Geometric Modelling in the Narrative of Metropolitan Areas: a View on Attraction Dynamics

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Abstract

The new urban geographies are characterised by a high variety of activities and functions that interact with each other with different complexity, creating polycentric, hierarchical and asymmetrical spatial organisations. The new urban geographies, made up of relationships, force a radical rethinking of mapping, more directed towards the exploration of new intangible realities and freed from the need for cartographic correctness; a voluntary act made up of choices, omissions, additions. Searching for and experimenting with new ways of representing metropolitan complexity, in particular the attractiveness of cities, we arrived at a modelling of three-dimensional geometric surfaces capable of describing the phenomenon with greater emphasis and communicative capacity. The analysis is carried out by adopting a bird's-eye viewpoint, which, although it tends to generalise by simplifying the complexity of a territorial system, allows its characteristics to be observed and understood globally. The geometric modelling of the attractive phenomenon is the result of a complex graphical-analytical algorithmic procedure based on a hybridisation between graph theory and sophisticated spatial analysis methodologies. The result obtained is a three-dimensional geometric surface of extension equal to the analysis area, whose Einsteinian-inspired morphology deforms as a function of the force of gravity exerted by the metropolis. The case studies are two regions with the main Italian metropolitan areas within them: Rome and Milan.

Keywords: maps, cartography, data visualisation, urban phenomena, metropolitan areas.

Introduction: view on the evolution of mapping

The evolution of 'mapping', from its origins to contemporary times, has always been characterised by the constant need to connect reality and its representation, regardless of the conditions of the historical, economic and social context in which this activity is performed [Cosgrove 1999; Salerno 2021]. Conceptually, the map is a model in its most formal sense, i.e. an artefact designed and used to replace reality with a less complex representation of it, be it physical or abstract, to allow measurements or actions of various kinds that would be impossible to perform in reality, highlighting the elements that are subjectively considered to be the most important in relation to a given purpose. This definition allows two fundamental and consequential issues regarding mapping operations to be made explicit. The first is the inescapable subjectivity in choosing what is important according to the purpose and the way the observer decides to examine reality, i.e. his or her point of view on the 'object' he or she is studying and representing. The second derives from the first and is as simple as it is fundamental: the map is not reality but an interpretation of it, more or less sophisticated, more or less correct, and sometimes even paradoxically misleading or dishonest [Bergamo 2021; Moretti 2021].
Technological progress, which has particularly affected the technical-scientific disciplines, has inevitably acted as a catalyst in the evolution of cartographic sciences, with regard to surveying techniques, cataloguing and representation, purposes, and the very object of reality modelled through

Fig. 1. Piece of the map made by Admiral Ahmed Muhiddin Piri in 1511 and found by the theologian Gustav Adolf Deissman in 1929 in the library of the Topkapı Palace in Istanbul where it is currently kept.

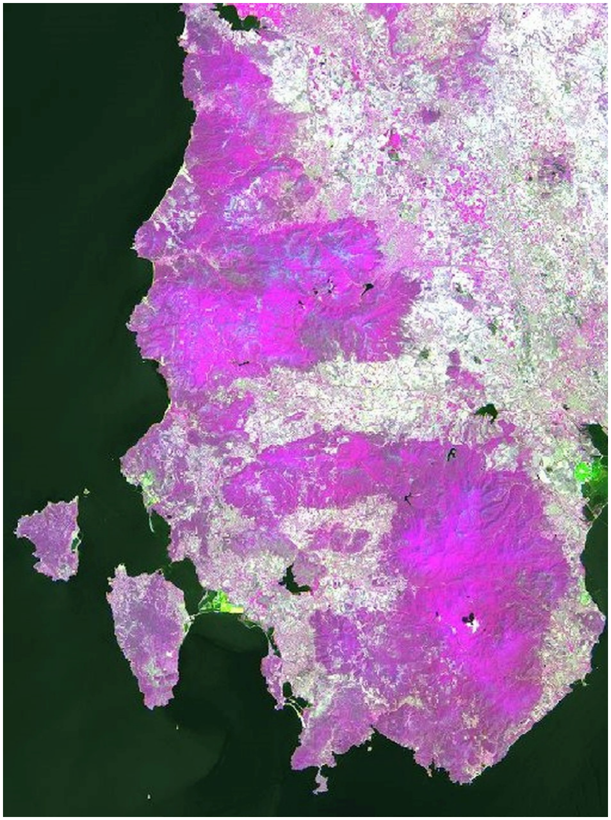


Fig. 2. Map from multispectral processing from Sentinel2 satellite acquisition, in particular the south-western coast of Sardinia with the vegetated areas highlighted in magenta colours using the NDVI algorithm (graphic elaboration by the author).

the cartographic medium [Llopis Verdú et al. 2019]. There has been a shift from modes of geographic data accumulation by direct knowledge and their subsequent processing within portulans since the 15th century [Lepore et al. 2017; Palestini 2021; Piscitelli 2011] (fig. 1), to contemporary means of massive and indirect acquisition and interactive data management through satellite remote sensing technologies [Buchhorn et al. 2020] (fig. 2).
Also from the point of view of purposes, the evolution of the cartographic medium shows a high dynamism: from the military and defensive purposes of the French three-dimensional cartographies of the 18th century realised through the 'Plans-Reliefs' technique [Salerno 2019], to the political purposes and representation of the 'Civitas' rather than the *Urbis* in the plan of Rome realised by Nolli in the mid-18th century using the *à poché* technique for the graphic restitution [Colonnese 2021] (fig. 3), to the strategic-commercial and softpower purposes of the European economic policy on a global scale of COPERNICUS [Zeil 2017], to an exquisitely artistic direction as in the 'Maps' project realised by the American graphic designer Paula Scher in which the conventional metrics yield the

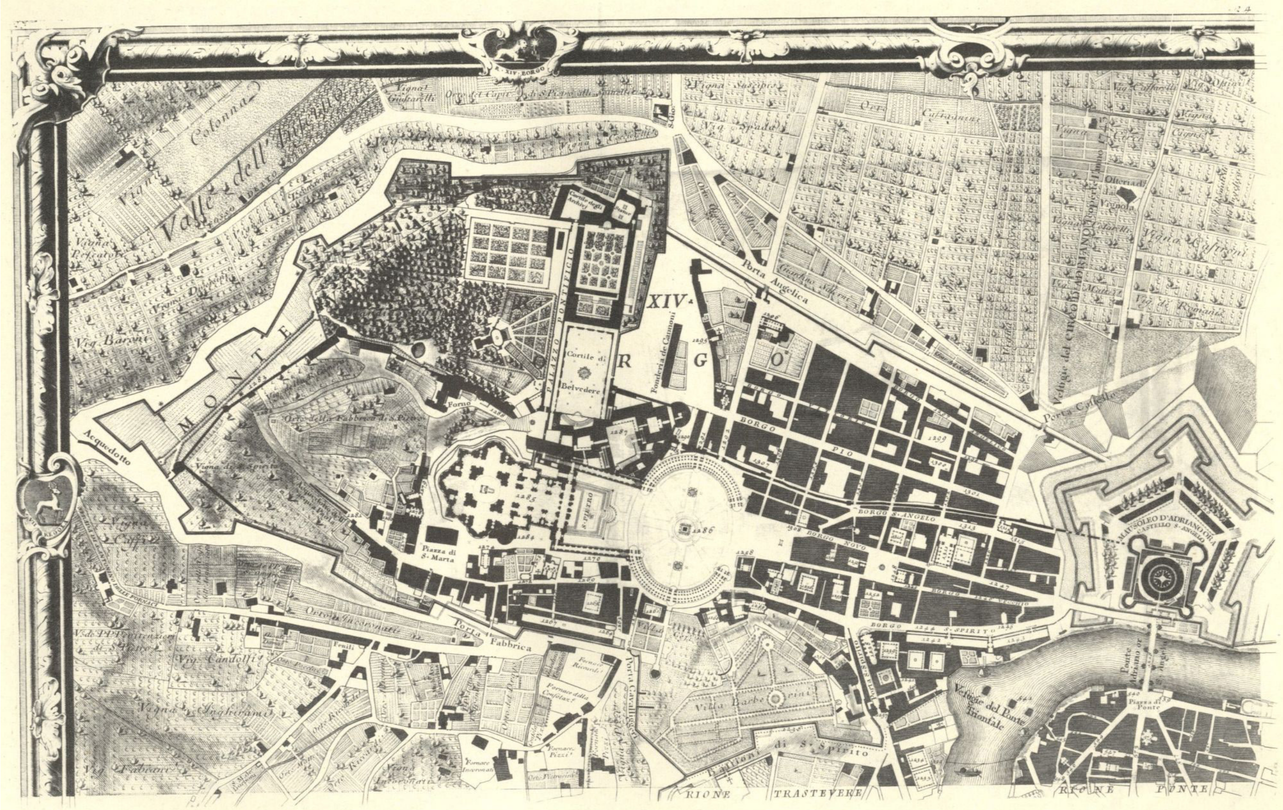


Fig. 3. Detail of the Map of Rome made by Giovanni Battista Nolli in 1748, detail of the Vatican area. Available resource online: <https://geoportale.cittametropolitanaroma.it/cartografia-storica/20/39/roma-nel-1748-9> (accessed May 23, 2023).

primacy of importance to the content and its communicative form [Palomba, Scandurra 2021].
Since geographer Roger Tomlinson created the first GIS system in the early 1960s, known as the Canadian Geographic Information System (CCIS), the computerisation of geographic data has rapidly spread and evolved in terms of analysis capacity and complexity [March, Scarletto 2017]. The development of new 'realities' parallel to our experience within cyberspace, the growing possibility of access to big data [Llopis Verdú et al. 2019] and the possibility of studying complex dynamic systems through algorithms and computer networks, have radically changed the point of view on the object to be represented; no longer physical spaces but increasingly extended and interactive virtual spaces characterised by increasing economic and social relational levels [Bergamo 2021]. In this direction, to accommodate the growing need for information correlated to the increasing territorial evolutionary complexity, the cartographic tool has evolved towards digital, dynamic [Bergamo 2021; Salerno 2021] and dematerialised forms [Valese, Natta 2021] capable of 'capturing' and representing also immaterial information through the use of infographic

visual codes addressed to all types of users [Zerlenga 2008] based on the map-diagram combination [Llopis Verdú et al. 2019] with a high information density but nevertheless capable of bringing out the peculiarities of the territory.

View on the representation of metropolitan areas

In the Italian national context, the metropolitan issue refers to the consolidation of asymmetric interdependence between contiguous municipalities, where asymmetry is given by the presence of a municipality that is larger than the others and whose force of attraction structures the surrounding territory [Calafati 2013]. The element that characterises all major urban agglomerations and in particular mega-cities [Blackburn 2019], i.e. those with a population over ten million, is their increasing attractiveness towards people, goods and energy [Wu et al. 2014; Batty 2011]. To this, the large flow of commuters who regularly travel from neighbouring or rural areas to access services or for work must also be added.
The discretion in the choice of viewpoint has over time produced numerous examples on the analytical and representative level of metropolitan areas. On the analytical level, several methodologies exist in the literature for the study of intra- and inter-metropolitan flows, mainly derived from ecological and economic sciences [Sen, Smith 2012; Fujita, Thisse 2013; Haynes, Fotheringham 2020]. Some authors have focused on defining the degree of polycentricity between different urban cores belonging to the same metropolitan area through the application of gravitational models [De Goei et al. 2010; Van Oort et al. 2010], while others have concentrated on studying the relationships between the centre and the surrounding area [Burger, Meijers 2012]. In this direction, several indicators of centrality and for the definition of sub-centres within the metropolitan constellation have been developed and used [Vasanen 2012; Veneri 2010; Veneri 2013; Roca Cladera et al. 2009; Krehl, 2018; Limtanakool et al. 2007]. However, the complexity of these models is often so high that the representations of their results are inaccessible if the user lacks the appropriate theoretical knowledge or more generally does not possess an adequate level of graphicacy [Cicalò 2020]. In this direction, the work of Craig Taylor as Senior Data Visualisation Design Manager of 'mapzilla' [1] is oriented. His elaborations, although based on a solid methodological and analytical foundation, are more oriented towards the development of infographics capable of filtering and representing with greater impact and understanding the territorial complexity and organisation of metropolitan systems from different points of view. For example, in the study of metropolitan spatial accessibility, the visualisation of travel times between the geographic centre of the metropolis and the peripheral areas is mapped by means of 'coral lattices' whose thickness is directly proportional to the level of accessibility (fig. 4). Considering the immateriality of the space of flows and the emergence of the network society at the beginning of the new century [Castells 2000], which conditions the everyday contemporaneity of human existence, the research conducted by the Senseable City Lab of MIT Boston [2] in particular on the project called 'Wanderlust', oriented towards the quantitative description of people's mobility, in particular towards attractive places, through their representation in spatial and temporal spectral form [Salerno 2021) appears equally significant (fig. 5). The realisation of the previous experiences was also possible thanks to the exponential technological development associated with an increasing accessibility to big data, which is acting as a catalyst for the evolution of graphic artefacts towards software-based workflows and algorithms for the construction of new augmented landscapes, necessary to probe the complexity of the non-human and to understand and govern its dynamics [Bergamo 2021]. In this direction, and focusing on the topic of research, it is possible to identify several researches in the literature that conceptualise the metropolitan area in the form of a network to be studied through graph theory and the metrics it makes available [Newman, Girvan 2004; Boccaletti et al. 2006]. According to this methodological approach, a metropolitan area can be schematised through nodes representing urban cores within the metropolitan constellation and links connecting them, with which different types of relationships are represented: commuter flow, goods, energy, political, economic or social relationships [Derrible, Kennedy 2011; Szmytkie 2017].

Case study and methodology

In line with the previous arguments, through this research we wish to explore new ways of representing territorial relations within contexts characterised by the presence of metropolitan areas, with particular reference to the force of attraction they exert. The case studies selected are the

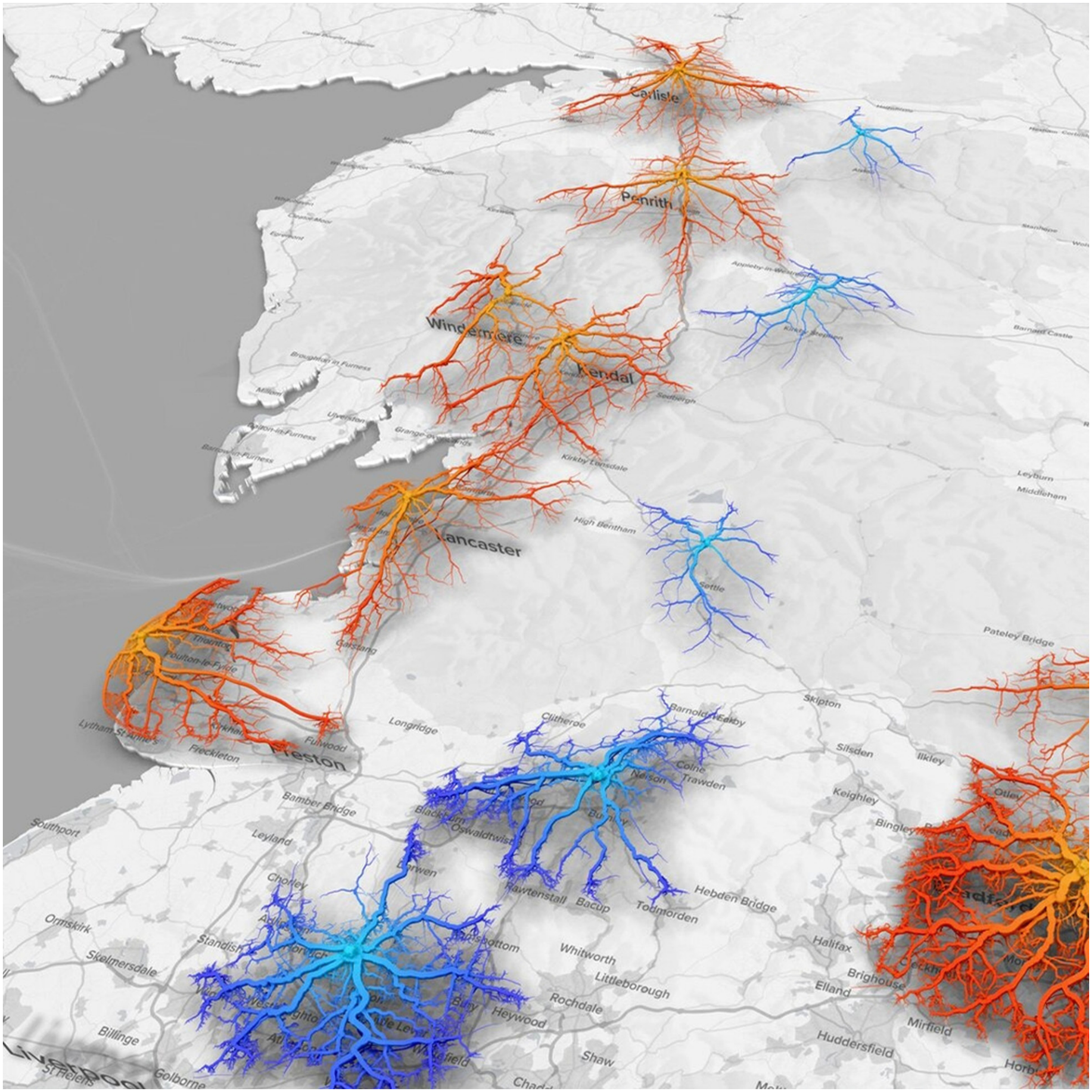


Fig. 4. The Coral Cities by Craig Taylor depicts spatial accessibility within 30 minutes moving from the urban centre to the suburbs by car. Online resource: <https://mapzilla.co.uk/work/the-coral-cities> (accessed May 23, 2023).

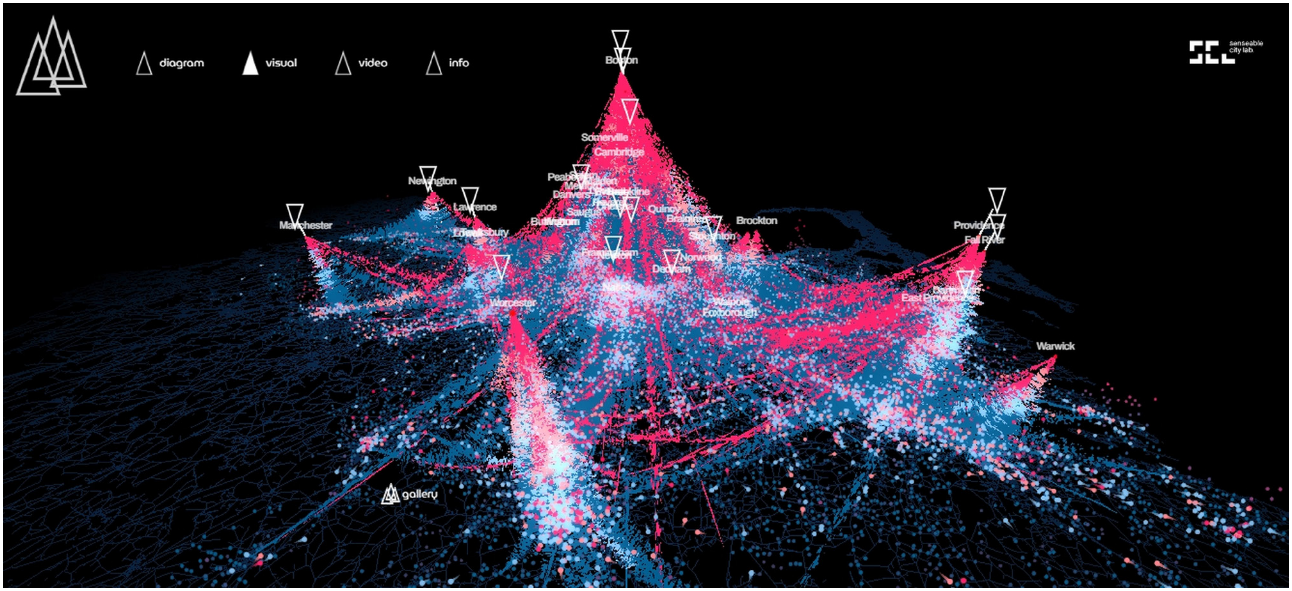


Fig. 5. Representation of 'The universal visitation law for human mobility' by Schläpfer., at the Senseable City Lab, MIT and published in the journal Nature [Schläpfer et al. 2021].

regions of Lombardy and Lazio because they are characterised within them by the two main Italian metropolitan systems, in terms of resident population and new economic wealth produced [UNHSP 2016]: Milan and Rome respectively. Exploiting the analytical potential of graph theory, it has already been shown how it is possible to determine in quantitative terms the attractive force of each urban centre within the study regions, by algebraically measuring this force as a function of the number of commuters travelling daily [Ganciu et al. 2018]; however, the aim of this research is not to investigate the dynamics in theoretical, methodological and quantitative terms, but rather to develop new ways for its representation, more suitable in representing the complexity of the phenomenon. However, for a better understanding of the research, it is considered necessary to briefly recall some elements of the previous works [Ganciu et al. 2018], which is the starting point for the development illustrated below. The data on commuting were extrapolated from ISTAT's National Commuting Matrix produced on the basis of the 15th National Population Census. This is a database of 4,876,242 records that condenses the interviews of 28,871,447 citizens who commute daily between their residence and their place of study or work, by any means of transport, whether public or private. Basically, it is a matrix in which each row shows the 'Source' and 'Target' municipalities, i.e. origin and destination, and how many people move between each municipality. Quantitative analysis was done by transforming the commuting matrix into a Graph (G) consisting of a non-empty set of nodes denoted as 'V(G)', a set of links denoted as 'E(G)' disjointed from 'V(G)', and the relationships (Ψ) associated with each link connecting two nodes [Boccaletti et al, 2006]; this transformation was performed using a specific library of the Python language called NetworkX, and importing the corresponding file into the Gephi software for subsequent calculations. The nodes represent municipalities; the presence of a link between two nodes indicates the existence of a flow of commuters between them; the magnitude of this flow, i.e. the number of people moving between two municipalities is associated with the link as a weight (w).

In order to determine the attractiveness of each municipality, the main measures of centrality were carried out, such as the degree of the weighted node '*Deg(w)*', which represents the sum of all weighted links converging on each node in the network, in other words the total number of commuters entering and leaving each municipality in the network, formally:

$$Deg(w) = \sum_{j=1,n} e_{ij}^{w}$$

with 'e' representing the weighted link between nodes (i) and (j). Given that each municipality is usually represented by a flow of incoming commuters as well as outgoing commuters, by exploiting the directionality of the links it is possible to calculate incoming commuters through *DegIN(w)* and outgoing commuters to other municipalities through *DegOUT(w)*, thus for each municipality in the study the following relationship also applies:

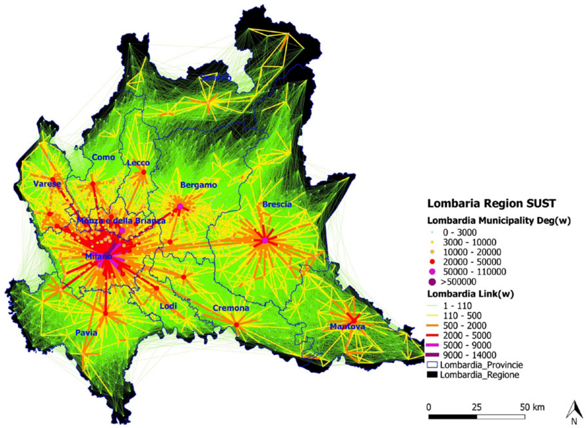
$$DegTOT(w) = DegIN(w) + DegOUT(w)$$

(figs. 6, 7).

In the research development presented here, starting from the above equation, it is possible to derive the net commuter flow for each municipality, meaning the algebraic difference between incoming and outgoing commuters for each municipality:

$$Fn = DegIN(w) - DegOUT(w)$$

Therefore a positive net flow (*Fn+*), will indicate attractive urban centres, vice versa a negative net flow (*Fn-*) will instead indicate those municipalities that give up commuters on a daily basis, to the advantage of the larger centres. Subsequently indicating with 'z', the net flow of commuters (*Fn*) as described above, the variable can be treated as the third spatial coordinate to be used in modelling the geometric surface that, as mentioned, will represent the commuter attraction capacity of each municipality. The proposed methodology also has the capacity to consider the cumulative effect produced by the proximity between urban centres by assuming that the attractiveness of a given territorial portion containing two or more administrative units possesses an attractiveness greater than the sum of the respective municipalities taken individually.



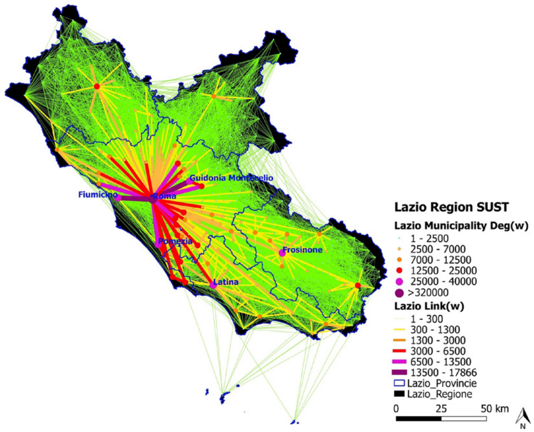


Fig. 6. Map of commuter mobility in Lombardy based on 2011 ISTAT data [Ganciu et al. 2018, p. 11]. The presence of a link indicates a flow of commuters between two municipalities, the number of travellers is expressed by weight (w).

Fig. 7. Map of commuter mobility in Lazio based on 2011 ISTAT data [Ganciu et al. 2018, p. 11]. The presence of a link indicates a flow of commuters between two municipalities, the number of travellers is expressed by weight (w).

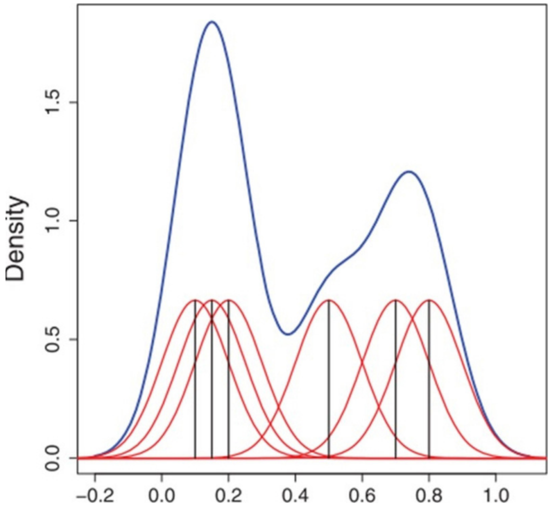


Fig. 8. Geometrical-mathematical example of cumulative estimation through KD [Chen 2017].

The geometries of the municipal administrative units processed in vector format (shp.file) were taken from the ISTAT database. Subsequently, again by means of GIS, a specific algorithm was used to calculate for each municipal geometry its centroid, which can be summarily indicated as the geometric centre of gravity of the municipal polygon considered, making it possible to obtain a new vector file composed of point primitives characterised by triples of Cartesian coordinates, with the values of 'X' and 'Y' expressing respectively the East and North co-ordinates of each centroid with respect to the chosen reference system, in this case based on the RDN2008/UTM zone 32N, and the third coordinate 'z' representing the value of (Fn). The cumulative attractiveness was also calculated within QGIS, through the use of the native algorithm for calculating Kernel Density [Chen 2017], which is influenced both by the degree of proximity or closeness of the centroids, but also of course by the relative variable 'z'. In particular, the KD also known as the 'Parzen window' [Parzen 1962], is one of the best known approaches to estimate the probability density function of a data set. Since the KD is a non-parametric density estimator, i.e. it does not require the assumption that the underlying density function belongs to a parametric family, it allows the algorithm to autonomously learn the distributional form of the density from the data (fig. 8). Figure 8 illustrates a simple example that explains the logic behind the KD algorithm: there are six observations, located at the positions indicated by the black lines. The individual observations make it possible to determine the probability of the density expressed by means of the Gaussian in red colour, which when added together yield the cumulative density estimate expressed by the blue curve.
Through the KD, positive and negative interference areas were then identified for each of the two study regions within a 20km radius of each municipal centroid; this radius was chosen because it is representative of the average distance of each centroid from its closest neighbours. The result of the analysis is provided through a raster model with a cell resolution of 250m × 250m. Each pixel of this raster thus contains a numerical value representative of the cumulative attractiveness of the municipalities that was subsequently used to model a geometric surface that interpolates all the values contained within the pixels obtained from the KD. The algorithm used for the interpolation is Inverse Distance Weighting - IDW [Lu, Wong 2008; Choi, Chong 2022], which based on the 'bird's eye' approach, as described in the introduction, was configured to consider the entire available dataset. In other words, given 'n' values present in 'n' pixels, the output raster was generated by interpolating n*(n-1) values, allowing the value of the variable to be estimated over the entire regional surface area. Again, the result is provided through a raster model for each of the two regions, with a resolution of 250m × 250m. This graphic-numerical matrix was finally treated as a sort of DTM (Digital Elevation Model) for three-dimensional modelling and visualisation, (figs. 10-12), making it possible to effectively visualise the attractive force of the urban poles through maps that will have the peculiar characteristic of being deformed by the attractive force that the metropolis exerts on the surrounding territory.

Results and conclusions

The geometric surface modelling presented in this research confirms previous research that identifies the cities of Milan and Rome as massive poles of attraction capable of significantly influencing the regional context in which they are located. In fact, by means of a 'drapping' operation (a technique that can be summarised by imagining spreading

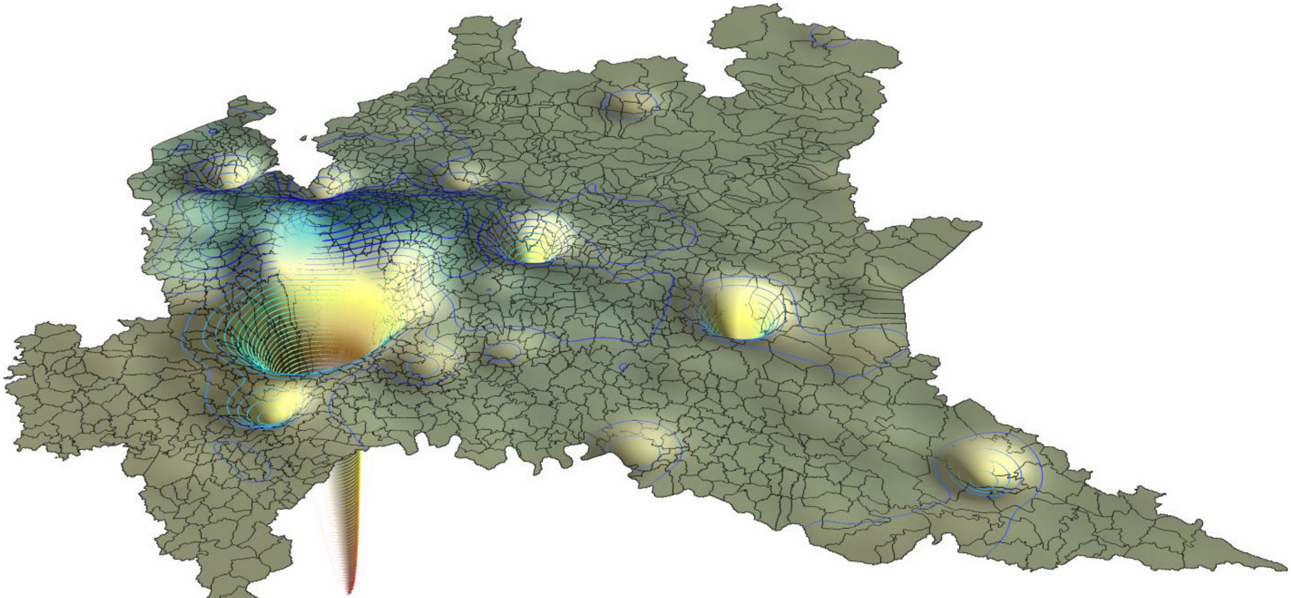
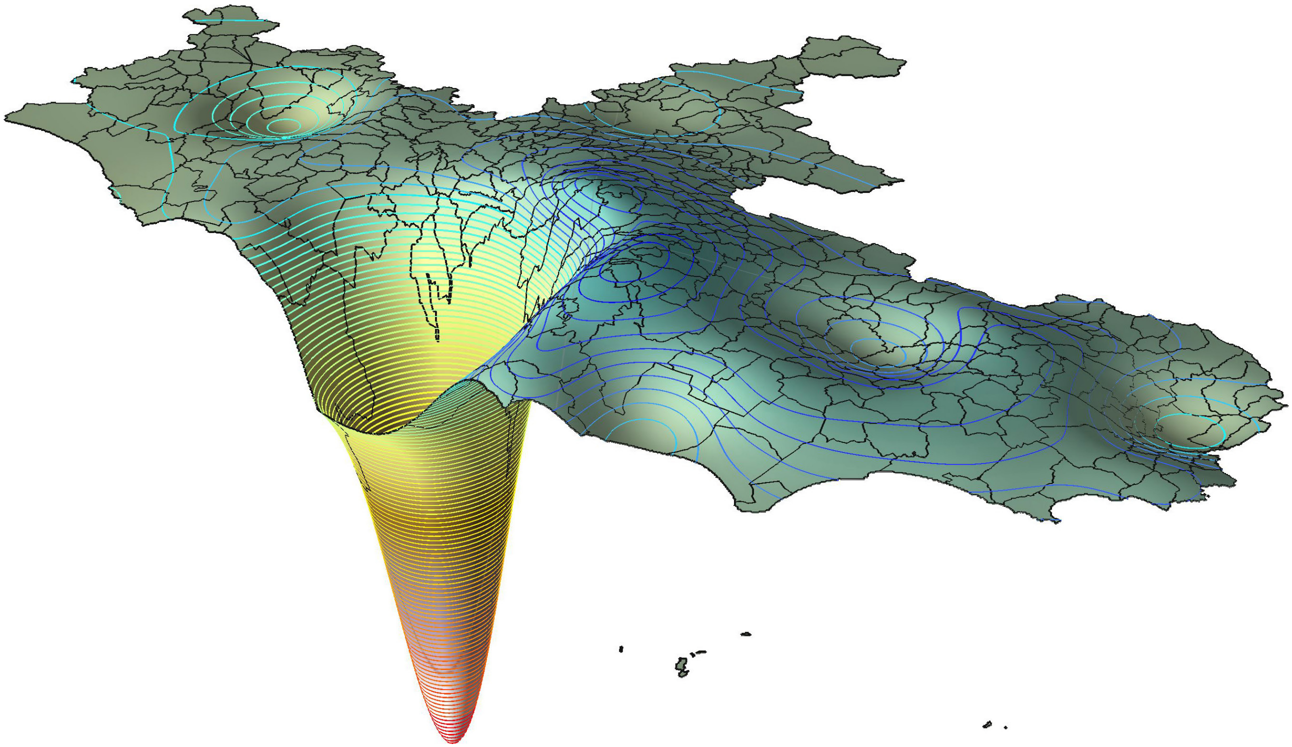


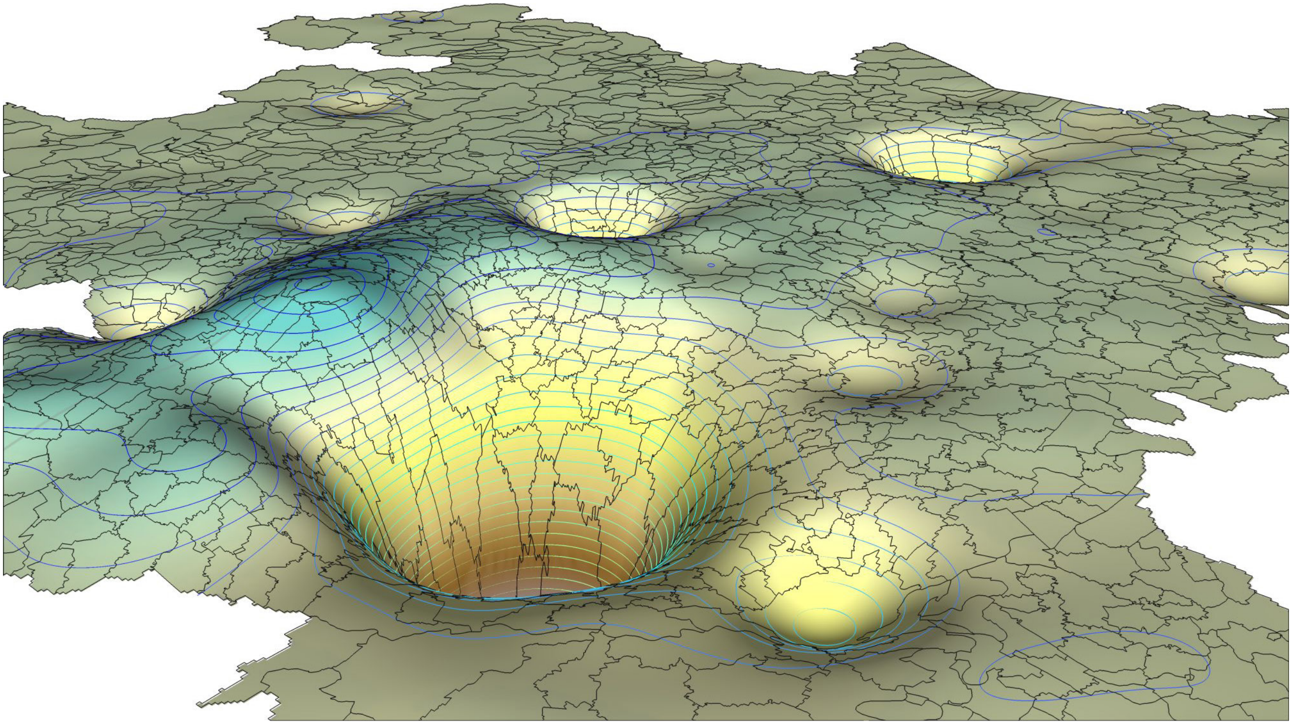
Fig. 9. Representation of cartographic deformations generated by the attractive power of cities in Lombardy (graphic elaboration by the author).

Fig. 10. Representation of cartographic deformations generated by the attractive power of cities in Latium (graphic elaboration by the author).



a transparent sheet over a solid) between the vector layer containing the municipal polygons and the calculated geometric surface, it is particularly evident how the geometries of the polygons tend to lengthen considerably once they have passed a threshold of attraction that we could metaphorically indicate as a sort of 'event horizon' (figs. 9-12). Modelling shows how beyond this limit the strength of the city grows exponentially, and this form of representation was particularly effective in the two proposed case studies, making the phenomenon more comprehensible than previous two-dimensional visualisations based solely on network visualisation [Ganciu 2018]. However, while this representation fulfils the objective of the research by bringing out more clearly the attractiveness of the cities and the power relations between the different urban centres within the two regional contexts, it is still unable to answer the legitimate question of how far the city's attractiveness extends in areal terms. In other words, further development is needed in the forms of representation in order to identify the solution that will make it possible to visualise the overall extension of the ramifications that, starting from the metropolitan pole, branch out over the entire surrounding territory and through which commuters move. In fact, it can be observed, comparing the results obtained in this research, that in the area containing Rome, the 'gravitational cone' is less deep but much wider than that observed in Milan; this apparent anomaly can be explained by considering that the proposed modelling is sensitive to the number of incoming commuters, manifesting the phenomenon through the depth of the cone, but is also sensitive to the interactions with the entire regional context according to a 'bird's eye' approach, evaluating the attractiveness of all the

Fig. 11. Detail of the cartographic deformations generated by the attractive power of the Milanese metropolis in Lombardy, one can observe how in the vicinity of the gravitational cone the deformations of municipal polygons increase dramatically (graphic elaboration by the author).



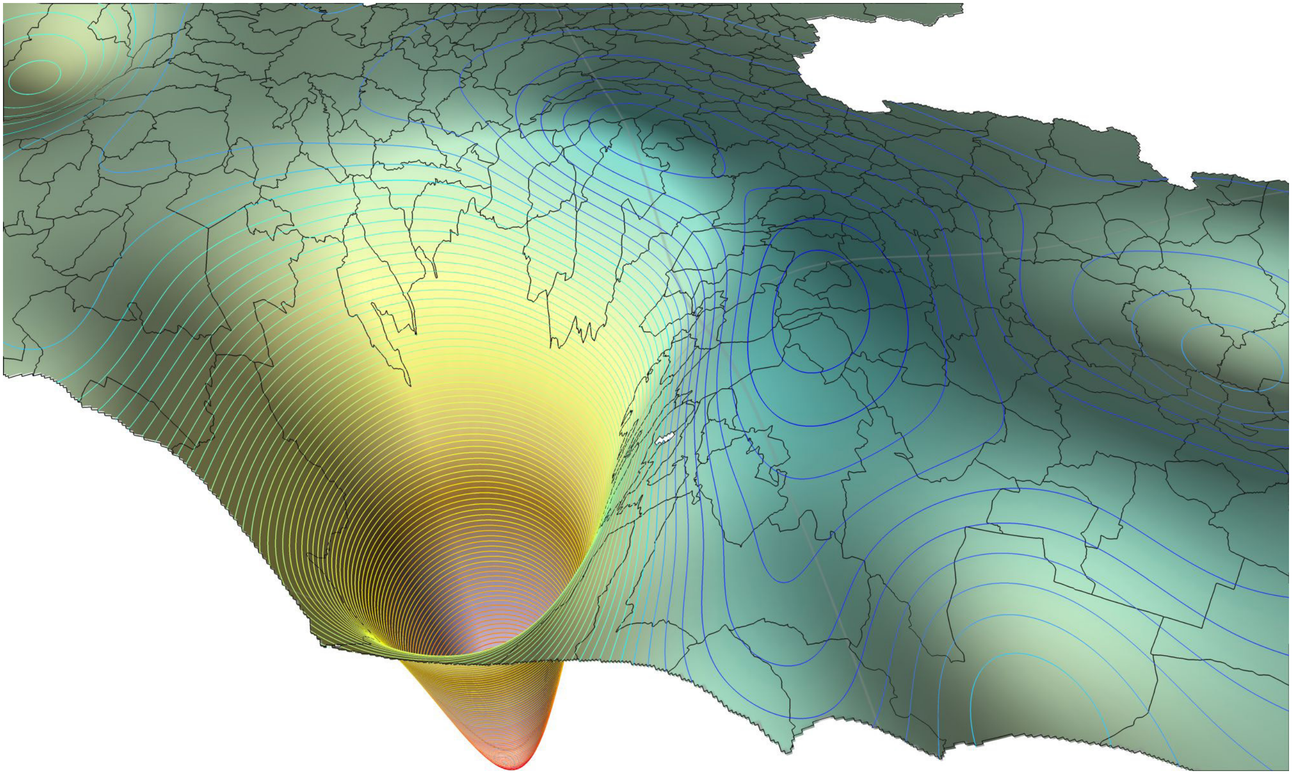


Fig. 12. Representation of cartographic deformations generated by the attractive power of cities in Latium, again it can be observed how in the vicinity of the gravitational cone of Rome the deformations of municipal polygons increase dramatically (graphic elaboration by the author).

municipalities and their ability to resist the attraction of the metropolis. If in fact in Lazio there are no powerful gravitational centres with significant capacity to balance the attraction of the Roman metropolitan area, the same cannot be said in the case of Lombardy with the presence of Milan and other urban centres capable of resisting the attraction of the great metropolis.
In conclusion, as discussed in the introductory section, the proposed mapping is the result of a well-considered but nevertheless arbitrary choice of the variables to be considered in the study, of the methodology for analysing and representing them, and of the point of view adopted by the researcher to observe the object being studied. This consideration, in the light also of the other examples illustrated, is of fundamental importance in the ability to nonetheless observe with a critical eye the graphical result obtained beyond the methodological complexity. In other words, it is not possible to exclude the possibility that variations in the point of view on the metropolitan phenomenon may generate different results, although this seems unlikely given the further confirmations obtained in this research. The proposed methodology has proven to be a useful tool to represent complex even intangible phenomena such as the attractiveness of cities, however, further development is deemed necessary to satisfy new research questions such as determining the territorial extent of influence.

Notes

[1] <www.mapzilla.co.uk> (accessed May 22, 2023).

[2] <https://senseable.mit.edu/> (accessed May 22, 2023).

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