Geometric and Instrumental Measurement in Representation

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Abstract

The recurring references to the topic of measurement in perspective studies between the seventeenth and eighteenth centuries highlight a return to objectivity in the method of perspective representation at a time when more emphasis was placed on its perceptive aspect. The present contribution examines a series of treatises, which contributed to achieving the perspective construction system of the measuring points, together with the evolution of a number of instruments used to measure architectural structures and spaces.
The manuscript on perspective by Jacques Alleaume, written in 1627 and posthumously published by the mathematician Estienne Migon (La perspective speculative et pratique, 1643), contains one of the first methods to represent segments in perspective, which are assigned by values of inclination and length.
The identification of the measuring points through the progressive generalisation of the problem is first found in the Cours de mathématiques by Jacques Ozanam (1693) and then in the work by Brook Taylor (Linear Perspective, 1715), whose most noted contribution was the systematisation of the reverse perspective method.
The claim by Ozanam to extend the use of the Universal Instrument to practical geometry (1688) reflects the general focus on the topic of measurement with regard to the fundamentals in theoretical mathematics, the scientific value of the representation methods, and the instruments to survey architectural structures.

Keywords: Drawing, Perspective, Measurement, History of Representation, Survey Instruments.

Perspective between perception and measurement

«Aveva il senso della misura ma non quello delle proporzioni.
Per questo non fece molti errori in vita sua, ma tutti colossali.»
[Mauro Parrini 2009]

The most evident purpose of perspective is to create three-dimensional perception of a represented subject. Unlike cylindrical projections, in order to obtain measurements from conical projections it is necessary to apply inverse procedures that are so complex that the role of measurement in perspective may be overlooked as a result, even though it has always had a strong tradition in architecture. This aspect is connected to artistic and instrumental applications, which developed over time and have relevance even today, such as digital photogrammetry that has been increasingly spreading and used for various purposes than ever before.
The present contribution pinpoints some key moments in this evolution, referenced in books on the history of perspective and survey, and in studies on the relationship between architecture, science, and technique. The application of these innovations to surveying methods demonstrates the versatility of this field of study, which encompasses all methods of representation of the space on a plane [1].
The close relationship between measurement and architecture is often mediated through representation, because representation is recognised as the preferred method to quantify the value of a spatial concept from a physi-

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cal, aesthetical or functional point of view. Currently, the theme of measurement has a tangible feedback in the challenges of contemporary architecture, which combines historical visions and current designing proposals, also through their poetic interpretation [2].
The search for configuration of the proportional ratios between architectural elements seems to be at the core of the contributions by Filippo Brunelleschi (1377-1446). Besides the extraordinary and demonstrative importance of his two lost perspective panels (their age is still debated today, between 1413 and 1424) [3], he was interested in the aspect of measurement –not only perception– concerning the new (or rediscovered) representation method of three-dimensional reality on a plane.
His choice of a symmetrical subject for his first experiment, the Florence Baptistery, allowed Brunelleschi to overcome the discrepancies connected to the method that was probably used to perceive representation, i.e. specular representation. The first method of perspective representation in the Renaissance and many of its subsequent echoes in the arts seemingly cannot be compared with the configuration methods that are found mainly in the current figurative world, which is often specifically focused on going beyond the symmetry rules of classical architecture and its derivatives [Cohen 2001].
From the point of view of perspective representation, the use of measurement was likely introduced with the perspective construction using measuring points, besides the processes of reverse perspective, even if evident signs of this connection can be noted in the entire history of representation.
Of the wide and unending debate on perspective between theory and practice in the Fifteenth and Sixteenth centuries, it is relevant for the purposes of the present study to mention the discussion on the introduction of the distance points, as the forerunner of the methods more generically adopted by the measuring points.
Within pictorial art, Leon Battista Alberti (1404-1472), in his treatise *De Pictura*, was the first to illustrate a perspective method that, even if limited to one particular case, was able to offer painters the possibility to create a measurable space [4]. The checkerboard floor, projected on a three-dimensional space, provided a model for painters to draw accurately, while at the same time leaving them considerable freedom, as much like metrics gives to poetry. In fact, the aim of the great architect and humanist was notoriously to elevate figurative arts to the same dignity as liberal arts. He sought to achieve this by changing the concept of the bottega (workshop) itself, which started to adopt a more scientific approach using perspective constructions.
In Alberti's construction, it is possible to define the decreasing distance in perspective between the equidistant parallel lines, perpendicular to the picture plane, through the rabattement of the observer's position. Apart from a number of significant precursors [5], from this moment onwards painters started applying this same method in various paintings. This can be verified thanks to one parameter described in Alberti's text and based on aligning the intersection points of the checkerboard floor: this is not yet an intentional use of the distance points, but there is undoubtedly the same geometrical reasoning at work.
There are plenty of transgressions in the paintings by those artists who understood and valued the corrective effects of perspective, but could not renounce a certain degree of autonomy in their art. In fact, there are often compromises to conciliate the contradictions resulting from diminishing one dimension in the attempt to represent reality, which then alter even the most evident perspective rules. In some paintings, the parallel lines do not converge in one vanishing point or, more frequently, there is an alteration of the measurements in depth, as one can observe in the works by some of the greatest practitioners in perspective art, such as Paolo Uccello[6].
The present contribution will not explore the complex evolution of the discipline's theory and practice, but it is worth noting that in the sixteenth century the transgressions to the norm became increasingly more elegant, while measurement was becoming consolidated even more so as an element that could be controlled through perspective. According to some critics, distance points were first used to their full extent by Jean Pélerin, known as Viator [Pélerin 1505] [7] but there are earlier illustrations in works by Leonardo da Vinci and Piero della Francesca [Piero della Francesca 2017].
Jacopo Barozzi, known as Vignola (1507-1573), and Egnatio Danti (1536-1586) described the two constructions [Barozzi da Vignola 1583, pp. 69, 100] [8], i.e. the *'albertiana'*, based on the intersection, and the *'pierfrancescana'*, using distance points. Combining the two methods (fig. 1) shows that the intersection rule (on the left, with the distance point *D*, as a rabattement of *V*) and the rule of the distance point (on the right, with the profile of the picture plane π and the point of view *V*) lead to the

same result. The text by Vignola and Danti also describes a device used to unveil secret perspectives [Barozzi da Vignola 1583, p. 96], which, even if notoriously recognised as an approximate method to generate or interpret perspective anamorphosis, is still effective to reproduce the proportion of an otherwise deformed drawing (fig. 2).

Seventeenth-century contributions between measurement and projection unification

A number of significant events in the Seventeenth century led to the revival of measurement in perspective, thus affirming a common basis for what are classified today as methods to represent space on a plane.
Among the pioneering texts, critics recognise the text on perspective by Guidobaldo Bourbon del Monte (1545-1614), written in 1600 [Del Monte 1600], with direct perspective constructions of figures (second book), which pioneered the studies on recognising projective relationships between the projections of the same object.
Today the role of Girard Desargues (1591 1661) is also commonly acknowledged for his text [Desargues 1636], in which he explained a method based on identifying the reduction of measurements in perspective. Without using distance points or other points outside the picture plane (as stated in its title) and based on dimensional perspective scales, Desargues utilised the only proportion that is maintained in all types of projections: the cross-ratio. Also, the contributions deriving from Desargues' text on conical projections in 1639 [Desargues 1639], intended as central projections of a circle, introduced the improper elements (points and lines at infinity) that constitute the essential premise in projection unification [Docci, Migliari, Bianchini 1992].
In 1605, Simon Stevin (1548-1620) [Stevin 1605] recognised –even after the rabattement of both the picture plane and the horizontal plane that crosses the horizon line– the characteristics of the perspective image are maintained, and he identified that the projective ratios between the real figure on the geometric plane and its perspective image are also maintained [9].
Regarding the relationship between perspective and the measurements of the represented object it is important to pinpoint the premises and the systematisation of the construction using the measuring point system. The manuscript on perspective by the engineer Jacques Alleaume

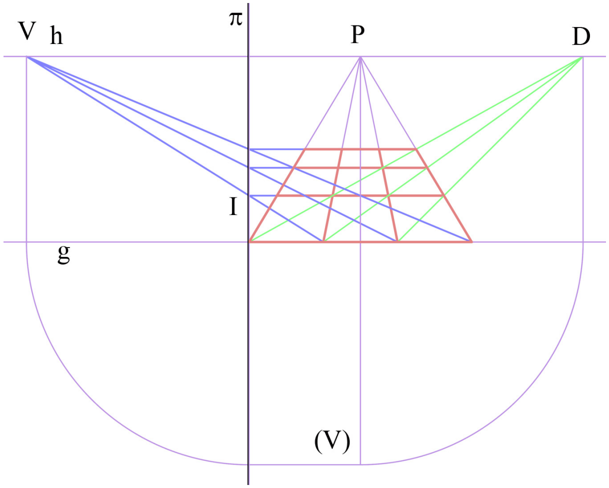


Fig. 1. The intersection rule (on the left of the drawing) and that of the distance point (on the right of the drawing) lead to the same result.

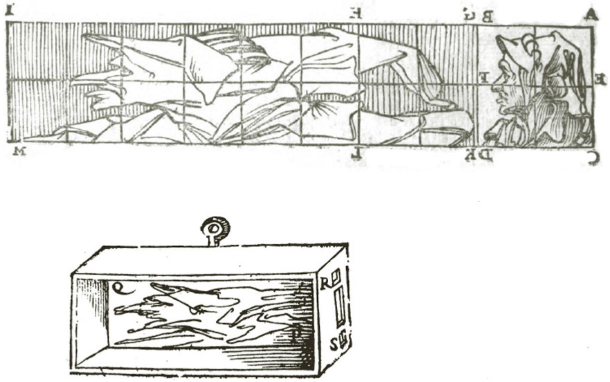


Fig. 2. Pitture segrete [Barozzi Vignola 1583, p. 96]. An approximate construction of anamorphosis.

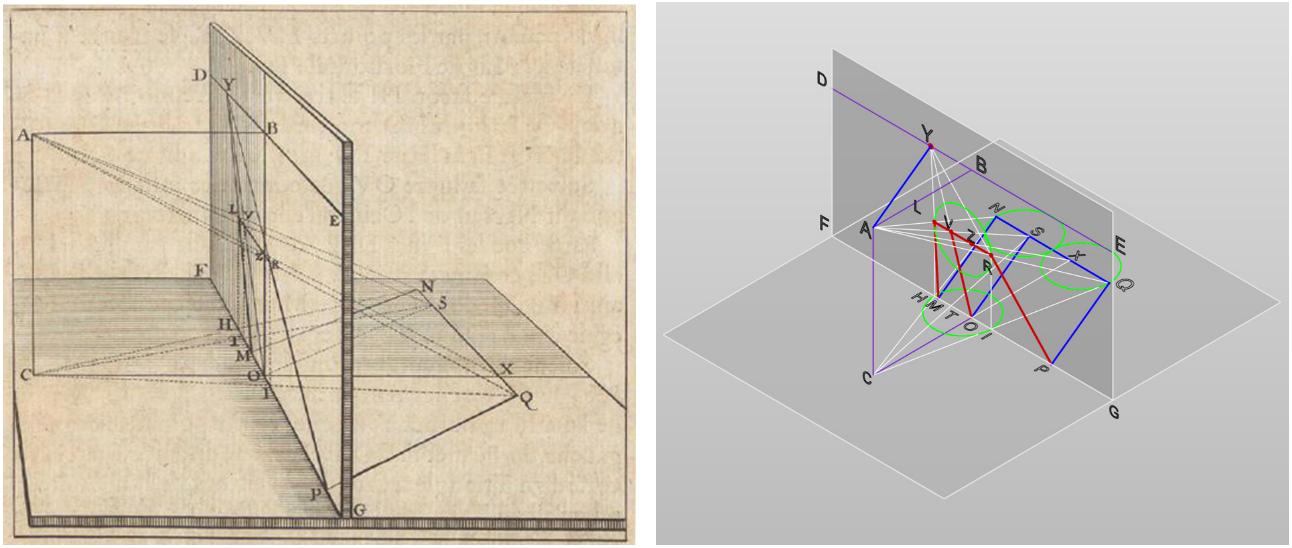
(1562-1627), titled *Introduction à la perspective, ensemble, l'usage du compas optique et perspectif* [10] is considered an extremely significant contribution to this topic. After its author's death, the text was not printed in its original version and it is now lost. Mathematics professor Estienne Migon bought the book's rights and published it with some adaptations and additions [Alleaume, Migon 1643] –that cannot be clearly differentiated from the original text– in order to give credit to Alleaume for the perspective method by Desargues [11]. In his text, articulated in eight propositions and eleven problems, there are even pioneering elements of the measuring points in *proposition V* and *problem VII* [Vagnetti 1979, pp. 385-398].

Proposition V [Alleaume, Migon 1643, pp. 39-43] illustrates a spatial system aiming to demonstrate the concurrence of the images of the parallel lines in one point of the horizon line (fig. 3). The process is complex but it does not offer general elements useful to recognise the precise ratios of measurements in perspective, because the provided example refers to the particular case of one segment placed at a distance from the picture plane equal to the main distance. In fact, the observed doubling and halving (e.g. *HI = 2 NQ*) do not seem to aim towards measuring functions, but more towards the construction of similarities able to lead to the demonstration of the concurrence in one single vanishing point (*Y*) of the parallel lines.

In *problem VII* [Alleaume, Migon 1643, pp. 128-130] there is a greater focus on the theme of measurement in perspective, as it sought to provide a method to construct segments of a desired length and angulation (fig. 4).

This contribution provides the following an interpretation of this proposition, which reveals its original structure, but also its limited general application. With the given line *NL*, with *N* as its intersection point with the horizontal line and *L* as its intersection point with the horizon line (i.e. its vanishing point), the perspective of one of its segments of a 16-unit length (it could be any unit of measurement, in this case feet), starts from any point *M* on it. From *N* count 16 units (or multiples) along the ground line –scaled appropriately– and identify point *O*, which connects with *L* and allows *MP* to be drawn, which represents in perspective a 16-feet long segment, parallel to the horizon line. To project this measurement on the half-line *ML*, consider that, because *ML* forms an angle

Fig. 3. On the left, Proposition V [Alleaume, Migon 1643]. On the right, three-dimensional elaboration with the proportional ratios due to the position of NQ, (graphic elaboration by the author).



of 58° with the ground line, one can consider *MP* as the side of an isosceles triangle, the other side of which, *MR*, along *NP* has to be determined. This triangle *PMR* must have two equal angles at its base, the sum of which is supplementary to 58°, i.e. 122°, which divided by two shows that *MPR* = *MRP* = 61°. For this reason, from point L count 61 notches on the horizon line, to identify point *Q* and draw the line *QP*, which intersects the line *NL* at the desired point *R*. In this way *MR* = *MP* = 16 feet. *Problem VII*, actually, seems to provide completion of the entire text's programme stated in its title [Alleaume, Migon 1643], which refers to an original method of perspective construction that does not use the main point or the distance points, but it utilises the horizon line. In fact, this method's validity seems to be connected with the accurate construction of the chart described previously in the text, particularly with the graduation of the horizon line [Alleaume, Migon 1643, from p. 73 ff.) that allows one to delineate the images of the lines through the knowledge of their inclination with respect to the picture plane.
It appears then that one cannot find any general guidelines, contrary to what was pre-empted by Simon Stevin, of whom Alleaume had been a scholar of [12]. In fact, Alleaume seems uninterested in these topics, but focuses only on the creation of a graphic scale to draw the perspective of predominantly linear elements [13]. Unfortunately, it is not possible to find out what was in Alleaume's dissertation on the perspective compass, which was omitted from the 1643 text, because it should have been discussed in one of two other volumes that Migon never managed to write. The possible connection between this tool with the graphic chart described previously is destined to remain hypothetical as well as its potential link with other optical tools studied and designed by Alleaume himself [Molhuysen, Blok 1912], whose original work still remains largely in the dark.
The observations on measurement in perspective by Jaques Ozanam (1640-1718) seem to have an extremely different origin than Alleaume and Migon's. Ozanam notably studied vastly diverse mathematical applications with the aim to make the discipline of mathematics more accessible, also through illustrations of its most paradoxical and amusing aspects [14]. In a complex educational text [Ozanam 1693], Ozanam studied perspective in great depth without simply listing known elements, in fact the text shows the first introduction of measuring

Fig. 4. Problem VII [Alleaume, Migon 1643]. Graphic elaboration with elements of linear and angular elements (graphic elaboration by the author).

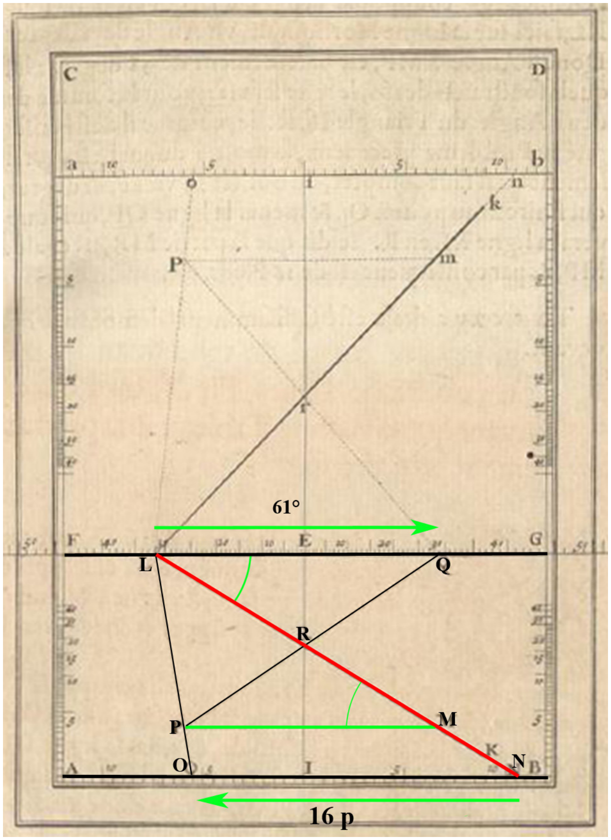


Fig. 5. On the left, Chart 7, fig. 14 [Ozanam 1693]. On the right, application of point D to obtain equal segments along AV (graphic elaboration by the author).

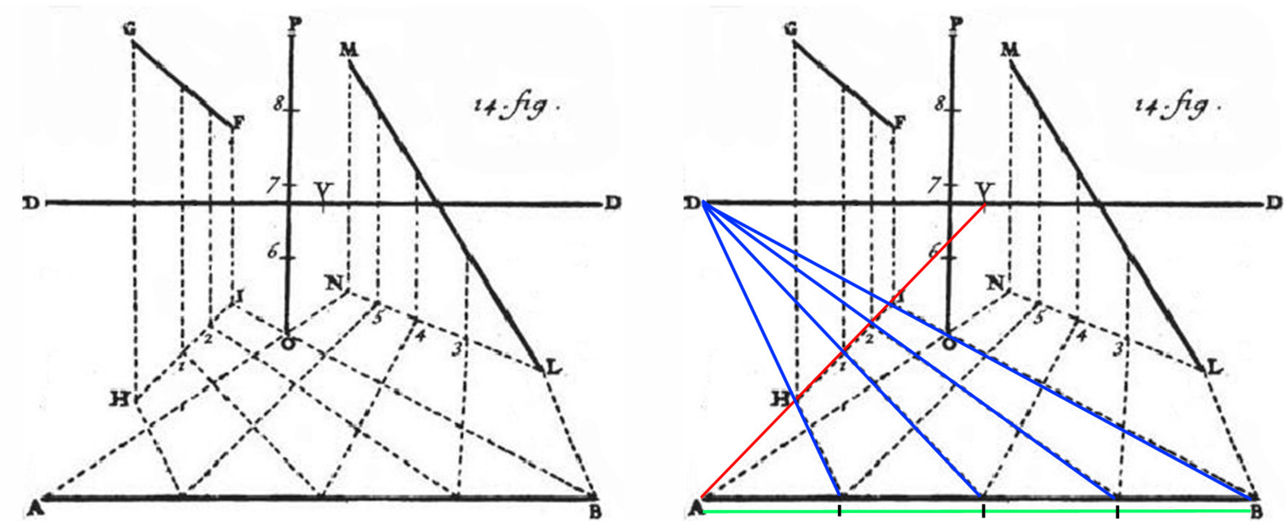
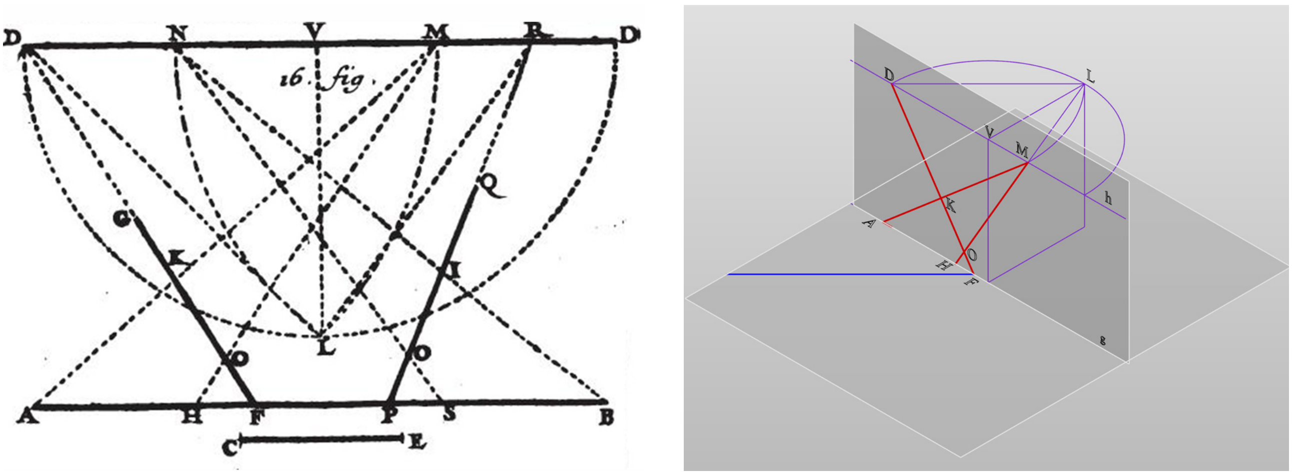


Fig 6. On the left, Chart. 8, fig. 16 [Ozanam 1693]. On the right, three-dimensional elaboration with the spatial ratios between D and M, measuring point (graphic elaboration by the author).

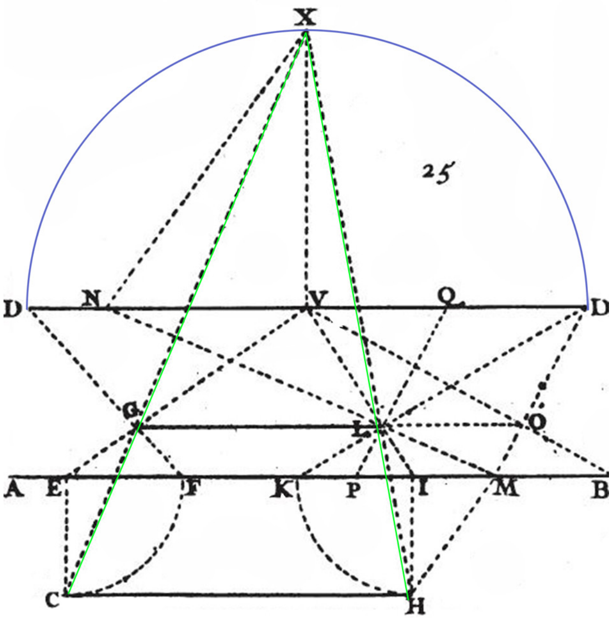


points [Andersen 2007, pp. 407-409], achieved through a method of progressive generalisation.
Having identified generic auxiliary vanishing points that allow the division of one segment into equal parts without, however, checking the real measurement of the segment and its parts [Ozanam 1693, chart 6], Ozanam attributed to the distance points the property of enabling the measurement of the direction orthogonal to the picture plane (fig. 5) [Ozanam 1693, chart 7, fig. 14], as shown in the earlier studies between the Fifteenth and Seventeenth centuries, cited previously.
The decisive passage is provided by the identification, through the rabattement of the viewpoint, of the vanishing point *M*. This direction enables the measurement of the segments on the lines at an angle of 45° with respect to the picture plane (fig. 6) [Ozanam 1693, chart 8, fig. 16], as for example segment *OK*, measuring *AH*: this is how the measuring point was created. It was destined to be key in the field of architectural representation.
It should be observed that Ozanam also developed the construction for the rabattement of the viewpoint and its use in perspective constructions (fig. 7) [Ozanam 1693, chart 8, fig. 25], illustrating the projective relation generated between the real object (segment *CH*) and its perspective projection (*GL*), the extremities of which are aligned with the rabattement of the viewpoint *X*. This scheme might have been inspired –or it could be simply a coincidence– by construction in the text by Philippe de La Hire in 1673 [15] [Andersen 2007, p. 344], identifiable by a homology, which has its centre in the rabattement of the viewpoint and the axis in the ground line.
The question is still open, whether the method of the measuring points was applied, or even invented, previously in the artistic field. The experiments of reverse perspective on seventeenth-century paintings show a degree of license when it came to the geometric norms, which is comparable to the artistic license in previous periods and is based on the necessary comparison between the objective and perceptive data of the represented object. This remained valid even in the moment of greatest diffusion of geometric logic. A rigorous result can be obtained even without the aid of measuring points, as it is true with distance points, and the effectiveness of these methods would be possible to prove with evidence remaining recorded on the supports, canvasses or plastering of the paintings, on which the paintings were carried out [16].

Measurement and dis-measurement: instruments for a scientific and transgressive representation

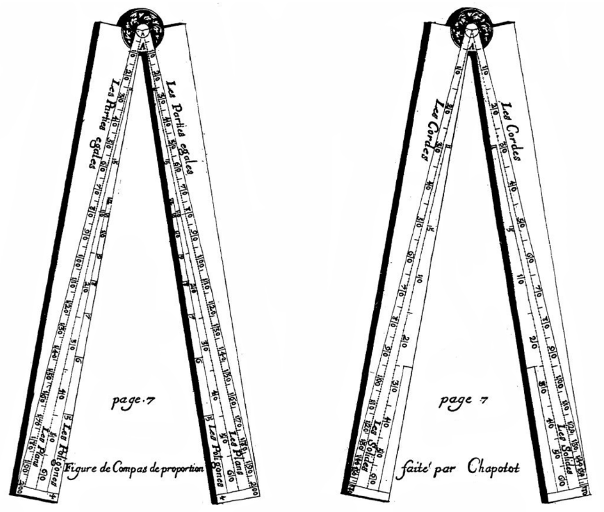
Ozanam's interest in the topic of measurement becomes evident in his texts on practical geometry and survey, such as his monographies dedicated to a number of measurement instruments. The text on the proportional compass, *L'usage du compas de proportion* [Ozanam 1688a] provides an extensive explanation of the instrument (fig. 8), which Galileo Galilei (1564-1642) had illustrated in 1606 [17] and as an instrument which is useful to carry out measurements and calculations applied to all the processes related with the art of war, ballistics or measurement of distances and places.
In the same year, Ozanam wrote a book on the universal instrument [Ozanam 1688b], which was constituted by a small rectangular board with fixed mobile rulers (fig. 9) used to measure angles even on vertical planes, to draw pa-

Fig. 7. Chart 12, fig. 25 [Ozanam 1693]. Graphic elaboration with the projective relation that is generated between segment CH and its projection GL (graphic elaboration by the author).



rallel and perpendicular lines and to draw plans, as it can be observed in the book's IX chart (fig. 10), where it explains how the instrument can be used to enlarge drawings through its homothetic relations.
Ozanam dealt with methods of measurement in his book *Traité de l'arpentage, et du Toisé* [Ozanam 1699], which were not particularly original, but extremely practical and very popular at the time. In his other book *Méthode de Lever les Plans et les Cartes* [Ozanam 1693], Ozanam examined the theme of surveying with a more systematic approach, with a thorough description of other instruments, such as the semicircle and, again, the universal instrument. These tools are based on the principles of ancient instrumentation for territorial and astronomical measuring (for example the Jacob's staff and the quadrant), and on the transposition of the angular measurements of natural vision into objective angular and linear measurements: this scientific foundation can also be found in the projective principles of the geometric linear perspective.
Jean François Niceron's book, *La perspective curieuse*, [Niceron 1638] contains an interesting chapter on the use of distance points, measuring the direction orthogonal to the picture plane. In it, distance points were given the function of measuring the distortion applied in the

Fig. 8. Jacques Ozanam, L'usage du compas de proportion, 1688 [Ozanam 1688, p. 7].



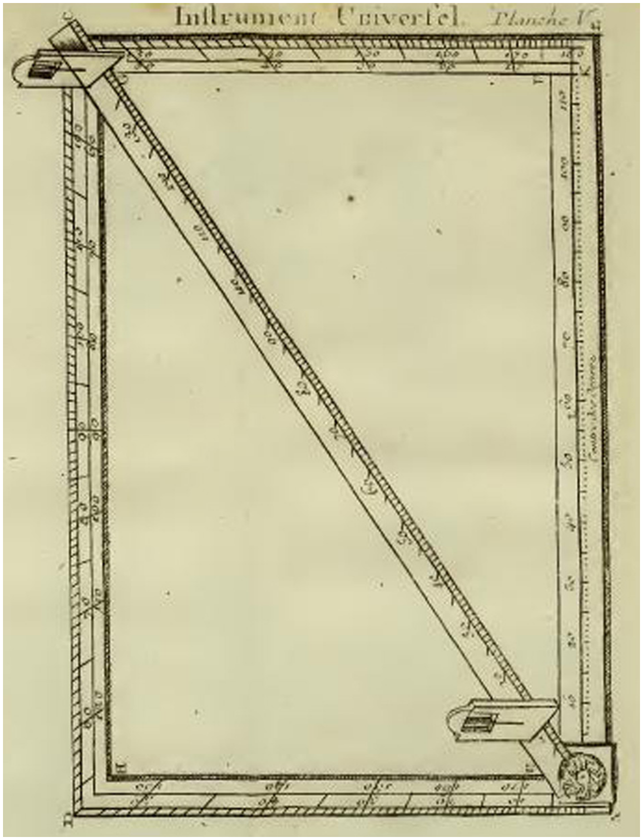
anamorphosis (fig. 11) and lead to an evolution of the approximated solution by Vignola and Danti [18].
To achieve a reversible relationship between the object and its perspective representation, it is important to quote the work by the English mathematician Brook Taylor (1685-1731) [19]. His text, written in 1715, titled *Linear Perspective or a New Method of Representing justly all manner of Objects as they appear to the Eye in all Situations* [Taylor 1715] was a theoretical and practical treatise on perspective, and the theory of shadows and reflections. The book was criticised for its complex and succinct language, which prompted Taylor to compile a second edition four years later, longer than the original text [20], which described a construction of the measuring points that resembled that by Ozanam [Taylor 1719, fig. 17]. The new text included two sections: the first one dealing with perspective construction and related topics, while the second section of the book was dedicated to reverse-perspective constructions (fig. 12). Lastly, the book had two appendices dealing with perspective on curved surfaces and the theory of Isaac Newton on colour. Taylor's research on the inverse problem might have gone on to influence the work of Johan Heinrich Lambert in 1759 [21] and contained the principles of photogrammetry that form the basis of the tools that are still used today in architectural surveying [22].

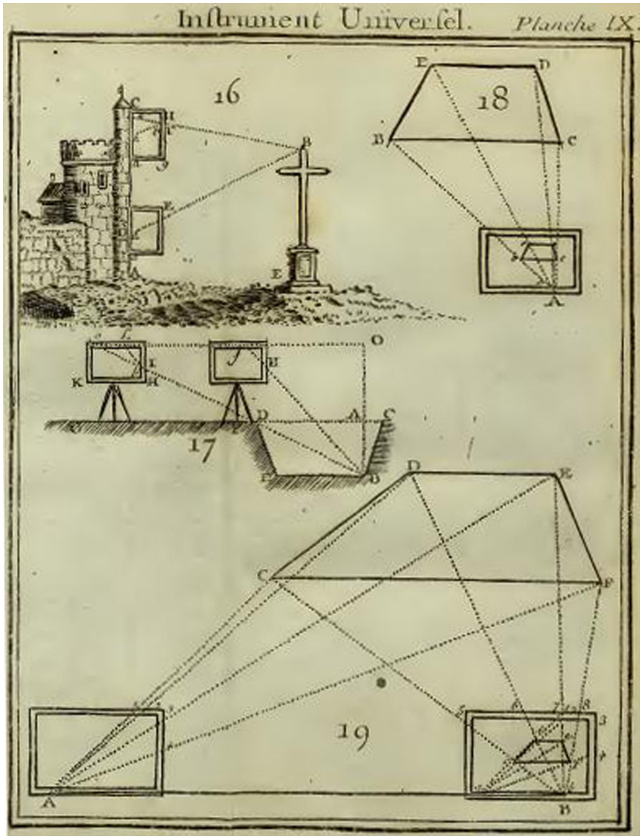
Conclusions

Between the Seventeenth and Eighteenth centuries, the importance of the perspective function of controlling measurements was gradually restored alongside the creation of perceptively significant images. There were many relevant contributions for the progressive focalisation of the problem, as demonstrated by the indications in the texts by Guidobaldo Burbon del Monte [Del Monte 1600], Simon Stevin [Stevin 1605] e Girard Desargues [Desargues 1636; 1639]. Fundamental milestones were the introduction of the perspective measuring systems in the text by Jacques Alleaume and Estienne Migon [Alleaume, Migon 1643], the identification of the measuring points by Jacques Ozanam [Ozanam 1693] and the systematisation of the reverse method by Brook Taylor [Taylor 1715] and Johan Heinrich Lambert [Lambert 1759].
An introduction to the measuring points can be observed in the use of the distance points, as measurements

Fig. 9. Jacques Ozanam, L'usage de l'instrument universel, planche V, 1688 [Ozanam 1688b]: the universal instrument.

Fig. 10. Jacques Ozanam, L'usage de l'instrument universel, planche IX, 1688 [Ozanam 1688b]: the application of the universal instrument to the surveying of spaces, and its application to scale drawings.





for the direction orthogonal to the picture plane, as well as in the continuity of theoretical studies and their instrumental application in surveying.
Measurement is in the end still one of the fundamental elements of perspective. Projective implications allow us to extend the range of our interpretations, because the reciprocal relationship created between the two projections, orthographical and perspective, leads to a relative understanding, as each projection represents one version, distorted in its own unique way, of the same three-dimensional object [Cohen 2001, pp. 54, 55]. There is therefore a strong –although contradictory– relationship between the most rigorous developments in perspective and the most peculiar ones [Barozzi Vignola 1583; Niceron 1638], both belonging to the same scientific field, which opened up to the infinite implications of representation with regard to conceiving and interpreting spatial phenomena.
The wide diffusion of these perspective innovations and their application to the instruments and methods used in surveying demonstrate, yet another time, the versatility of perspective theory as well as the strong relationship between drawing, architecture, science, and technique.

Fig. 11. Jean-François Niceron, Chart 13,1638 [Niceron 1638]: anamorphosis correctly built with the aid of the distance to points, used in this case not to make measurements proportionate, but instead distort them.

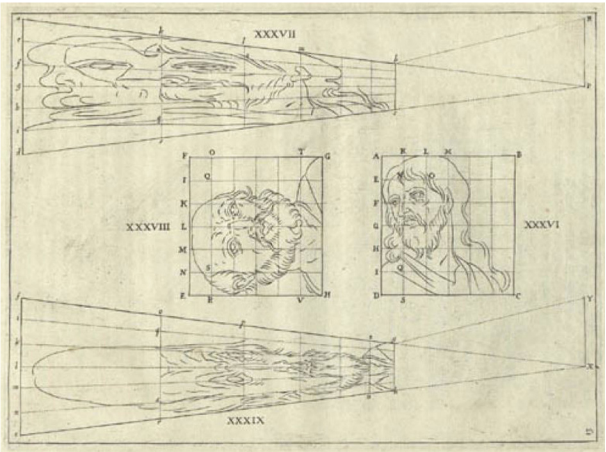
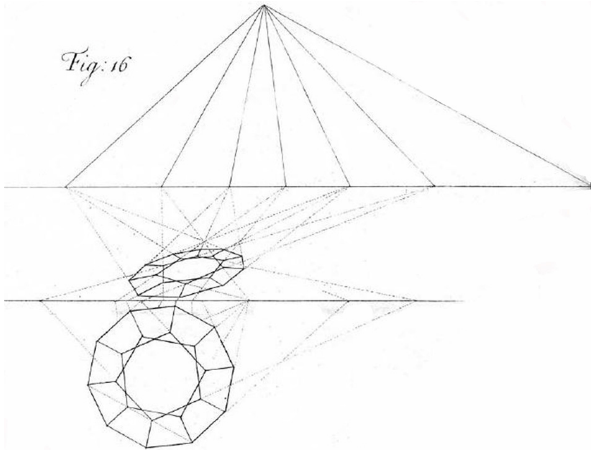


Fig. 12. Brook Taylor, Fig. 16, 1719 [Taylor 1719]: perspective and reverse perspective.



Notes

[1] Among the many reference texts, see: Docci, Maestri 1993; Sgrosso 2001; Giordano 2001; Camerota 2006; Paris 2014.

[2] Consider the significant title of the collections of essays on the relationship between architecture and art by James S. Ackerman [Ackerman 1991] and the exhibition held in New York *Measure. Exhibition. Storefront for Art and Architecture*. New York, August 14th, 2015-September 19th, 2015.

[3] The present contribution can provide only few references to study this topic in further detail: Damish 1987; Camerota 2006.

[4] For the dating of the Latin and vernacular editions, see: Sinisgalli 2006.

[5] See the Holy Trinity by Masaccio, Florence, Santa Maria Novella, 1426-1428.

[6] As in the Miracolo dell'ostia profanata, Urbino, Palazzo Ducale, 1465-1468

[7] For the Viator's text, see Brion-Guerry 1962. An illustration of the distance point method by Leonardo can be found in the Ms. A, Institut de France, Paris, see: Massey 2003, p. 163.

[8] The text *Le due regole della prospettiva pratica* was compiled by Vignola between 1530 and 1545 and pubblished by Egnatio Danti in: Barozzi da Vignola 1583.

[9] Rabattement, *Theorem 5, proposition VII; Theorem 6, proposition VIII*.

[10] AlleaumeJ. (1627), *Introduction à la perspective, ensemble, l'usage du compas optique et perspectif*, (Ms).

[11] For the known sequence of events, see: Amodeo 1933, pp. 7, 8, 33-37; Vagnetti 1979, pp. 385 and 398; Sgrosso 2001 pp. 269-270; Andersen 2007, pp. 418-427.

[12] Alleaume was also a scholar of the mathematician François Viète (1540-1603). See: Molhuysen, Blok 1912.

[13] The work by Migon can be considered an anticipation of the measuring points according to Vagnetti [Vagnetti 1979, p. 398] and Anna Sgrosso [Sgrosso 2001, p. 269], who inserted the image of a graduated circle that cannot be found in any of the two consulted [Alleaume, Migon 1643; 1663].

[14] Ozanam, J. (1694). *Récréations mathématiques et physiques, qui contiennent plusieurs problèmes d'arithmétique, de géométrie, de musique, d'optique, de gnomonique, de cosmographie, de mécanique, de pyrotechnique, et de physique. Avec un traité des horloges élémentaires. Paris: Jean Jombert*. For the work by Ozanam, see: Càndito 2015.

[15] Philippe de La Hire, *Nouvelle méthode en géométrie pour les sections des superficies coniques et cylindriques*, Paris 1673. See: Andersen 2007, p. 344.

[16] Marks can be detected with instruments that are not limited to photographic shooting. For some studies on this topic, see: Valenti 2014; Bartoli 2015.

[17] Galilei described it in its pamphlet *Le operazioni del compasso geometrico et militare*, 1606, but the instrument had been described previously, for example by Fabrizio Mordente and by Muzio Oddi. See: Mordente, F. *Modo di trovare l'astrolabio*. Venezia 1567; Oddi, M., *Fabrica et uso del compasso polimetro*. Milano 1633.

[18] De Rosa 2013, pp. 13-17. For the anamorphosis: Camerota 1987.

[19] Brook Taylor is mostly known for the famous formula, that bears his name, presented in *Methodus Incrementorum Directa et Inversa*, London, 1715.

[20] The modern edition of the two editions by Taylor 1719 can be found in Andersen 1992.

[21] Lambert 1759 see: Loria 1921, pp. 43-48; Vagnetti 1979, pp. 441-443; Kemp 1994, pp. 167-170; Andersen 1992; Giordano 2001, pp. 53-61.

[22] For the contribution by Aimée Laussedat del 1899 and the evolution of photogrammetry, see: Paris 2014.

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