Observations on Geometry and Cartography: or on the Perception and Representation of Space

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Introduction

In recent years I have found myself reflecting more and more often on the relationship between the geometry of representation and cartography, trying to find a link between these two subjects that could take into account my two paths of life and of research, developed along parallel lines, apparently without points of contact. It is clear that this reflection did not refer to the need of giving meaning to my personal choices, something that can only be of interest to myself, but rather of understanding if there were, and in what terms, a historical, epistemological link between these two disciplines with very indistinct boundaries. For years I have collected and studied maps simply for the pleasure of doing so, and for years I have taught and studied Projective and Descriptive Geometry for academic purposes, as well as, of course, taking great pleasure in doing so.

With this brief and unstructured essay, I would like to clarify this connection, in the hope of soliciting further reflection on all the areas of research related to the representation of space, and which do not refer only to the technical informatics data of drawing (a trend becoming more and more risky today with image digitization, computer graphics and virtual 3D) but to the complexity of the entire process from conception to realization.

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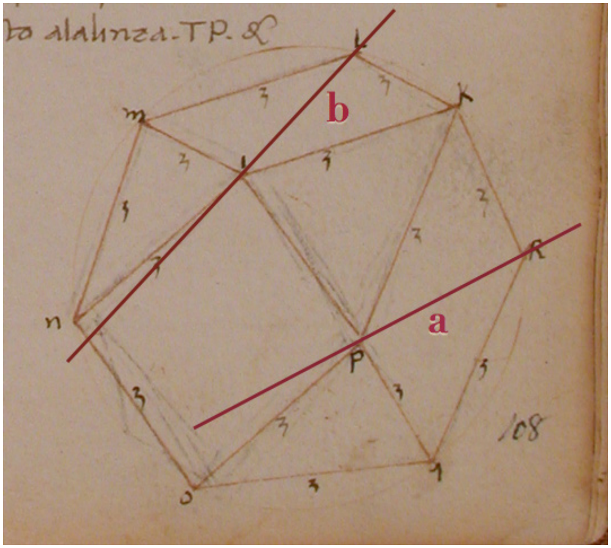
Looking at maps, not only ancient ones but those of every age and place, one immediately realizes that these images contain enormous quantity of elements, all to be analyzed: symbols, colors, the presence or absence of geographical and astronomical indications, possible grids and their forms, graphical rendering, lettering and much more. Due to the complexity of the images, the study of maps cannot be reduced to the analysis of the variations of the geographical contents of the area represented, as has often been done, and is still done, having cartography been considered a sort of attribute of geography. Geographical content may also be an investigative factor, but it is not the most important one. The risk is to reduce the complexity inherent in maps, but I would say, in every image, to only one of its aspects: content. Let's imagine what would happen if we wanted to judge a painting by reducing it to its content and to its correspondence to whatsoever real, literary or historical fact. For example, if we wanted to analyze Masaccio's *Crucifixion* by measuring its correspondence to the evangelical dictates, we would not understand anything about the painting's chromatic, compositional and spatial rendering revolution, nor its communicative complexity or its historical contextualization.

Fig. 1. Drawing of a cuboctahedron. From: Piero della Francesca, Abaco, f. 108r. Biblioteca Medicea Laurenziana, Ashb. 359, Firenze. The line segments a and b have been added for illustrative purposes (graphic elaboration by the author).*



What makes the study of maps extremely complex and intriguing is the elaboration produced by the human mind in passing from the spatial to the two-dimensional datum, which is essentially the compositional process, and I have no qualms about using a term borrowed from the world of art.

Perception and representation

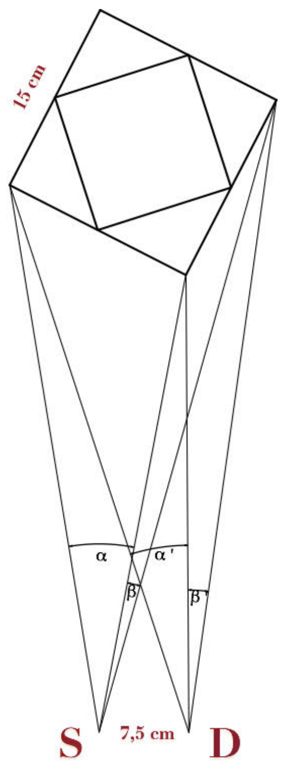
Drawing a map is an ancient way of putting order into the surrounding world using graphic techniques, in other words, it is a cognitive operation that makes it possible to move and orient oneself in space. In this definition, which I used a few years ago for a multimedia encyclopedia project at the Museo Galileo in Florence, Italy, are condensed the two moments of cartographic production that connect maps to psychology and geometry, "putting order" and "using graphic techniques" [Valerio 2008].
These two expressions also identify two aspects of the entire cognitive process: the "perception" of space and its "representation." These are two separate operations that relate to two different activities of the human mind.
The first (putting order) is a purely psychological operation and can be expressed in various, even descriptive, textual forms ranging from live sensations to reminiscences; we could define it as a narrative of spatial sensory sensations. An example of this way of describing space is given to us by Lucretius, in the famous passage on optical illusions: "A portico, / Albeit it stands well propped from end to end / On equal columns, parallel and big, / Contracts by stages in a narrow cone, / When from one end the long, long whole is seen,– / Until, conjoining ceiling with the floor, / And the whole right side with the left, it draws / Together to a cone's nigh-viewless point" [Lucretius Carus, *De rerum natura*, IV, 426-436].
The reading of this passage and the comparison with its coeval pictorial representations have given rise to long-standing discussions and disputes between the supporters of the theses of Erwin Panofsky [Panofsky 1961], who thought of Renaissance perspective as the cultural product of a society, and Decio Gioseffi [Gioseffi 1957], a proponent of the universality of perspective, which he believed to be already known and used in the ancient world, and which translates, univocally and correctly, perception into representation. Many questions would have been solved if the participants in the controversy had kept this fundamental distinction in mind [Vagnetti 1979].
The second operation (using graphic techniques) concerns the way in which a sensory datum is translated into a drawing, the

graphic representation of space, and is a purely cultural product, as demonstrated by the variety of graphic renditions produced over time and in various geographical and cultural areas.
Distinguishing these two moments (perception and representation) is fundamental in order to historicize and contextualize a map or any other graphic representation of space, from the architectural to the geographical dimension.
It is important to underline that Lucretius describes in words the sensation while observing a long portico from one of its extremities; nevertheless, the graphic outcome of this sensation is not predetermined, there is no univocal graphic response to that perception and, in fact, artists his contemporaries do not provide a single unique solution. Even the geometry of the ancient world never attempted to find a solution that could turn the aporia between observed reality and represented reality, because a solution does not exist. All the attempts made over the last few decades to demonstrate the knowledge of Renaissance linear perspective on the part of the ancients (making Lucretius' "perceptive" text a "prescriptive" text) have failed, for the impossibility of finding a single rule to be recognized as a graphic model of spatial representation. For the simple reason that the concept of "projection" is lacking in ancient world [Valerio 1998]. Moreover, as we have said, but it is important to repeat that it has been claimed that the Renaissance rule of perspective, which is none other than a method, an algorithm, a mechanical device, coincided with the "exact" representation of real space. If this were the case, the realization of the photographic camera (which is nothing more than a perfect perspective tool) would have solved all the problems of two-dimensional representation of space.
Rules of an axonometric nature (objectively parallel straight lines represented as parallel straight lines) or rules of perspective with floors, walls or trabeations whose extensions converge in several distinct points, or lines that move "obliquely" on the plane of representation coexist and share the same expressive validity, in the ancient world as in the modern one. We are faced with simulations which attempt to deceive the eye, knowing it provides us with deceptive sensations that only the mind can recompose.

Exactness and error

One of the outcomes of the confusion between perception and representation is the search for the "exactness" of the image, which involves the evaluation of any possible "error."
Exactness is a very ambiguous term that is often identified with numerical precision, making it a characteristic value of scientific disciplines (which would include the representation of space, seen *sub specie geometrica*) as well as an weapon for historical judgment: where there is no exactness there are errors. I find no worse way to approach historical, epistemological and scientific research studies than to search for or report errors.
Exactness in the representation of space lies not in the metric correspondence between the image and reality but in the "precision" of the description. And here I can only refer to what Italo Calvino wrote in his lecture on exactitude in *Six Memos for the Next Millennium* (*Lezioni americane*) [Calvino 1988]. The author addresses the issue of exactitude through the many aspects offered by literature, philosophy and essays;

Fig. 2. Diagram relating to the binocular vision of a cuboctahedron placed at the distance of an arm from the eyes of the observer (drawing by the author).



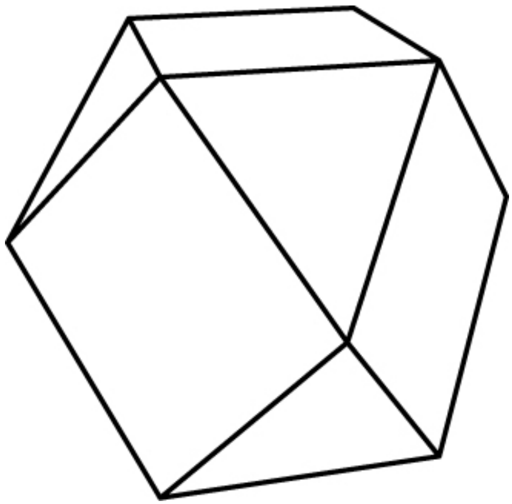
for him, exactitude had nothing to do with the mathematical (or geometric) correspondence between reality and image but rather with the "precision" with which a certain reality is described, or represented, as we would say. Even the "vague" and the indeterminate in Giacomo Leopardi can be achieved only through "a highly exact and meticulous attention to the composition of each image" [Calvino 1988, p. 60], transcending the contradiction in terms between "vague" and "precise."
The exactness of an image, to use Leopardi's literary vision, which I would make my own, lies precisely in the accuracy with which the spatial context that is the subject of reflection and representation is described graphically. It makes no sense to find the metrical correspondence of a cartographic image or drawing with reality if the "quantity" (understood as metrical correspondence) is not one of the author's objectives. In the study of images, I would find it more appropriate to replace the search for "quantities" with the search for "qualities." We must realize that if the evolutionary method is applied to cartography, all the pre-geodetic maps, prior to the second half of the eighteenth century, are wrong. At the same time, all the projective representations before the mathematical-geometric coding by Monge and Poncelet can be defined as wrong, or without a method, arriving at the paradox that all the scientific theories of the past that have been surpassed by subsequent ones are incorrect. Our past becomes a history of human errors, overcome by the "magnificent and progressive fates" depicted by Leopardi [Leopardi, *I Canti, La ginestra*, v. 51].

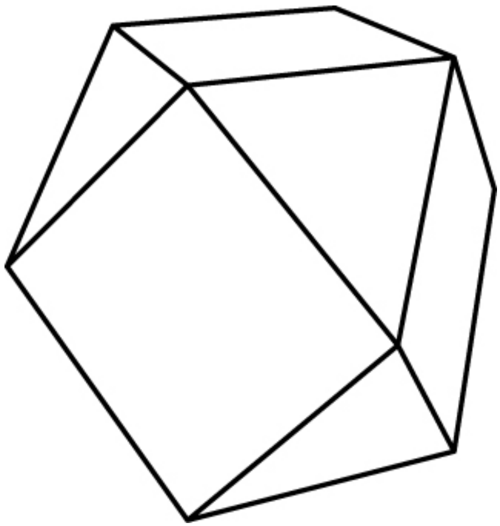
Philology and drawing

The right questions to ask would be: what does one want to represent, and with which technical means? It seems clear to me that these questions could be asked in any age and in any place, giving different results depending on the societies and cultures that produced those images, and it seems evident that an answer to these and other questions, including the analysis of the sources used and of the derivative due to the inertia of the images, involves themes that we could define as a "philology of drawing." A fascinating theme that has only been tackled in recent decades, since when the drawings and diagrams present in scientific texts have been analyzed with the same meticulous precision and methodology similar to those used for the analysis of texts. Here I would like to mention the works of the

Fig. 3. Perspective of the cuboctahedron from the point of view S (left eye, see fig. 2), (drawing by the author).

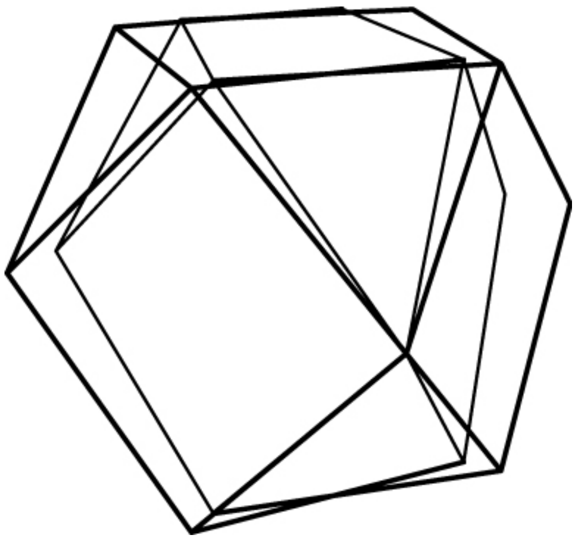
Fig. 4. Fig. 4. Perspective of the cuboctahedron from the point of view D (right eye, see fig. 2), (drawing by the author).





Commissione Nazionale per gli scritti di Piero della Francesca [Piero della Francesca 1995; Piero della Francesca 2012] and of Revil Netz on the Archimedes Palimpsest [Netz 2009]. Even when dealing with simple geometrical schemes, as in the cases mentioned above, we are facing themes of the representation of flat or three-dimensional figures, because the aim of the draftsman is not that of being exact in the graphical representation but readable, "precise" in the description; the task of the historian (of the philologist of drawing) is to read, understand and communicate the construction of an image [Valerio 2012b].
A case that to me seems exemplary of philology applied to the study of a drawing and of the need to not identify an error, but the motivation behind a derogation from the rules, is provided by the "spatial" drawing, (I would not know how else to define it, if not with this contradiction in terms), of a cuboctahedron, one of the 13 Archimedean polyhedra with six square and eight triangular faces, drawn by Piero della Francesca in his *Abacus Treatise* (fig. 108r).
It seems impossible that one of the founding fathers of linear perspective, as well as one of the most refined painter-theoreticians in the study of plane and solid geometry, to represent this solid in an apparently incorrect manner [Piero della Francesca 2012, pp. 126, 127].

Fig. 5. Superposition of the two perspective views of the cuboctahedron from the point S and from the point D (see figs. 3 and 4), (drawing by the author).



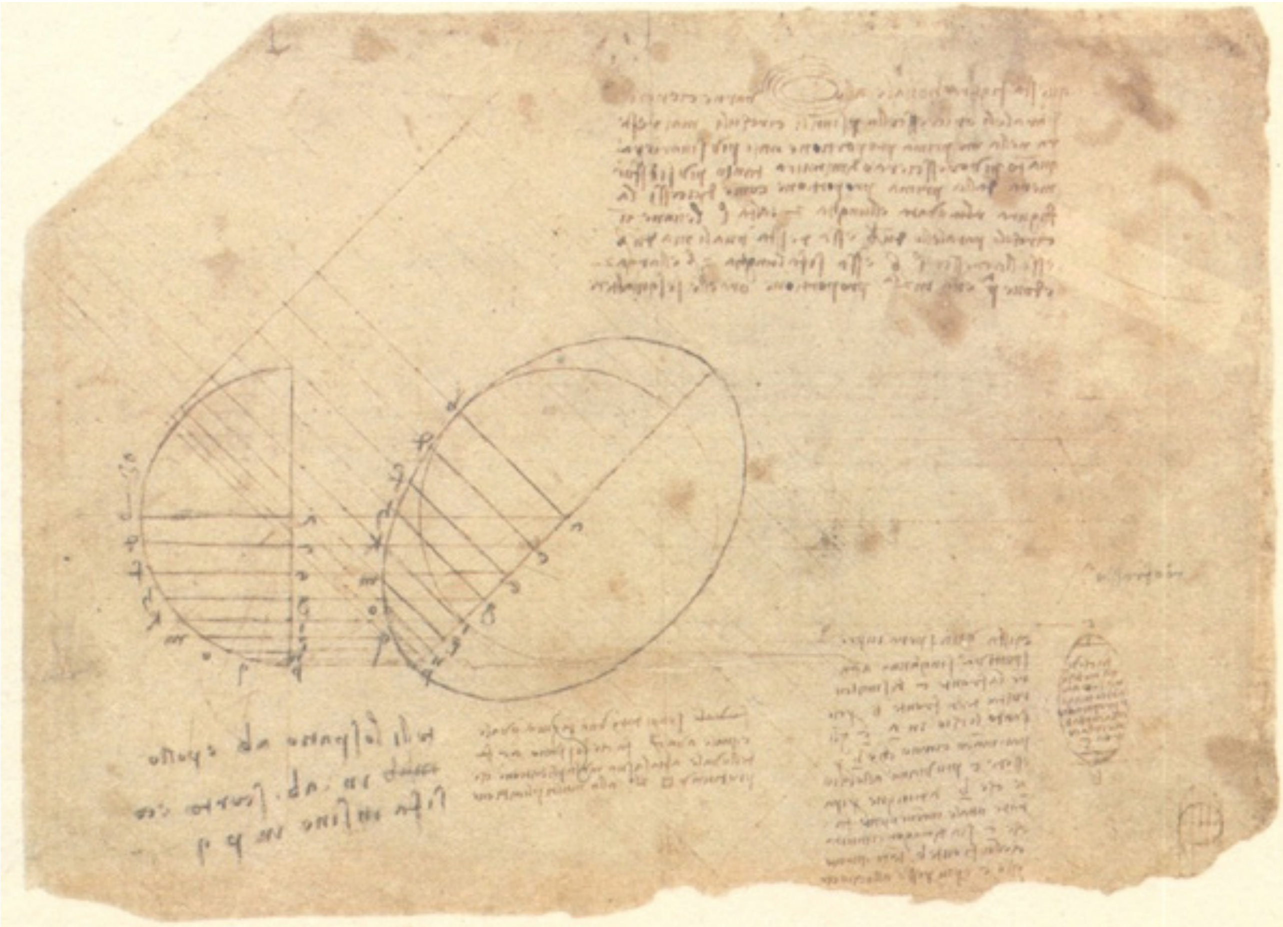
In this figure, two objectively parallel straight lines, deducible from the diagonals of two of the square faces, the lateral one (*a*) and the upper one (*b*), are not coherent (fig. 1); Piero does not draw parallel lines, as in a correct axonometry, that he knew well and applied, nor lines convergent towards the bottom, as in a linear perspective. The two straight lines converge towards the observer, generating the effect of an "inverted perspective". A possible solution to this dyscrasia can be found in binocular vision, whose parallax, for small objects placed to a short distance from the eyes, such as an arm, generates two different and divergent images.
In this condition (fig. 2), while the left eye (*L*) sees the left face under a greater angle and the right face foreshortened, the contrary occurs with the right eye (*R*). The perception of the object as a whole occurs through the sum of the angles α and β' and is greater than the vision of a single eye (figs. 3, 4), the angle α being greater than α', and the angle β' greater than β.
It is likely that Piero had a cuboctahedron in front of his eyes and that he was drawing it while holding it up with one hand. The use of models of geometric solids is known and attested to during the fifteenth century also by certain iconography, just think of the drawings realized by Leonardo for Fra Luca Pacioli's *De Divina Proportione*, published in 1509, and Piero della Francesca's drawing is a confirmation of this use.
The two faces of the solid seem less foreshortened in comparison to the vision seen with just one eye, giving rise to a sort of natural "inverted perspective" of perceptual origin (fig. 5).
Piero never adopted, not even in the *Libellus de quinque corporibus regolaribus*, a representation of solids through linear perspective, but mainly used an empirical system of parallel line construction, comparable to present-day axonometry. That Piero's main intention was the "legibility" of the solids and, at the same time, the display of their geometrical properties can also be seen in the persistent tangency of the vertices of regular solids to the circumference that identifies the circumscribed sphere, which is a real property of solids, but which is not preserved in the plane image.
For this reason, as we have said, Piero abandoned the use of perspective, of which he was a master, because this would have created foreshortenings and deformations that, given the complexity of some solids, would not have facilitated understanding them: he preferred the precision of description to geometric exactitude.
Of course, this justification for Piero della Francesca's inverted perspective, illuminating in this historical context, cannot fit all the cases of inverted or reversed perspective [Florenskij 1984; Scolari 2005].

Awarness

Another crucial theme that results from the application of philology to the study of drawings, and I do not make any distinction between maps and geometric drawings, is that of the "awareness" on the part of the author of what he is realizing: it is not enough for something to happen, to occur, for there to be an awareness of what is being done or observed.

The perfect ellipses drawn by Piero della Francesca in his paintings and by Leonardo in some of his drawings are not the conscious result of the projective transformation of the circle, but are the result of the application of graphic rules that operate according to projective mechanisms of which both Leonardo and Piero della Francesca were not fully aware [Valerio 2006]. Piero never mentions the word "ellipse" in any of his manuscripts, and when Leonardo finds a graphic system for creating an ellipse from a circle

Fig. 6. Drawing of an "ovata" (ovate) figure. From: Leonardo da Vinci, Codice Atlantico, f. 318b. Reverenda Biblioteca Ambrosiana, Milano.



(incidentally, perfectly drawn) he calls this form "ovataovate" and "ovale" (fig. 6) [Leonardo da Vinci 1973, f. 318b].

Geographical space and architectural space

I would now like to go back to the general theme of the representation of space with no other attributes of scale or content. There are historical moments in which reflection on the representation of space leads to a closer link between maps and, in general, the drawing of architectural and pictorial space become aspects of the same problem of representation.
It is worth analyzing at least two of these periods in Western history that have coincided with the birth of a real cognitive revolution, where drawing became a heuristic tool for revealing reality as well as for representing it: the Renaissance and the Enlightenment.

Everything in the right place

The revolution that took place in the Renaissance on the theme of drawing the space is closely linked to the discovery of a scientific text lost in the Western world and to a particular cultural *humus* which developed around the rediscovery and reinterpretation of ancient scientific culture: namely, Claudius Ptolemy and the early fifteenth-century Florence [Edgerton 1976; Valerio 2012a].
The Alexandrian scientist, active in the second century AD, was the first to write a treatise dedicated to the compilation of maps, a text dedicated to the drawing of images (this redundancy is desired) of the Earth. This was a disconcerting novelty in medieval cartography: to draw a geometric grid within which to place human and natural geographical features (cities, rivers, populations, regions, mountains). Each point on the earth's surface was not defined in relation to another on the basis of a description given by literary sources or by itineraries (egocentric descriptions), but on the basis of a coordinate reference system covering the entire surface of the terrestrial sphere (allocentric geometrical reference). These two terms in parentheses will be explained below.
Of course, all this implies the measurability of the world referred not to the approximate measurements of distances traveled on foot or by ship but to the position of celestial bodies, stars and the sun in primis. For these reasons, and perhaps not wrongly, Jacobus Angelus, the first translator of Ptolemy's *Geographia*, preferred to adopt the term *Cosmographia*, which was maintained in the fifteenth-century printed editions. If it is true that a careful reconstruction of Ptolemaic procedures does not lead to the definition of a true linear perspective based on the basic principles of projective geometry, it is also true that Ptolemy's entire geographical work suggests the possibility of placing objects in space using a reference grid, and the deformations that this grid undergoes are controllable, making it possible to respect the relative positions of objects in space: it allows, in short, the creation of a one-to-one relationship between a flat image and the space it represents [Valerio 1998]. The transition from the representation of geographical space to that of architecture and painting was made by the first Florentine humanists, who saw in the Ptolemaic text not only an operational tool but also a method that can be applied just as well to the regions of the world as to all objects placed in space, giving rise to the birth of linear perspective.
The relationship between geography and perspective is suggested to us by a refined theorist, Leon Battista Alberti, who was the first to describe the method of perspective in painting based on the definition of a square "drawing grid," a sort of system of geographical coordinates (or Cartesian coordinates *ante litteram*) whose drawing, according to the rules he indicated, allows objects to be placed in space with their respective positions and dimensions. Objects are "geometrically" foreshortened and not in an intuitive or simply perceptive manner, as was the case in ancient and medieval perspective. A proof of this mental attitude, of Ptolemaic origin, can be found in the language used by Alberti, who betrays his geographical debt: "*Quo pacto omnes, pavimenti parallelos descriptos habeo. Est enim parallelus spatium quod intersit inter duas aequidistantes lineas de quibus supra nonnihil tetigimus*" and a few lines after, referring to the heights of men, "*Ex quo fit ut picti homines in ulteriori parallelo steterint*" [Alberti, *De Pictura*, I, 20]. Further on, in the same text, just as he would do in the vernacular text, he does not abandon the use of the term "*paralelo*", nor those of "*latitudine*" and "*longitudine*" for indicating the dimensions and shapes of bodies.
For the first time, in the Western world, a process for the construction of a drawing that made it possible to biunivocally transform three-dimensional space into its two-dimensional representation. Cartographic drawing and architectural drawing are both based on the concepts of measurability and position. Pictorial space and geographical space are subject to the same laws of projective transformation, and the interconnection between the two representations of

space runs through the entire arc of scientific experiments from early Humanism to the height of the Renaissance, which saw painters, mathematicians, engineers, architects, scientists and astronomers engaged on the crucial theme of the relationship between a flat image and its three-dimensional counterpart [Kemp 1990].

It should not be forgotten, however, that at the basis of pictorial and cartographic representation of space during the Renaissance there are two different perceptions of space, defined conceptually only in recent years by studies on spatial cognition, a branch of cognitive psychology: one called "egocentric," whose reference is in the observer (as in the case of pictorial perspective), the other called "allocentric," with an external reference (as in the case of Ptolemaic cartography), and which sometimes coexist in the same representation without any contradiction because the synthesis takes place in the composition of the drawing. It is important to remember the distinction between perception (vision) and representation mentioned above.

In egocentric vision, the observer refers everything to himself and to his position while in allocentric vision, the eye of the observer is like that of God (*Apollo's eye*, as written by Denis Cosgrove with a well-turned metaphor [Cosgrove 2001]) who sees everything from a stable perspective not subject to the variability of points of view. They are two complementary, non-conflicting approaches, which have the same origin and which give rise to maps such as that of the territory of Verona in the mid-fifteenth century and of many other maps up to our day [Valerio 2019].

A perfect drawing

Three centuries later the humanistic revolution, in a utterly different historical context and with completely different motivations, this time the war to determined a profound reflection on drawing, the need to find shared ways of representation began to be considered, in order to make cartographic and architectural drawings comparable [Valerio 1987]. There is a text that can be taken as a manifesto of the new way of representing space on the two-dimensional plane of a drawing, and it is the *Mémorial Topographique et Militaire*, published in Paris in the autumn of 1803 [Mémorial 1803].

"*Cette représentation, c'est-à-dire, la manière de dessiner la topographie ou d'y suppléer par des notations ou signes de convention, […], était jusq'ici livrée à l'arbitraire; chaque école, ou plutôt chaque topographe, avait sa manière*." With these words, Joseph Pascal-Vallongue, general of the *Corps du Génie* and director of fortifications (as well as vice-director of the *Dépôt de la Guerre* in Paris), introduces the work of the commission specially convened at the *Dépôt Général de la Guerre*, made up of "*tout ce que les divers services avaient d'officiers ou d'employés les plus instruits en cette partie*". The commission was charged with the task "*de simplifier et rendre uniformes les signes et les conventions en usage dans les Cartes, les Plants et les Dessins topographiques*". We have to keep in mind that only about ten years earlier (1794), Gaspard Monge had given a name to an ancient and renewed discipline of drawing, *Géométrie Descriptive*, published for the first time in 1799 and mentioned several times in the *Mémorial*. The commission met in Paris between September and November of 1802, and within a few months the results were published in the fifth volume of the *Mémorial*. The seven volumes of the *Mémorial* published between 1802 and 1810 [Bret 1989] in which various technical provisions in the field of topography appeared (from advanced geodesy to surveying, from detailed drawing to military reconnaissance) together with historical studies for use in military art, closed an era of cartography and heralded in modern cartography and technical drawing.

Through a careful analysis and examination of contemporary production, uniform rules for drawing are established: clear and shared symbols relating not only to topography but also to mineralogy, hydrography and the distinction of troops for drafting battle plans. In short, all the aspects of cartographic production are analyzed, from the representation of orographic features to the use of colors, arriving at the definition of typographic characters, establishing their bodies and dimensions in various scales; the most suitable types of paper for the drafting of manuscripts and prints are also discussed, up to the techniques of engraving. In the *Mémorial-No.5* for the first time the perfection of a map, defined in no uncertain terms as a "*carte parfaite*," is associated with an exact correspondence to reality, not only as regards the metric aspect, which is not everything (here I refer to what I pointed out at the beginning of this paper), but also and above all the formal and communicative precision: topographical drawing, according to the Commission, must represent "*la nature elle-même revêtue de ses formes et de ses couleurs, mais réduite aux dimensions de l'échelle*" [Mémorial 1803, p. 41].

A drawing must allow those who observe or use it to immerse themselves in reality, to be able to experience it, we would say today, as a virtual reality.

A matter of scales

The only distinction that arises among all the drawings that somehow represent spatial problems that are geographical, urban or architectural is the scale of the drawing. Following the very recent introduction of the meter in Republican France (abolished with the downfall of Napoleon) and of the decimal system, the scales of drawings are determined in a univocal and universal way, as a direct relationship between them and reality, something that had never even been imagined before. Previously, the correspondence between a drawing and reality was filtered by modules and units of measure: a certain module of the drawing corresponded to a given real measure, the result was not a scale ratio but a modular ratio. This was a revolution in the way of conceiving drawing that no historian of architecture or epistemologist has ever given due attention to. Talk about the "scale" of a drawing before 1803 is a historical falsification. The scales we take from "ancient" drawings or maps (1:…) are the superimposition of our knowledge and our way of working on objects imagined and drawn with a completely different mentality.
The Commission of 1802 wanted to make drawings comparable, whosoever made them, or whatever the country in which they were produced, freeing them from local units of measurement, which gave rise to incommensurable scales, and referring them to a single and universal unit of measurement. We may call it a dream that ideally connects the Enlightenment of the late eighteenth century with the Positivism of the late nineteenth century. The Commission determined a taxonomy of drawings that groups, according to the scale, various types of spatial representations: from the "*Topographie de détaildetail*" (1:2,000 and 1:5,000) to the "*Topographie générale*" (from 1:10,000 to 1:100,000) and the "*Chorographie*" (from 1:200,000 to 1:1,000,000) and ending with the "*Géographie*" (1:2,000,000). Drawing, in its broadest sense, including that which "*suivent les ingenieurs des différens services, dans les plans et le dessins, et dans quelques cartes, relatifs aux traveaux publics*" is discussed in a special paragraph entitled "*Des projections et du dessin en general*" [Mémorial 1803, p. 16]. In a table created in order to determine the width of the types to be used for each scale they start even from the 2 to 1 scale (used for industrial drawings) to arrive in an uninterrupted sequence at the scale 1: 20,000,000 (fig. 7).
Representations of space, we could say, from the microcosmos to the macrocosmos, find in the *Mémorial* of 1803 their unity through scale ratio and the rules that govern its drawing.
However, even because of the political will to deny the scientific results born from the revolution, after the Congress of Vienna and during the nineteenth century, maps in scales based on local measures and not decimals continued to be published: just think of the scale of 1:86,400 (one inch for 1,200 Viennese *klafters*) adopted for the Austrian map of the Kingdom of Lombardy-Venetia of 1851, or the scale of 1:66,360 (one inch for a British mile) of the Ordnance Survey realized starting from 1817 [Seymour 1980].
The dream of the scientists of the French Revolution would only slowly come true during the nineteenth century, to then reach us, projected into a virtual reality without scales, but perhaps even further from reality.

In memoriam Anna Sgrosso (1927-2019)

Fig. 7. "Tableau présentant . . . les Types des hauteurs d'Écritures affectés aux Échelles adoptées". [Mémorial 1803, p. 98].

N.os des Triades.	N.os des Échelles.	DÉNOMINAT.ons	RAPPORTS avec la grandeur des objets, en Décimales.	RAPPORTS avec la grandeur des objets, en Fractions ordinaires.	VALEURS des RAPPORTS CI-CONTRE exprimés exactement en anciennes mesures.	HAUTEURS en décimil.es du type des écritures ou des noms de villes du 1.er ordre à chaque échelle.
		1 centimètre pour				
1	1	5 millimètres.	2.0	2/1	1 pied pour ½ pied.	"
	2	1 centimètre.	1.0	1/1	1 pied pour 1 pied.	"
	3	2 centimètres.	0.5	1/2	1 pied pour 2 pieds.	"
2	4	5 centimètres.	0.2	1/5	14 po. 4 lig. 8 pour 1 t.	"
	5	1 décimètre.	0.1	1/10	7 po. 2 lig. 4 pour 1 t.	"
	6	2 décimètres.	0.05	1/20	3 po. 7 lig. 2 pour 1 t.	"
3	7	5 décimètres.	0.02	1/50	1 po. 5 lig. 28 pour 1 t.	"
	8	1 mètre.	0.01	1/100	" — 8 lig. 64 pour 1 t.	"
	9	2 mètres.	0.005	1/200	" — 4 lig. 32 pour 1 t.	"
4	10	5 mètres.	0.002	1/500	14 po. 4 lig. 80 pour 100 t.	375 "
	11	1 décamètre.	0.001	1/1,000	7 po. 2 lig. 40 pour *id.*	250 "
	12	2 décamètres.	0.0005	1/2,000	3 po. 7 lig. 20 pour *id.*	190 "
5	13	5 décamètres.	0.0002	1/5,000	1 po. 5 lig. 28 pour *id.*	150 "
	14	1 hectomètre.	0.0001	1/10,000	" — 8 lig. 64 pour *id.*	100 "
	15	2 hectomètr.es	0.00005	1/20,000	" — 4 lig. 32 pour *id.*	75 "
6	16	5 hectomètr.es	0.00002	1/50,000	" — 1 lig. 72 pour *id.*	60 "
	17	1 kilomètre.	0.00001	1/100,000	" — 0 lig. 86 pour *id.*	40 "
	18	2 kilomètres.	0.000005	1/200,000	" — 0 lig. 43 pour *id.*	34 "
7	19	5 kilomètres.	0.000002	1/500,000	" — 1 lig. 72 p. 1,000 t.	30 "
	20	1 myriamètre.	0.000001	1/1,000,000	" — 0 lig. 86 pour *id.*	25 "
	21	2 myriamèt.es	0.0000005	1/2,000,000	" — 0 lig. 43 pour *id.*	21, 25
8	22	5 myriamèt.es	0.0000002	1/5,000,000	" — 1 li. 72 p. 10,000 t.	19 "
	23	1 grade.	0.0000001	1/10,000,000	" — 0 lig. 86 pour *id.*	16 "
	24	2 grades.	0.00000005	1/20,000,000	" — 0 lig. 43 pour *id.*	13, 60

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