

# Perspective Trials in the Manipulation of Space. The Bramante's Fake Choir of *Santa Maria presso San Satiro* in Milan

Giorgio Buratti, Giampiero Mele, Michela Rossi

## Abstract

The Milanese church *Santa Maria presso San Satiro* documents an experimental integration between perspective and architecture in which the accelerated perspective of the choir simulates a distance comparable to that in the transept. The effectiveness of perspective deception demonstrates the quality of local perspective science and justifies the perspective variables examination, comparing the previous hypotheses in light of a proper survey of choir inserted in the real architecture. The research is based on the interior space three-dimensional reconstruction of the virtual architecture designed by Bramante. Starting from a survey campaign, which integrated direct methodologies with laser scanning, it was identified the privileged point of view, reconstructed through the illusory space digital simulation of Bramante's architecture. The mounting of 24 point clouds and the subsequent transformation in mesh surfaces allowed us to verify the construction accuracy and to individuate some unknown anomalies hidden by the ornamental apparatus which make difficult to set the point of view in a unambiguous location. The virtual reconstruction highlights a theoretical point of view, different from those already hypothesized and explains the design that identifies the solid perspective of *San Satiro* as the first model of modern theatrical scenography.

Keywords: Bramante, Fake Choir, Solid Perspective, Architectural Perspective, Illusory Space.

## Introduction

The church of *Santa Maria presso San Satiro* in Milan, attributed to Donato Bramante on a documentary basis, is famous for the solid perspective of its choir. It evokes a depth that is comparable to the transept with the aim to simulate a Latin cross space in the first, exceptional example of integration between perspective and architecture. Bramante, already dubbed '*il Prospettivo*', would cement his fame with this wonderful spatial artifice, turning the perspective from a rational space representation into a perceptive deception. A wooden and stucco perspective apparatus, embedded in the back wall of the apse, transforms a small space into the illusion of a choir with a coffered barrel vault that extends the pictorial decoration of the nave. The Tau cross plan becomes a Latin cross as

a prelude to the central plan. This application of the perspective with a new intent, which was extraneous to its primitive concept of mathematical 'measurement', opened the way to further applications in theatre and to the *quadratura* interior decoration.

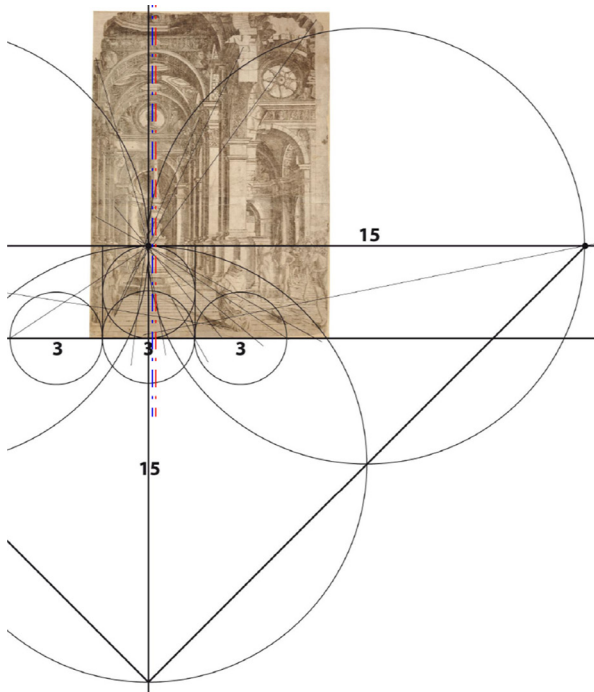
Like many other Maria's churches of the Renaissance, mostly with a central plan, the construction is linked to a prodigious event [Lotz 1955], which took place near the shrine of San Satiro, Sant'Ambrogio's brother. The original building was erected before 879 by Ansperto, archbishop of Milan [Buratti 1992, pp. 9-24] and according to the tradition, in 1442 the image of the Madonna was profaned by a stab and bled. The place became a pilgrimage destination, then Ludovico Sforza approved the construction of a new

church, which was built between 1478 and 1482 [Dalai Emiliani 1980].

Due to lack of space, Bramante created the choir in a solid perspective. The image of the apse recalls the Masaccio's Trinity with an unprecedented interpretation that was named 'admirable artifice' by his contemporaries. He applied the Donatello's stacciato in a full scale, visualizing a different space from the built one. The idea was new and it became a decisive model for the theatre, which was forced to contract the scene in the stage. The baroque scenography will define the theoretical principles later [Baglioni, Salvatore 2017].

This solution prompted many scholars to study the church and its perspective suggesting the point of view in different positions. The first hypothesis is that of Arnaldo Bruschi [Bruschi 1969, pp. 745-750], taken up by Filippo

Fig. 1. Pattern of the Prevedari engraving: the offset in the point of view highlights the depth staggering the elements along the visual axis (graphic elaboration by the authors).



Camerota [Camerota 2006, pp. 247-248], which identifies the height of the point of view at 2.60 m and fixes the distance of the observer in golden ratio with the width of the picture plane, defined by the nave width.

This hypothesis seems to be based on an arbitrary graphical reconstruction of the choir geometry, because it does not correspond to existing surveys that are correct. The second is Eros Robbiani's one [Robbiani 1980], quoted by Rocco Sinisgalli [Sinisgalli 2001, p. 264], which establishes the main point at a height of 2.10 m with the observer on the edge of the first bay, about 13.90 m from the picture plane. Also in this case the published drawing does not overlap the surveys drawings.

The different height of the point of view is surprising. In fact it coincides with the point of concurrence of the straight lines that simulate the depth of the fake choir, which can be easily derived from the restitution of the choir previous surveys [1]. These overlap each other and respect the current survey. No hypothesis followed the geometric survey published by Adele Buratti in 1992, which was focused on the church structure and elevation, more than to the articulation of the choir, only represented in cross-sectional drawing [2]. The other authors who studied the church did not make perspective investigations, with any references to other hypotheses.

Both theories fix the point of view at a higher level than the one described by the treatises, which Alberti measures in 3 Florentine arms, corresponding to about 1.75 m., comparable to eye level of a good size man. Robbiani justifies the excessive height with a detailed analysis of the analogies with the perspective in the Prevedari engraving [3], in which the horizon line is high if compared to the kneeling figure's eye, less than the others (fig. 1). The plate highlights a slight offset of the central vanishing point with respect to the church axis, underlined by a monument under the dome. As a consequence, the main centres of symmetry are not aligned with the perspective, as seems to happen also in the Milanese church because of the axis offset and the choir asymmetry that emerges in the survey.

The two hypotheses seem to be set on arbitrary restitutions, based on a perspective reconstruction that fixed the central point in the middle of the tabernacle because it was flawed by a symbolic prejudice or perhaps by a photograph from a higher point of view. It can be verified that it would raise the point of convergence of the coffers in the fake vault. No authors refers to a measured survey.

The importance of Bramante's example in the science development of perspective justifies therefore a new study. This is eventually based on an accurate architectural survey, adopting tools that are suitable to the object complexity to verify the position of the projection point of the fake choir.

### Architecture, simulation and construction

The research pursued the privileged point of view identification starting from the virtual reconstruction of the consolidated design hypothesis, according to which the choir simulates the transept length. The digital reconstruction of Bramante's illusory space was derived from a reworking of a theoretical model assumed from the instrumental survey with laser technology [4] (fig. 2), as part of a national research project dedicated to architectural perspective [5].

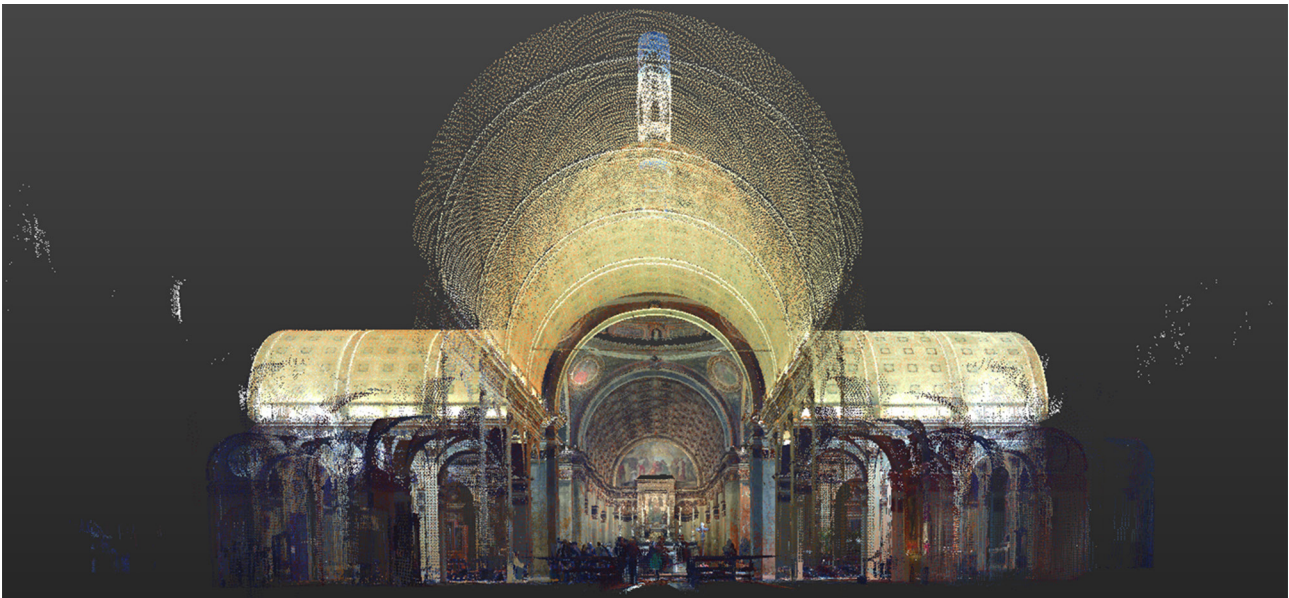
The hypothesis was to reconstruct the virtual apse theoretical model using the scan of a transept arm to collimate the perspective conspicuous points of the choir, and to

determine the point of view on the longitudinal axis identified by the false coffered vaults, looking for references that can be recognized directly on the mesh model.

Three-dimensional scans provide conspicuous and precise data, but point clouds do not constitute a three-dimensional model immediately available to the accelerated perspective geometric analysis, based on the collimation of fake real points of the choir with those of the ideal model. Therefore it was necessary to transform the point cloud into a mesh model [6], whose level of detail was a compromise between computational manageability and the precision required by the research, with a high polygonal density in the choir and less for the rest of the architecture (fig. 3). The superposition of photographic pixels, made available from the same scan, to the reworked mesh was fundamental to identify the conspicuous points to be collimated even where the density was lower.

In restitution it was decided to deprive the surfaces of the decorative paints, obtaining a 'neutral' model that would allow an objective and punctual reading of wall structures plastic articulation, to highlight and to facilitate the study of geometric characteristics and possible anomalies.

Fig. 2. The laser scanning point clouds (elaboration by Compagnia delle Misure).



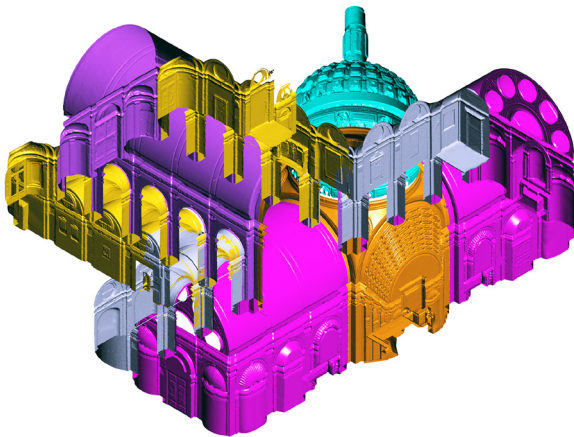
Both the verification of previous hypotheses and the point of view research are based on data provided by an accurate digital survey, integrated by a direct survey of the main elements, capable to offer immediate and precise indications (fig. 4).

The main steps were:

- the integration of 24 points' clouds produced by the scans required for the church interior survey;
- the reworking of mesh model with the stitching of gaps in the shadow areas;
- the restitution of the direct survey and its overlap to horizontal section of the points' cloud with a check for possible model construction 'errors';
- verification of geometric correspondence with respect to the axes for the identification of constructional anomalies:
- identification of collimation points of the solid perspective with the virtual choir;
- identification of the main straight line [7];
- the construction of virtual choir with the length of the transept, according to the theoretical design model;
- the definition of intersection area of the collimation lines;
- verification and comparison with a reference grid based on the construction metric unit.

The direct survey has highlighted the metric regularity in the rhythmic scansion of central nave and transept, with a difference of 2 Milanese feet in their width and the con-

Fig. 3. The mesh model of the church used in the reconstruction of the choir, after optimizing the point clouds (graphic elaboration by the authors).



sequent 'lengthening' of the cross vault, dominated by a dome resting on an oval cornice.

The instrumental survey has confirmed this basic construction precision, but also some important constructional anomalies hidden by the ornamental apparatus:

- the misalignment of the central nave compared to the choir;
- the different misalignment of the cross vault with respect to central nave and the choir due to the transept's slight deviation;
- a marked as much as inadvertent asymmetry of choir and pilasters that define the frame;
- the circular base of the dome, hemispherical, disguised by the oval cornice overhang inserted in the trapezoidal lay-out of the cross vault.

The greater transept width, already distinguishable in the point cloud due to the impossibility of collimating the depth lines of the nave with those of the choir, and the little deviation of its axis with respect to perpendicular to that of the nave are concealed by the different protrusion of the tambour cornice on the two axes and by the raising of one foot of the center point of arc and nave vault to keep the *cervello* element at the same height (fig. 5, fig. 6).

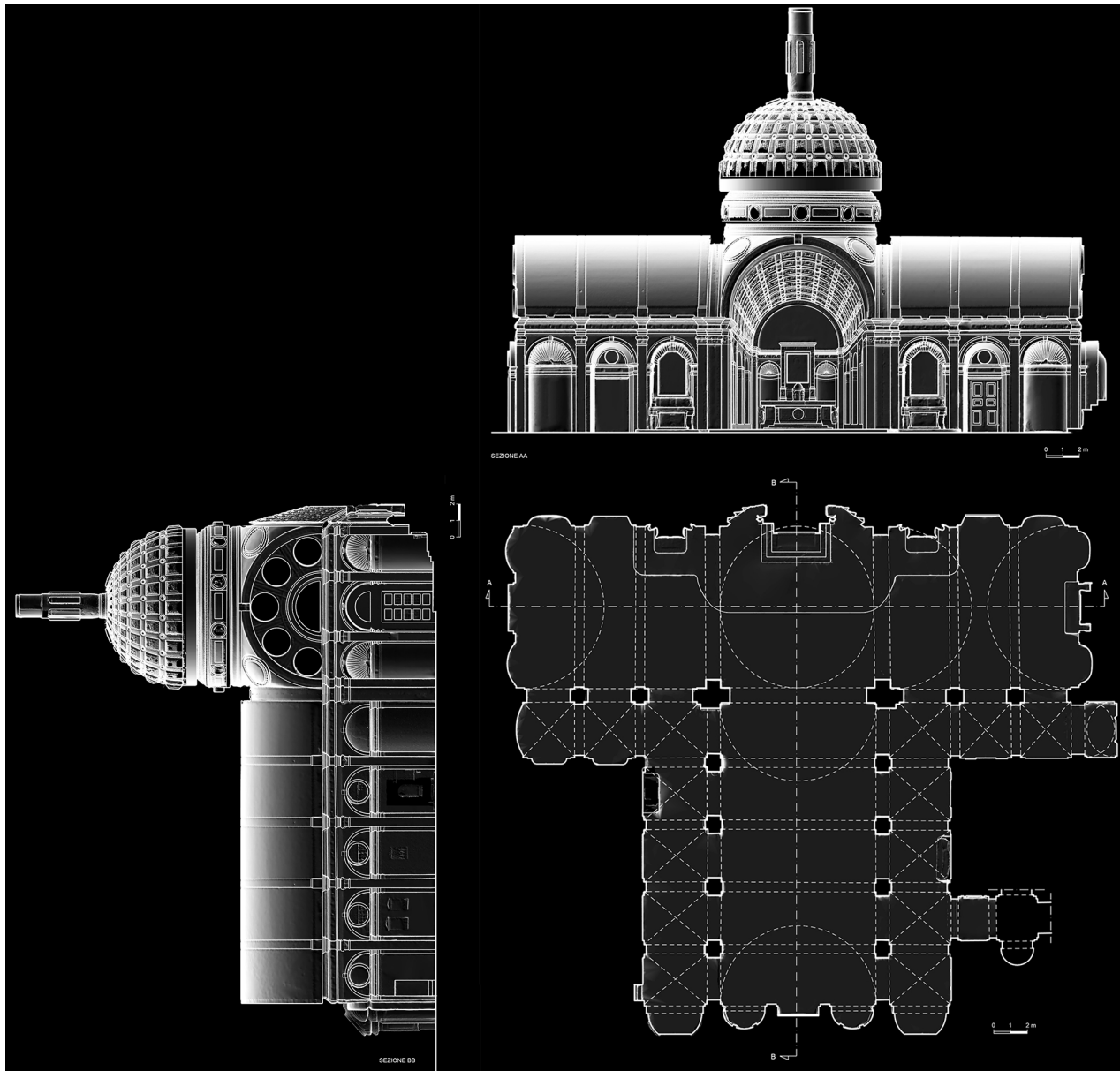
The false vault is divided into 10 coffered ceiling with the impost of the arch at the linear frame, which is lower than the diameter. Developed on a circumference the panels are 19, as a consequence of same arch elevation and of the choice of positioning the cornice in key place of the *paterna*, as done by Alberti in Sant'Andrea in Mantua, unlike the pictorial examples on the model of Masaccio's Trinity.

This solution contradicts the constructive custom of coffered vault, but it allows to have the wooden choir in correspondence of false capitals, avoiding the lacunars cutting and to optimize prospective deception. On the contrary, the fake painted coffered ceiling that decorate the vault of the nave resumes the pictorial model, with an inversion of roles between reality and illusion. (fig. 7)

The laser survey also points out the choir asymmetry, whose vertical axis is displaced by 5 cm between the frame (taken on the advanced plane and the background) and the misalignment of the two arches of the cross vault equal to about 16 cm, which makes it difficult to uniquely determine the main distance between the two halves, while the different width does not allow the direct reconstruction of the virtual choir from the scan of the transept.

The choir asymmetry is revealed in the three-dimensional collimation of choir conspicuous points with its virtual ar-

Fig. 4. Architectural survey of the church. Longitudinal and cross section (graphic elaboration by the authors).





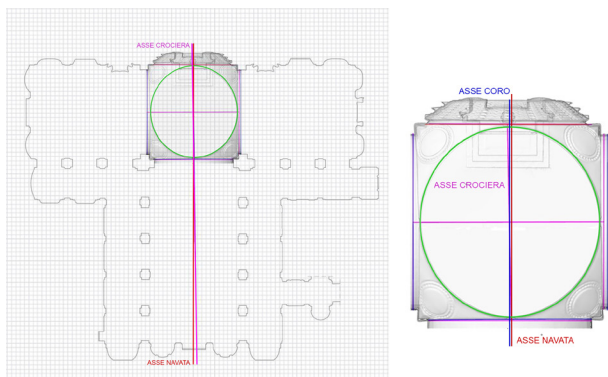
chitecture, rebuilt starting from the nave for the different width of the transept, which highlighted the geometric irregularity in the different intersection of the visual rays with the axis of the nave in the two halves. The deviation to the left recalls in proportions the one of the Prevedari engraving, already underlined.

This asymmetry, due to a 2 degrees difference in the angle between the base that defines the two depth planes and the frame, shifts their intersection to the left in the point of view, making difficult to determine it.

The survey documents the current situation of a building on which various kinds of morphological-conservative interventions have followed [8], that may have altered the geometry of the perspective elements, so before proceeding with the analysis it was decided to verify the specificity of restoration in relation to the purpose of the research [Grecchi 2015]. In particular, the last restoration (1983-1992) found the original terracotta floor 17 cm lower than the current two-colored marble floor, created in 1531 by eng. Cristoforo Lombardi [Marrucci 1987, pp. 23-50]. Other interventions that have affected the apse have been done on the wall of via del Falcone, because of its very small thickness required a first consolidation in 1662 and again in 1937.

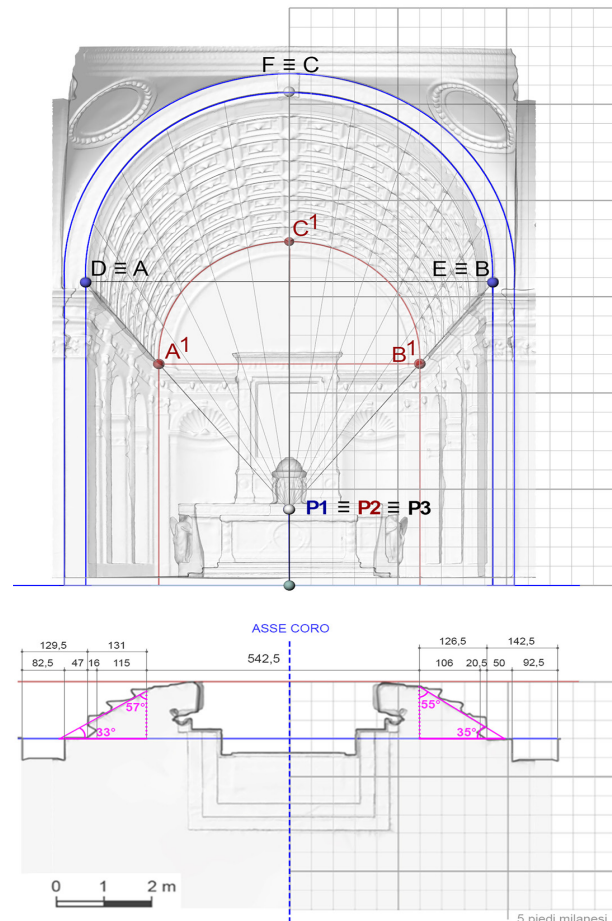
Also the wooden apparatus and the stucco of the choir have been restored several times, but do not seem to have undergone morphological changes, while the new floor has distorted the position of the observer's eye with

Fig. 5. Geometric and building irregularities (grid of Milanese feet), (graphic elaboration by the authors).



respect to the original drawing, which had not foreseen the presence of the base of the pillars, as it can be seen in the articulation of the false choir. From the survey it does not seem that the wooden apparatus was moved upwards when the floor was raised, therefore the main point of the perspective does not seem altered. However, the double survey identifies with adequate precision the main point in line with the point of concurrence of the

Fig. 6. Choir asymmetries: the angle in plan that compensates for the different width of the pillars keeps the altar in the middle (graphic elaboration by the authors).



false vaults alignments, establishing the horizon at a height very close to the ideal value suggested by literature, verified taking into account the floor rising, invalidating consequently the previous hypotheses [9].

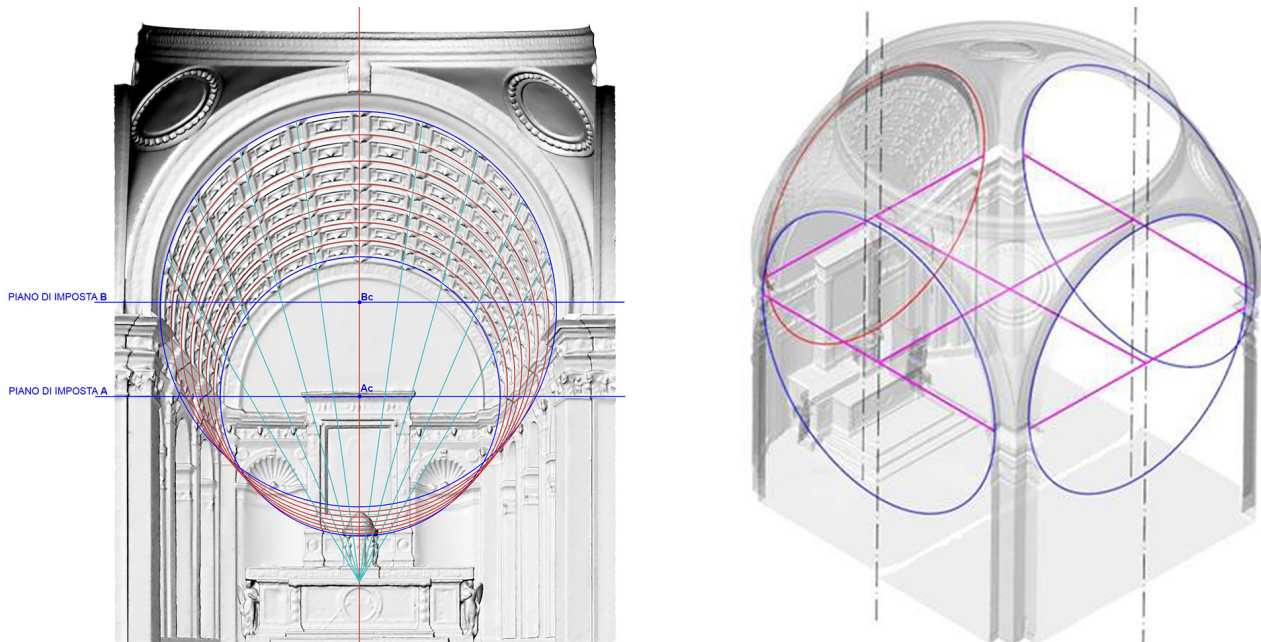
The point of view determination is more complex, placed on the horizontal axis at the height of the main point. In fact the possible alignments are not many and for the approximation of the fake choir plastic apparatus the perspective pilasters that simulate the pillars have significant dimensional variations. The maximum deviation can be found in the pilasters that delineate the choir and define the perspective framework, which differ by 10 cm (the left is 82.5 cm; the right is 92.5 cm) (fig. 8). Rather than constructive inaccuracies, this asymmetry is attributable to a series of compromising adaptations to the existing elements, aimed in finding the most effective solution and from which it is impossible to disregard the perspective reconstruction analysis for which there is no reference below the horizon level. In fact, as Robbiani pointed out, the choir floor is not in perspective, although it is beyond

the frame and it's justified by the need to guarantee visual continuity between reality and illusion.

### Design drawing, measurement and geometry

Retracing the hypothetical procedure of design drawing, the survey establishes the height of the main point P1 at 1.53 m from the current floor; that is 4 Milanese feet from the supposed level of the original floor, lower about 21 cm. The viewpoint is 16.97 m (equal to 39 feet from the frame and 42 from the background plane). The framework, beyond which the solid perspective of the virtual choir originates, is on the faces of the masonry pilasters that separate the architectural space from the perspective space and the distance of the point of view from the picture is therefore 1.5 times the presumed depth of the transept. The analysis of the survey using a square grid in Milanese feet allows to overcome the uncertainties produced by construction irregularities and to formulate a reliable

Fig. 7. Geometric analysis of the fake vault. The bow is raised and the vault has a beam in the key ridge (graphic elaboration by the authors).



hypothesis on the design process. Alberti's perspective theory influences the construction of the accelerated perspective. The reading of *De Pictura* points off a series of axioms and rules that see a rigorous design method in measurement, squared grids and cubic space, that the writer refers empirically and his illustrator of the Lucca manuscript clarifies through a drawing that represents a schematic plane that can refer to a space punctuated by a cubic tessellation [Alberti 1518, f. 23r]. During the Re-

naissance the use of the 'cubic space' was the architectural design control method.

The illustration of procedure for putting in perspective a cube of known dimensions divided into square modules, with the frame on one of the faces and the point of view at distance and height in proportion with the measure of the edge, highlights relations and coincidences between the perspective variables. The representation of a cube in perspective exemplifies the possibility of 'measuring' an en-

Fig. 8. Search for the point of view in the solid perspective of the fake choir (graphic elaboration by the authors).

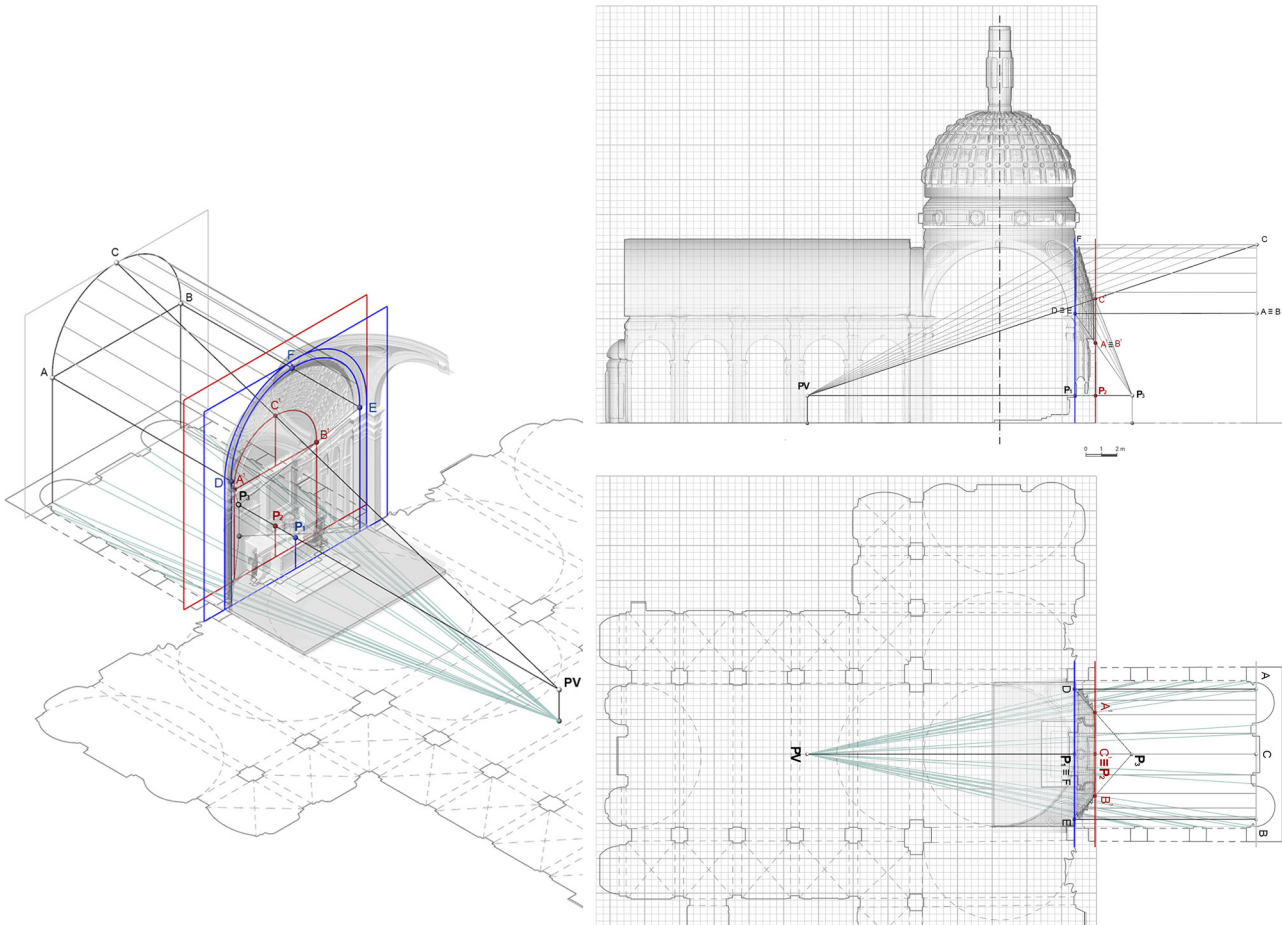




Fig. 9. Pattern of Albert's quadrangulus and geometric pattern in axonometric view. (graphic elaboration by the authors).

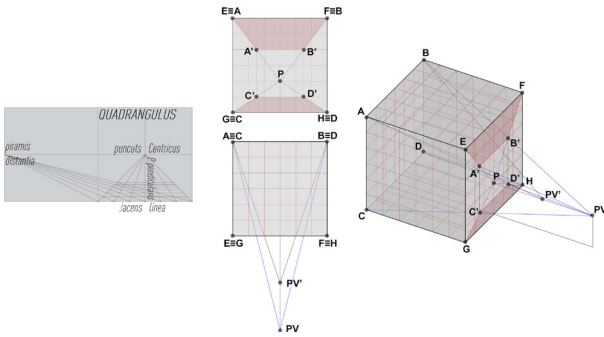
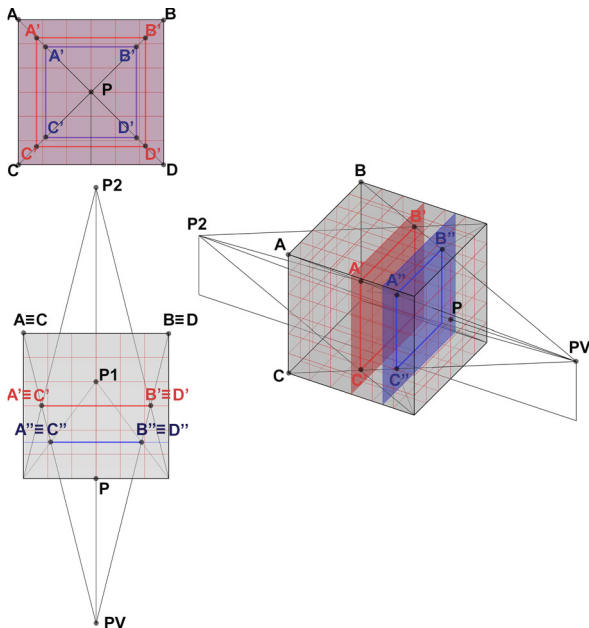
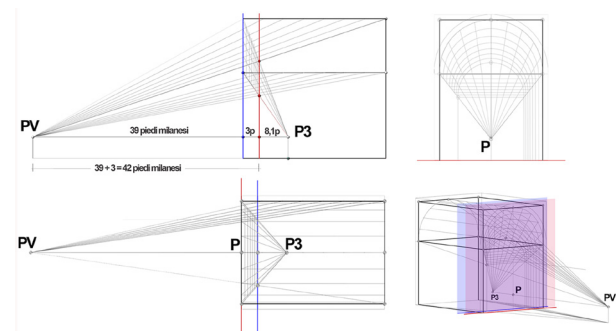


Fig. 10. Three-dimensional pattern of the three variables in the solid perspective: point of view, picture plane and background (graphic elaboration by the authors).



environment in which the plan and elevation are contained in the same square matrix scheme: the grid belonging to the frame is a contemporary representation of the plan and the elevation of the cubic room. The idea of a cubic space and its plan drawing are therefore governed by the same modular scheme of the square grid, in which *ichnographia* and *orthographia* can be traced back to a scheme that associates number and measure also in perspective. Figure 9 explains the relationship between the surfaces projected on the frame and the grid placed on it. The edge is 6 modules and the faces are divided into 36 (6 x 6) smaller squares, for a total of 216 cubic units, like in the explanatory graph of Alberti's words. If you place the point of view A at a distance from one of the faces of the cube [10] equal to the length of the corner (6 units) and at the height of 2 units, the projection on frame of the opposite upper edge face coincides with the second horizontal line from the top of the frame grid, while the corner projection opposite to the base cube framework corresponds to the first horizontal line of the same grid. If the distance of point of view is halved, the coincidence between perspective and grid occurs for edges opposite to the framework and orthogonal to the projection plane. With this matrix, if the point of view is placed at a distance from the picture equal to the edge of the cube and at a height equal to half, the problem of the *quadrangulus*, exhibited by Leon Battista Alberti in the *De Pictura* [Alberti 1518] and then taken up by Piero della Francesca in *De Perspectiva Pingendi* [11] incipit, is proposed again. The question became the paradigm for positioning the height of the point of view (3 Florentine arms = 4 Milanese feet), but here we can realize that the

Fig. 11. Perspective diagram of the variables of the solid perspective of the choir (graphic elaboration by the authors).

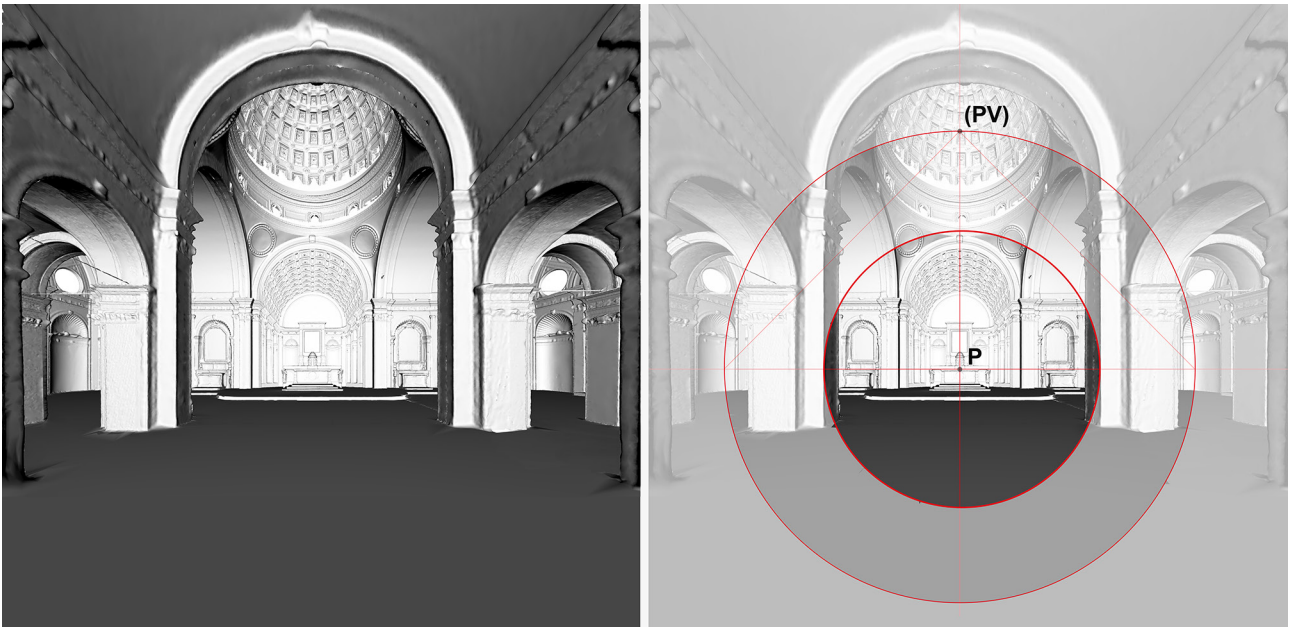


problem is of a completely different scope, since it explains the construction of the solid perspective, which no one before Bramante had dared to apply on a real scale.

This simulation allows us to hypothesize that Bramante applied an extreme simple process, derived from Alberti's perspective method, which lays the foundations for the conception of the Renaissance 'cubic space' [12]. In San Satiro Bramante shows a knowledge that integrates architecture and painting in a single method attributable to the square grid. This tool allowed to think space as a function of shape, number and size, to project it on a plane (linear perspective) or to contract it between two planes (the perspective frame and the background plane), generating a solid perspective. To contract the space in a scenographic fiction it's necessary referring to the relationship between the variables of accelerated perspective: the relative position of the point of view, of the frame and of the 'background plan'  $\pi'$  parallel to the frame. The conceived architecture contraction depends on this plan position. If the depth to be contracted is known and the

position of the  $\pi$  frame and the PV point of view is decided the second plane is fixed parallel to the frame at a distance of 3 feet (distance of the accelerated perspective of the fake choir) and the notable points A, B, C are connected with the point of view: the alignments intersect the plane  $\pi'$  parallel to  $\pi$  identifying the points A', B', C'. Joining these points with the corresponding ones D, E, F on the frame, the segments A'D, B'E and C'F are determined whose extensions intersect in P3, which is the point of convergence of parallels to the segments described by the contracted coffered ceiling of the vault. Once these four elements and the metric procedure that links them have been identified, it's easy to obtain all the points of the solid perspective. The constructive tracing of the wooden quarterdeck that keeps the floor horizontal may have been done in scale or in true dimension on the floor, reporting the fundamental elements of the perspective in plan and section (fig. 10). Once the tracing is prepared and the armour of the fake choir is built, one proceeds with the definition of the perspective position

Fig. 12. Virtual vision from the theoretical point of view: the central space unites nave/choir, transept and dome (graphic elaboration by the authors).



of the pillars, creating a simplified wooden structure on which to define the minor parts in stucco, and realizing the final painting.

## Conclusions

This is the goal of the check carried out on the architectural survey, verifying the theoretical design hypothesis on an ideal model that was reconstructed coherently with the real architecture (fig. 12). The consolidated thesis of the design reference to a central plan suggested the search for the position of the point of view in the 'theoretical' plan with transept arms, choir and nave of the same dimension, aligning the fake chorus with the real transept width. In this case PV would advance towards the centre of the arm of the Greek cross, bringing the observer closer to the middle of the nave in the centre of the imaginary space, thus in a conceptually significant position. Once again the survey removes any doubt: the particular proportion that the longitudinal section shows between the distance of the point of view from the picture plane and the theoretical depth of the choir underlines the correctness of perspective reconstruction, deduced from the virtual model.

This theoretical point of view satisfies both the suppositions of Architecture scholars and the statement of contemporary treatises, but also the perceptive assumptions pointed off in Robbiani's study.

The Bramante's design mastery, manifested in the elegance of the relationship between the real architecture and the perspective apparatus, shows that once acquired the instruments for space measurement, the architect exploited the visual representative potential by applying the perspective to interior decoration, as an integration of the architecture itself.

## Notes

[1] The known surveys that may have been at the base of two perspective reconstructions are those of F. Cassina (1840-62), E. Road (1884) and F. Manspero (1938).

[2] Performed by the G.M.S. from Milan.

[3] The image is attributed to Bramante on the basis of an inscription inserted in the engraving which indicates he is the author.

The meaning of 'real measure', which was originally associated with perspective, was later extended to the space and turned into the theatrical game between reality and fiction suggested by Bramante's work. The same method that Alberti describes in the exercise of the *quadrangulus*, was applied inside the room to create a virtual space that, like the two Brunelleschi's plates, becomes 'real' because the image has the same measure of the reality.

The geometric identity between the object and its image is what makes the measured representation 'true'. Thus the perspective materializes the artifice through which the image interacts with the real space, creating a figurative room that extends the building without continuity solution. The key is the integration of the point of view of the *architectura picta* in the real room, with the application of some expedient devices to hid the critical points in the boundary between the real and the virtual space.

The geometric pattern deduced from the metrological reading of the survey reveals an idea of space functional to the scenic perception and its perspective representation, confirming the importance of geometry in the control of the visual space. The 'cubic lattice' solves the solid perspective problem with a simple pattern, through which the solid perspective of San Satiro became the first model of modern theatrical scenography. In this way Bramante fixed the empirical bases before Guidobaldo del Monte's coding [Del Monte 1600] and the further application to the baroque theatrical scenography, according to the practical indications of Nicola Sabbatini [Sabbatini 1638].

Thus, at the end of the fifteenth century, in Milan, the perspective becomes an useful element for the definition of the architectural space. After this Bramante's 'admirable artifice', the quadratura reworked its application to theatrical scenography.

[4] The survey was carried out with a Faro laser scanner by the research group.

[5] PRIN 2010-11 National Research, principal investigator professor R. Migliari, with the participation of research unities of Rome, Milan, Turin, Venice, Napoli-Salerno, Florence-Bologna, titled *Architectural Perspectives: digital digital preservation, content access and analytics*.

[6] The mesh model was processed using Geomagic Studio 12 software.

[7] The main principal ray or observer's Optic axis is the line perpendicular to the pictorial plane, passing through the point of view; in this case it is a horizontal line and the only variable that we can determine exactly.

[8] Cfr. Cultural Heritage Information System of the Lombardy Region, Sheet OARL-1j560-00062.

[9] Leon Battista Alberti states 3 braccia fiorentine (*1 braccio* = 58,3626 cm) that correspond to 4 *piedi milanesi* (*1 piede* = 43,5185 cm).

[10] The cube side is the plane of representation.

[11] Piero della Francesca. (1476 ca). *De Perspectiva Pingendi*. Book I - XII. "Da l'occhio dato nel termine posto il piano asignato degradare" [Nicco Fasola 1984].

[12] The author explains the problem by placing a square on the geometric 6 x 6 Florentine arms lattice; the picture plane lays on a cube side and the point of view has the same distance of the cube side length (6 arms), the height is equal to the half of the cube edge (3 arms) [Alberti 1518, f. 23r].

## Authors

Giorgio Buratti, Department of Design, Politecnico di Milano, giorgio.buratti@polimi.it

Giampiero Mele, eCampus Department, eCampus Telematic University, giampiero.mele@uniecampus.it

Michela Rossi, Department of Design, Politecnico di Milano, michela.rossi@polimi.it

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