobe Photoshop CC* from the previously mentioned Studio Behind 90, Adobe Photoshop includes by default a tool to switch between the equirectangular projection and a flattened vision. Finally, there is the Sketching in Space project [Habakuk, Zöllner, Müller 2010] that uses an interactive installation based on a cube and wireless tools designed ad-hoc.

For the cubical projection using geometrical-mathematical methods, the research gives a partial result with raster-based solutions, leaving a gap of disciplinary advancement for a vector-parametric answer. There are also methods that could be adapted to give partial solutions since the cube is used as the basic reference to draw spatial forms but always giving just incomplete solutions. Some examples are the *Prisma T-homólogo de un Cubo* method used in the *Homologías entre figuras de tercera categoría* [Fernández Rodríguez 2002, p. 2]; or the drawing of the cube in perspective that makes an "*aplicación de retícula con figuras en paredes laterales y localización en su interior de dos estructuras poliédricas valiéndose de la retícula de las paredes y el suelo*" [Hernández Falagán, Signes Orovay, Berdié Soriano 2015, pp. 10-12]. All these knowledges, undoubtedly valuable as a starting point, do not give a fully, systematized, organic and elegant solution to the immersive creation with cubical projection. In order to do it, a solution that includes the classification of lines and the elaboration of grids must be proposed as, for example, the complete immersive drawing method based on geodesics as used for the equirectangular projection [Araújo 2018a, p. 17].

Even if the current developments for cubical perspective are not exhaustive to solve the problem, as indicated in the first part of this work [Rossi & Olivero 2018, p. 36], a close approach is the comparative study of both surfaces in computer science for environmental mapping, such as Isocube: Exploiting the Cubemap Hardware [Wong, Wan, Leung 2007], Converting to/from cubemaps [Bourke 2016] or Environment Mapping and Other Applications of World Projections [Greene 1986]. However, as could

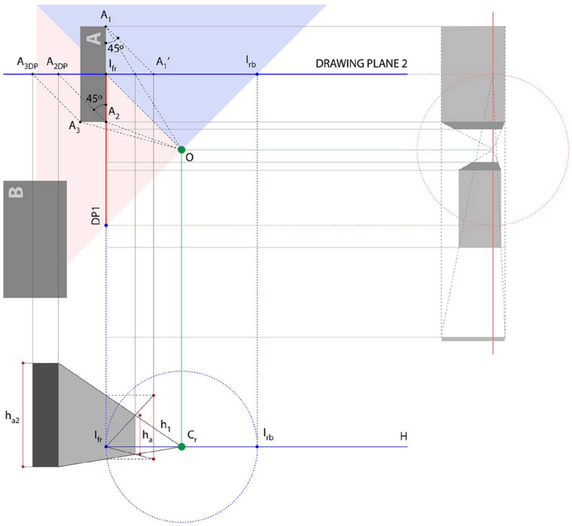

Fig. 5. Construction of the conical perspective rotating the observer 90° to its right (graphic elaboration by Lucas Fabián Olivero).

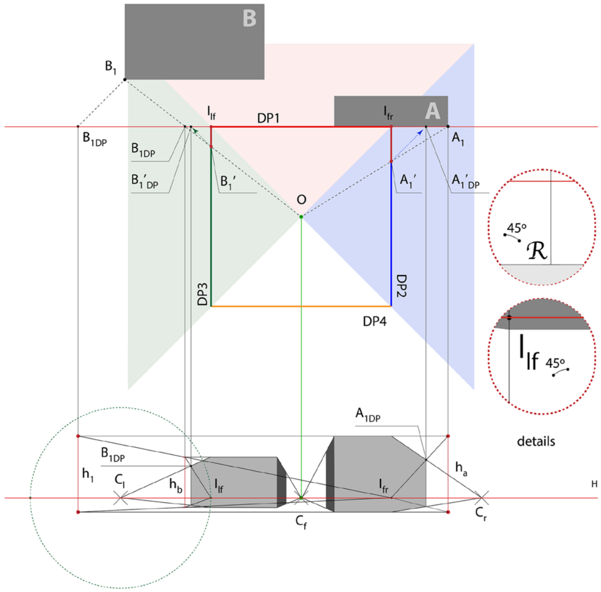

Fig. 6. Final composition of the scene solved with the use of conical projection (graphic elaboration by Lucas Fabián Olivero).

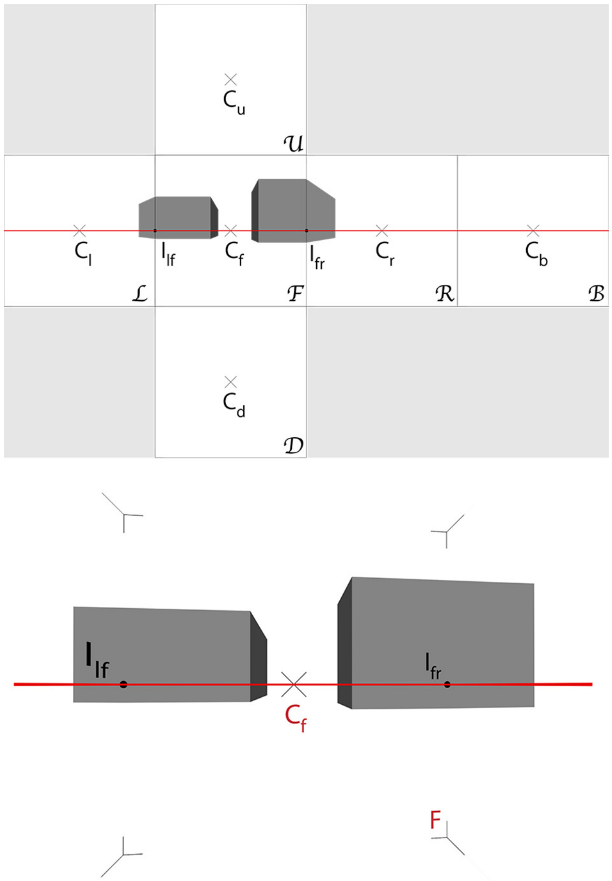

Fig. 7. Final construction placed into the cubemap and its immersive visualization (graphic elaboration by Lucas Fabián Olivero).

be natural for this area, the study is focused on the performance and efficiency in terms of resources for rendering, evaluating, for example, graphical benefits such as reflections and shadows for 3D modelling. In terms of texture, for example, each pixel of the sphere (inscribed or circumscribed in a cube), finds its correlation with a pixel of the texture of the cube. The result is just validated by a correct visual appreciation without a metric parameter.
As a result, the content collected for the cubic projection using geometric / mathematical methods does not complete a holistic solution. The empirical methods instead, manage to cover well the how-to procedure but leaving out of focus the scientific precision. So the current state of art shows a big presence of the trial and error method and as a consequence, the obtained result can be only for exhibition purposes mainly, which has, at least, a doubtful utility for scientific purposes.
This characteristic seems to be inherited from the historic conception and use of immersion. In fact, in those cases the goal was to increase the emotional impact of the users as pointed in Virtual Art: "Immersion can be an intellectually stimulating process; however […] in most cases immersion is […] a passage from one mental state to another. It is characterized by diminishing critical distance to what is shown and increasing emotional involvement in what is happening" [Grau 2003, p. 13]. Therefore, one risk is the propagation of a not normalized representation that could lead to a black box use [Araújo 2018a, p. 16]. As a result, those who make use of this representation system may not really possess the knowledge of what they are doing, rather they may be are mere virtuosos in the use of some software or tool. Nevertheless, the whole state of art testifies the growing interest in those techniques that join unify the analogical drawing with the VR technology.

Basic descriptive geometry in cubical projection

Hereinafter, a generic example of representation in cubical projection using a geometrical / mathematical method is presented. At the base of all the reasoning there is classical descriptive geometry. The objective is to find the correct projections for a projection that will result fragmented since it exceeds the normal 90° vision cone that each cube face contains. In order to clarify some concepts that will be used next, a conical perspective example with a central vanishing point is recall (fig. 1).

The used method suggests to find the correspondent position of each point in the perspective (bottom part of the figure) by defining rays from C_1 to C_4 to the observer O and intersecting the drawing plane (DP). In particular, C_3 and C_4 result in real dimension in H because they are in contact with DP.
The problem is how to find the depth of elements that are not in contact with DP such as C_1 and C_2.
In a first method, C_{1DP} and C_{2DP} are going to be used, which are the intersections of rays C_1O and C_2O with DP. Then C_3 and C_4 are projected toward the central point. The projections of C_1 and C_2 are in the intersection with the extensions of C_{1DP} and C_{2DP}.
Furthermore, there is also another possibility which suggests to project the searched point with an angle of 45° towards the drawing plane. What are we doing actually is to use known vanishing points (those corresponding to 45° lines). In the specific case of C_2, the diagonals intersect DP in C_{2DP45} and C_4. Applying the reasoning previously explained, the real height of the object h_c is positioned in H in the extension of C_{2DP45} or C_4. From there, C_{2DP45} is projected to the vanishing point I_1, or to I_2 if C_4 is being used. Any of the two diagonals give the same result, which can be graphically verified also for C_1.

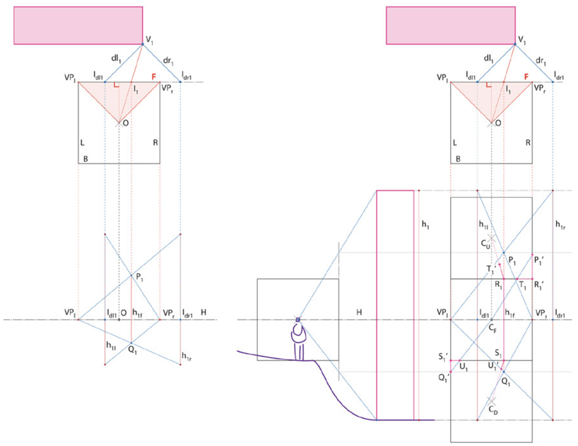

Fig. 8. *General layout of the complete algorithm for horizontal and vertical lines parallel to the faces of the cube (graphic elaboration by Lucas Fabián Olivero).*

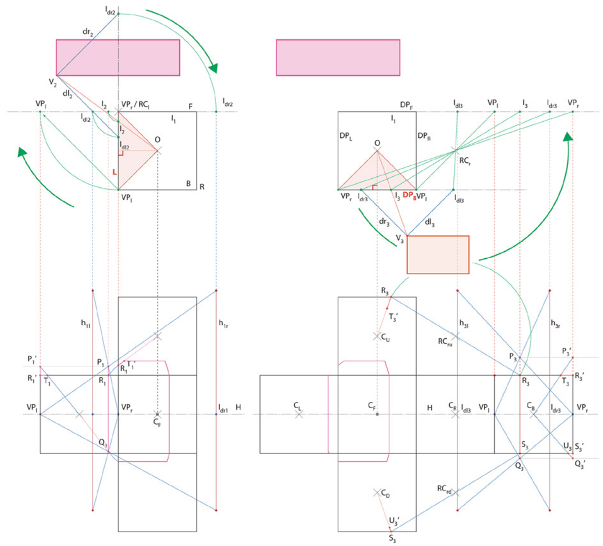

Fig. 9. Application of the algorithm to a practical exercise. Resolution of a line contained in one face (left) and of a line divided in three segments (right) (graphic elaboration by Lucas Fabián Olivero).

In conclusion, any of the three ways must verify that the projection of C_2 is at the intersection with C_{2DP}.
Let us now present the generic scene composed for two buildings A and B (fig. 2) to be represented in cubical projection. The perspective is a composition that uses one central vanishing point called C_f (fig. 3). Both buildings have the same height h_1. The facade of building A is parallel and coincides with DP1. Building B is also parallel to DP1 but it is located at a certain distance d_b. A generic observer O is defined located at a distance d from DP1 and perpendicular to it.
Now to construct the perspective, every vertex of buildings A and B are extended to O. Any intersections with DP1 is, at its time, extended to the bottom part until the horizon line H. Since A_1 is in contact with DP1, it results that $h_1 = h_a$. Then h_a can be translated to H in real dimension and positioned directly according to its relationship with the observer (or H). To find h_b instead, the previous method with the diagonals will be follow projecting, for example, B_1 at 45° towards DP1. As a result of this construction, the complete scene is obtained with zones that exceed a 90° field of view.

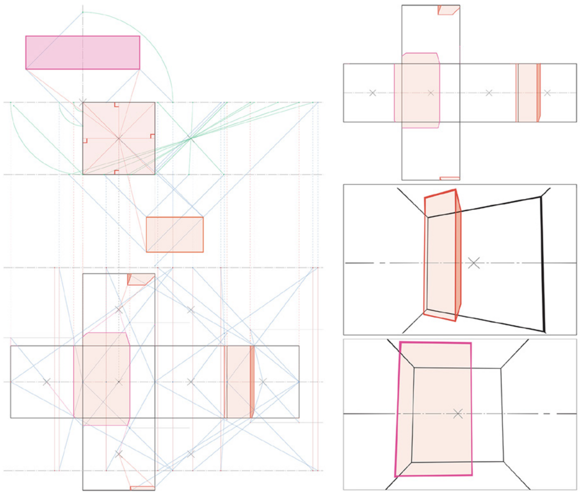

Fig. 10. Application of the algorithm to a vertex positioned in planes different than the frontal one: projection in L and R (left), projection in B (right) (graphic elaboration by Lucas Fabián Olivero).

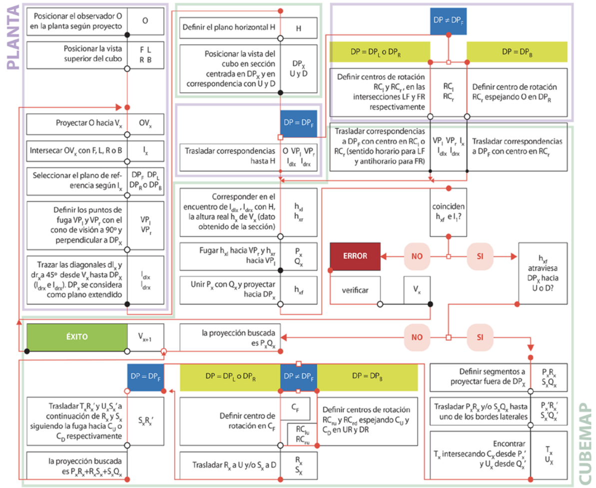

Fig. 11. Complete resolution of a practical exercise applying the algorithm (left). Final cubemap and VR visualization (right) (graphic elaboration by Lucas Fabián Olivero).

Next, the recently created scene is positioned in the open cube map and then proceeded to the immersive navigation. As each face of the cubemap contains a field of view of 90°, the outside zones are incorrectly visualized (fig. 4). Nevertheless, if the observer turns to its right, it is possible to reconstruct building A which is actually the only one in its visual field (fig. 5). In this case is used drawing plane DP2, perpendicular to DP1 and the vanishing point Cr.
The part ahead DP2 is solved in the same way at the beginning of this section, that is translating A_2 and A_3 with an angle of 45° to DP2. The real height of the building is in correspondence with point I_{fr} (intersection of A with DP2). Using I_{fr} and projecting from C_r the searched heights are obtained in correspondence with A_{2DP} and A_{3DP}.
Placing this last content on the right face R of the cubemap centred on C_r the immersive navigation results in a correct visualization (fig. 6). Thus, the use of a drawing plane orthogonal to the first scenario gives as a result the correct anamorphosis.
In order to complete the scene, it is now proposed a direct method. Coming back to the initial scene, we want to find the intersections without the cumbersome need to rotate every 90°. To this end, the projections in the four planes of drawing DP1 to DP4 located around the observer are studied (fig. 7).
A_1 is used to verify the new method with the already elaborated perspective. After established new drawing planes also new intersections have appeared, and therefore new projections. In the intersection of ray OA_1 with DP2 has now appeared A_1'. This point is in front of DP1, so, to find its correspondence is translated 45° towards the extension of DP1, which gives $A_1'_{DP}$. In perspective, $A_1'_{DP}$ effectively matches the intersection already built during the previous steps. Iterating the process for B_1 the effective end of building B is also found. Now joining all the constructions in the cubemap and using the immersive modality, the correct composition of the anamorphosis can be verified (fig. 8).
As last step, once a complete drawing has been done, digitalized and cut with the right proportions, it will be necessary to add some metadata trough digitally manipulation and do certain mandatory passages required to assemble the model using software such as Hugin or PanotourPro to recompose the cube. A first work-flow guideline for this part can be found in *"CubeME", a variation for an immaterial rebuilding* [Barba, Rossi, Olivero 2018].

The complete algorithm

"*A partir del estudio geométrico y algebraico de la homología entre figuras de tercera categoría podemos proporcionar a la Informática una serie de algoritmos que nos permitan explotar las posibilidades de esta transformación geométrica*" [Fernández Rodríguez 2002, p. 1]. As corollary of the applications, a first complete algorithm for drawing on the six faces of the cube is presented (fig. 8). The general workflow is defined through the use of the already presented 45° diagonals and the searched height is found in their encounter. Once got it the height, a double verification is made using the correspondence given by ray OP_x (fig. 9). As practical application is solved an example composed by two buildings with the same height located in the front and back of the observer (fig. 10) where is also includes the solution to represent fragmented heights, that is, contained in more than one face of the cube (figs. 9, 10). The correctness of the whole construction is verified with the proper anamorphosis in the immersive visualization (fig. 11).
Is still missing the resolution of lines or planes that do not have their vanishing points at the centres of the faces (or, lines that are neither horizontal nor verticals, parallels to the edges of the cube). It is also announced that this total resolution will be the object of future publications.

Conclusions

Cubical projection is presented as a complex representation that nourishes from the concepts of the classical perspective as a starting point. This new way of adaptation enhances undoubtedly immersive graphics. However, is still lacking to develop a complete classification of lines, the resulting projections from the intersection of planes with the cube, as well as a method to locate points from live survey.
Some innovative aspects of this kind of representation are synthesized in: first, the system seeks to enhance, define and organize in a technical way a geometrically defined immersive hybrid model, exclusive potential for the moment of the digital modeling. In fact, thanks to the scientific use of technology, the usefulness of immersive installations has been extended and made more complex, with the fundamental difference (regarding the historical panorama) that the user can interact and add content in real time.
Second, being at the base of these models the analogical drawing, the system constitutes an instrument to understand and manipulate the space. Can be seen that this upgrade of the traditional methods supports an extended application. In fact, since the support is not limited to 90º of visual field but open to the entire surrounding vision, can be meekly studied the relation of the object and its spatial insertion and the building in its urban context. The produced environment also gives (and at the same time) a base to upload interactive contents with the possibility of visualization in real scale (thanks to the use of VR glasses).
Third, although the huge visual field covered, thanks to the use of anamorphosis and digital technology, no bulky support is needed. In effect, technology comes to complete a universal and 'pocket' access through mobile devices and the use of Internet.
Finally, being the whole process in direct correlation with the technical definition of anamorphosis, perspective and descriptive geometry, as well as mathematics and computer sciences; we have a package of resources more than enough to follow the path of a possible and innovative system of representation.

Acknowledgments

With special and deeply dedication to Vito Cardone.

Notes

[1] See <https://www.microsoft.com/en-us/p/sketch-360/9p89s2qlh11t> (accessed 2018, February 17).

[2] See <https://www.studiobehind90.com> (accessed 2018, February 17).

[3] See <https://www.oniride.com/360art> (accessed 2018, February 17).

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