

The Points of Concurrence Theory in Guidobaldo del Monte's Scenography

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Abstract

Theatrical scenography was one of the privileged applications of perspective in the Renaissance. The court theater, characteristic of those years, is structured around a frontal perspective installation developed in depth. This particular perspective application makes the scenography a privileged place for experimenting the projective transformations at the origins of the relief perspective in which the real space contracts and transforms itself into the illusory space of the scene. This study regards the De scenis, the sixth book of Perspectivae libri sex, written by Guidobaldo del Monte in 1600, entirely devoted to theatrical scenography. The treatise elaborates a scientific method of universal validity to construct scenes, based on the theory of the points of concurrence explained in the first book of this work, a theory that deeply influenced the history of perspective and descriptive geometry. In Guidobaldo's work, the problem of controlling the projective transformations of space is resolved by reducing the relief perspective to a set of flat perspectives in relation to each other and controlled through the theory of points of concurrence. Applied to the art of scenography, this theory reveals itself in all its generality, since it allows the representation of classes of lines generically oriented in space and, at the same time, resolves the problem of measuring angles and lengths with projective reasoning.

Keywords: Perspective, Scenography, Guidobaldo del Monte, Relief perspective, Points of concurrence.

Introduction

During the sixteenth century scenography was one of the privileged applications of perspective. The theories of linear perspective, which had involved mathematicians and humanists in the late fifteenth century, became, during the Renaissance, the main form of representation of reality. This theory manifests itself in every form of art and finds, in scenography, a particularly fertile ground, capable of giving an unprecedented aspect of centuries-old art [Mancini 1966, p. 9]. In fact, court theater is structured around a frontal perspective installation where mainly urban models are represented, in relation to the work to be staged. Vasari recounts how this perspective model, first conceived flat, was extended to three-dimensional space thanks to the contributions of Baldassarre Peruzzi, who widened the depth of scenic space,

[Mancini 1966, p. 25] introducing, as is defined today, relief perspective in scenography.

Thus, this art appears in the treatises of perspective, published in the sixteenth century, such as those of Serlio, Barbaro, Vignola-Danti and Sirigatti, where it is taught to construct the scenes from an operational point of view.

Guidobaldo's contribution is at the apex of the scenographic production for the court theater, written in years when mathematicians had begun to take an interest in perspective [1], at a time when so much in scenography had been experimented and so little theorized.

The *Perspectivae libri sex*, a monumental work written by Guidobaldo del Monte in 1600, is a fundamental text in the history of perspective since, as noted, it forms the

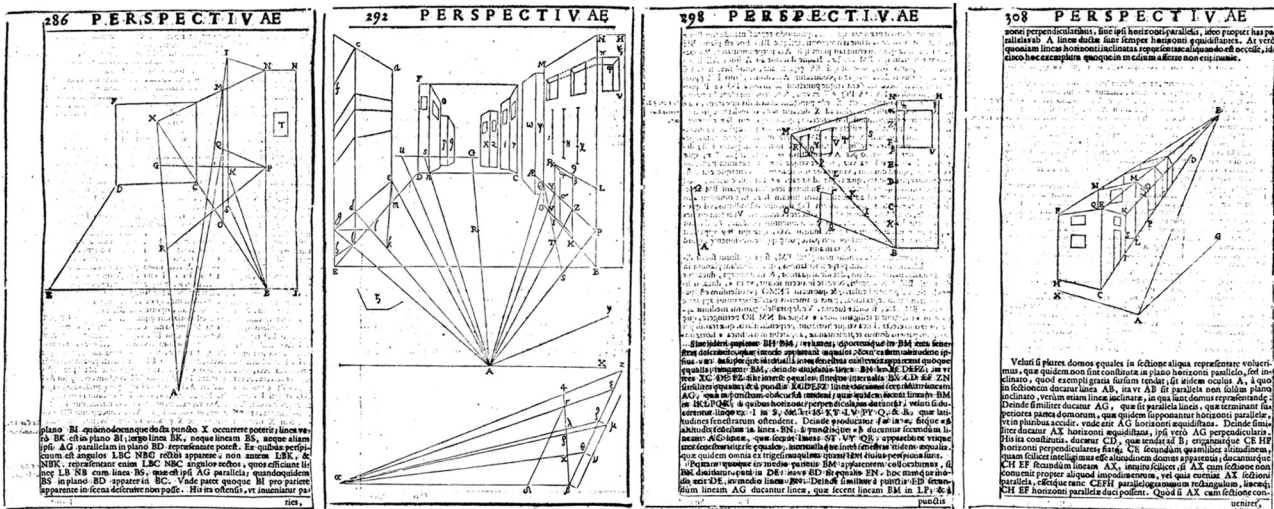


Fig. 1. Guidobaldo del Monte, *Perspectivae Libri sex*, illustrations of *De scenis* [Del Monte 1600].

projective foundations that will lead to the theorization of modern perspective. The sixth book, *De scenis*, is entirely devoted to theatrical scenography and focuses on the practical application of the theoretical principles explained in the previous volumes (fig. 1).

Guidobaldo's work was intended to bring the mathematical universe closer to pictorial practice, becoming an occasion for direct experimentation of apparently abstract theoretical principles. Scenography must be understood in this regard. In continuation of the perspective tradition of the Renaissance, scenography was considered a perspective laboratory in real scale in which to experiment procedures and validate results. Scenography is the place where projective transformations at the basis of relief perspective materialize, where real space contracts and transforms itself into the illusory space of the scene. The three-dimensionality of the constructions is also reflected in the images that illustrate the treatise, which describe the contraction of the scenic box in a sort of natural perspective that invites the reader to reason in three-dimensional space [Field 1997, p. 173].

Control of the projective transformations of the scene is resolved by Guidobaldo through repeated application of the theory of points of concurrence. This flawless reason-

ing allows the representation of entities in a generic position and is structured around two main moments:

- definition of the projective transformations of the scene;
- construction of perspective on the lateral wings.

Projective transformation of the scenic space

Construction of the scenic box was a direct consequence of stage floor declivity. In fact, to avoid this plane appearing particularly flat in the eyes of the observer, the stage undergoes a declivity of a few degrees, which is also useful for the actors allowing them to amplify the acting space. This change of position introduced into the stage space a perspective artifice. All the scenographic planes, namely the lateral wings, must be in relation to this artifice, giving the spectators the illusion of seeing a regular space or, at least, a plausible space with respect to those known in common visual experience [Baglioni, Salvatore 2017, pp. 1-12].

Consequently, Guidobaldo's treatise stems from the need to reduce the scenic space, demonstrated by *reductio ad absurdum* [2]. This reasoning considers the overall contraction of the scenic box a result of a set of projective transformations of all the planes that compose it, degraded one by one

through the application of the theory of points of concurrence. The desire to experiment the validity of this theory in perspective practice, animates our mathematician's interest in theatrical scenography: "Since the scenic apparatus seems to partially vindicate the analysis undertaken [...], we will briefly mention a few things that also concern this topic; and we will illustrate in the following way [...] that the procedure obtained, specific and universal in reproducing the Scenes, is a result of the principles that we have explained" [Del Monte 1600, p. 283; Sinisgalli 1984, p. 219].

Between the XXVIII and the XXXII propositions of the first book of this work, it is explained how to represent the perspective of a straight line oriented in a generic position in space, through construction of its points of concurrence, by means of a procedure still used today for the vanishing point construction of a given line (fig. 2). "The point on section, where a line is conducted from the eye equidistant to a set of other parallel lines, is a point of concurrence" [Del Monte 1600, p. 44; Sinisgalli 1984, p. 64].

Therefore, Guidobaldo considers the edges of the wings that define the scenic space one by one; he imagines that they are perpendicular to the front of the scene, as in reality, and projects them to the lateral wings, each intended as picture planes. Then, he obtains the point of concurrence of these straight lines, constructed by the intersection of the 'visual ray' with the respective picture planes, sections in the treatise, namely the planes of the wings. Since these edges are parallel to each other, the respective perspective images will share the same point of concurrence and because they will have to appear perpendicular to the front of the scene, this point will be given by the intersection of the perpendicular straight line to the front itself, with the planes of the wings, opportunely imagined, extended.

The final result is the relief perspective of a parallelepiped space, contracted in a pyramidal trunk, in which the actors must move (fig. 3).

In the contraction of the scenic box the relation between scenography and relief perspective is revealed. However, the principles on which this form of representation is based, are still far from the treatises of the nineteenth century, which will give theoretical foundation to the perspective transformations in the projective space. Indeed, Guidobaldo reduces the relief perspective to a set of flat cases, resolved by the reiterated application of the points of concurrence theory. The solution is legitimate but resolves a very special case, in which the object of the projective transformations is a portion of the space, namely, a container delimited by a series

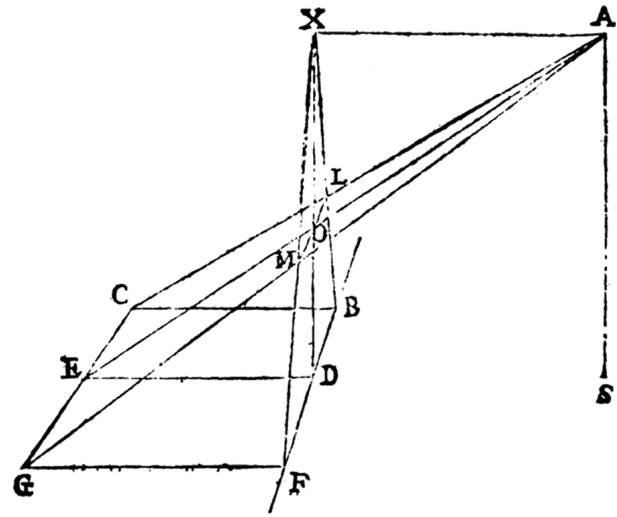


Fig. 2. Guidobaldo del Monte, *Theorema propositio XXIX*. One of the images that describes the theorization of the point of concurrence [Del Monte 1600].

of flat surfaces that establish its boundaries. Thus, the relief perspective concerns the scenic box, affecting the architecture only with respect to the construction of the facade of the buildings and their moldings [3].

The definition of the point of concurrence of the lines perpendicular to the picture plane is exemplary in terms of theory and therefore legitimates the procedure in use. However, its construction is not easily applicable in the operational practice; this point was generally inaccessible due to the reduced court size. This obstacle is the starting point for the experimentation of refined projective procedures with an aim toward resolving the problem. In the treatise, this is preceded by a significant description of the operational practices for construction of the perspective intersections in use at that time.

All methods described in the text, both those practiced by Guidobaldo and others, operate on a plane, defined today as 'projecting plane', determined by the visual ray (projecting line) and by a chosen point on a lateral wing, through which the perspective will have to pass. The materialization of the visual ray, by means of a taut rope between the observer and the backdrop plane is common to all the described methods. Therefore, Guidobaldo describes how it was customary to construct the perspective of a straight line, *PQ* in figure, pass-

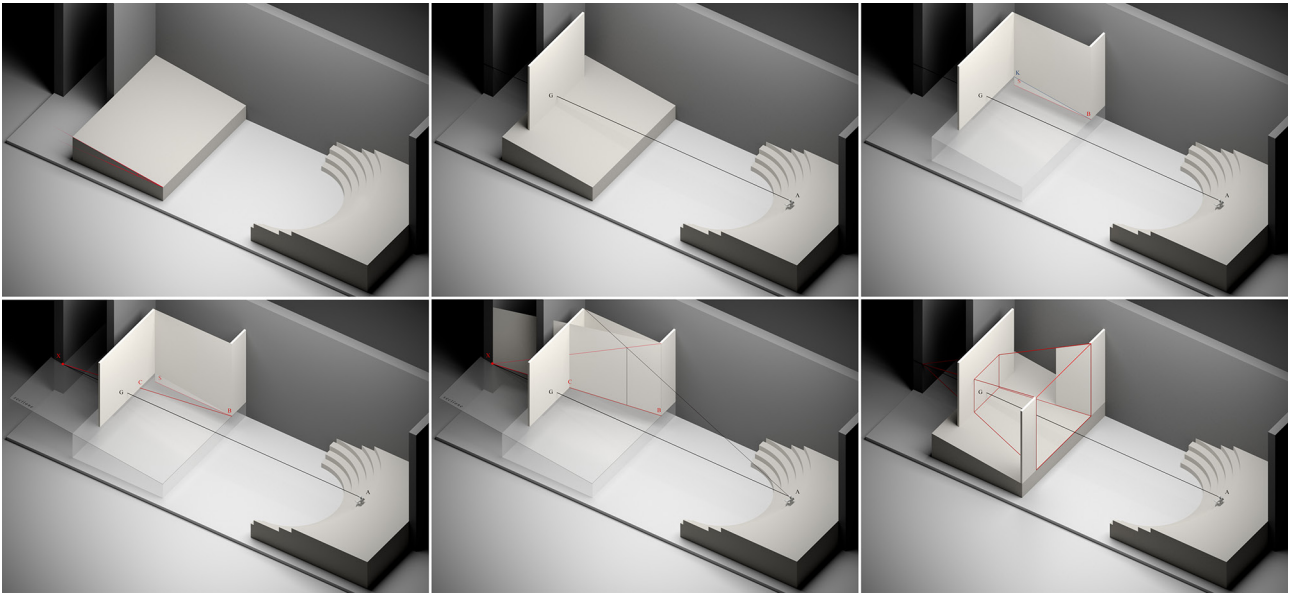


Fig. 3. Phases of projective transformations of the scenic space (graphic elaboration by the authors).

ing through an assigned point P on a lateral wing, with a series of taut ropes. Given the visual ray AG , a second taut rope was placed between point P and point R , chosen in a generic position on AG [4]. With a third rope, fixed this time in the eye A position, it was possible to project one of the infinite points of the straight line PR on the lateral wing, which is the picture plane in this construction, determining point Q , useful to construct the sought after perspective PQ (fig. 4). This procedure, similar to that described in the *Commentarii* by Egnazio Danti on *Due Regole della prospettiva pratica* di Vignola [Vignola Danti 1583, pp. 90-92], presupposes that the projective operations are performed on the projecting plane, because the perspective images of all lines that belong to this plane appear in its intersection with the picture plane. It was possible to reproduce this projective procedure with ropes and cords, or with light sources, generally mentioned in the form of candles in this and other perspective treatises of the time. These light sources represented the sought after perspective traces through the projected shadows of the taut ropes. If the problem with the use of the ropes consisted of their flexibility over long distances, the problem with the use of the torches was due to their feeble

illumination capacity; in fact, at the same distances, they would hardly have been able to project a clear shadow.

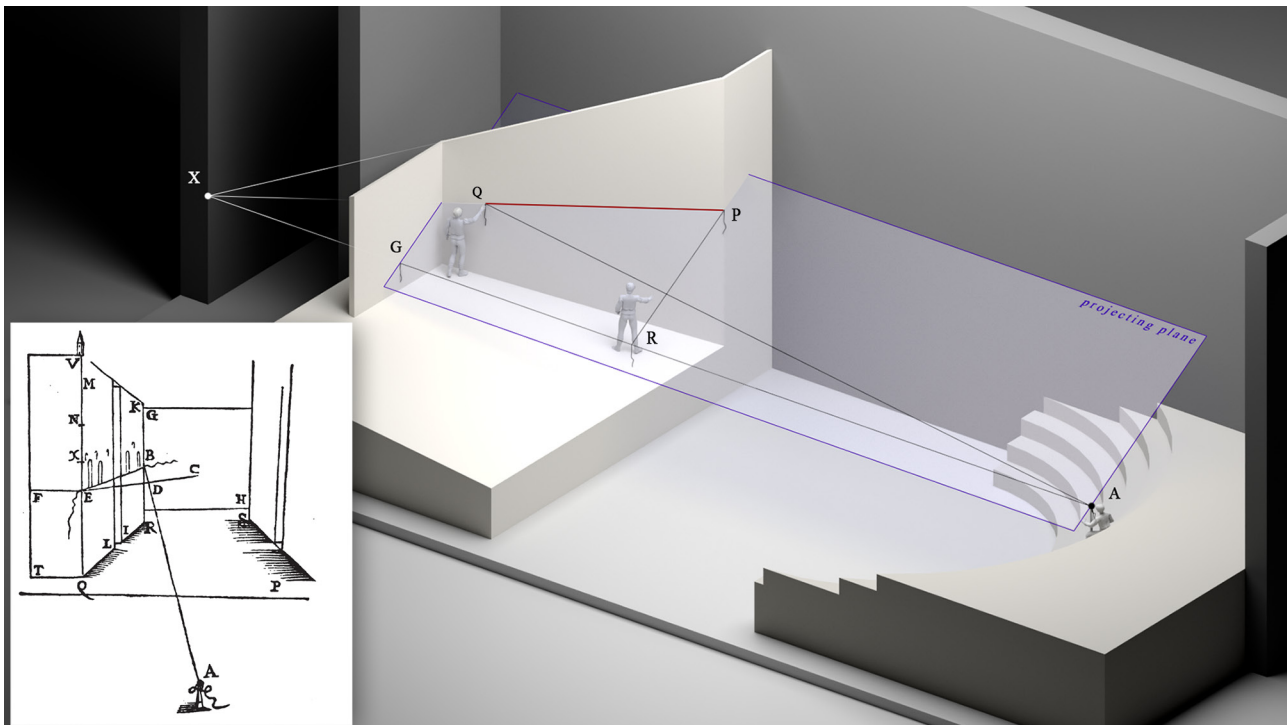
Combining theoretical synthesis and procedural agility, the method proposed by Guidobaldo resolved this problem because it liberated the projective operations from the observer position, projecting straight lines that lie on the same projecting plane from any point of this plane, with the use of only one rope: the visual AG ray.

Having to represent the perspective PQ of a straight line that is perpendicular to the picture plane, as in the previous case, Guidobaldo imagines an observer, whom we define as an auxiliary, who is free to move in half of the scene. The visual AG ray and a P point on the wing are given. The auxiliary observer will have to move on the scene, observing the AG line and P point together, and going up or down until the two entities merge into a single image. If the perspective images overlap, the eyes of the auxiliary observer are placed on the projecting plane belonging to P and AG . It follows that the PQ perspective will be confused with the image seen by the auxiliary observer who, in that position, can easily recognize one of the infinite points of the perspective PQ ,

so determined [5] (fig. 5). The construction could have been executed by replacing the auxiliary observer with a light source free to move on the projecting plane. Thus, this light could have been positioned with respect to the line to be projected and to the picture plane at convenient distances to ensure the projection of a sufficiently sharp shadow. Once the method has been explained and the procedure illustrated, the problem is presented in marked operational terms. "In order not to make a serious error" [Del Monte 1600, pp. 289; Sinisgalli 1984, p. 222], the borders that delimit the stage floor and establish the entire contraction of the scenic box must be constructed before the others. Therefore, once the declivity is fixed, the design of the stage could be resolved using two different procedures, derived by two different constraint conditions. The first establishes the height of the observer, consequently obtaining the foreshortening

of perspective; the second, on the contrary, fixes the foreshortening of perspective, given by the shape of the trapezium of the stage, thus obtaining the height of the observer. The first procedure follows the construction previously described in the case of the perspective PQ of a straight line although, in this case, point P belongs to the stage plane. Using this procedure, it was impossible to plan the width of the stage *a priori*. It was only possible to foresee that the greater height of the observer would have caused greater width, due to the increase in the distance between the point of concurrence and the observer, and vice versa. However, the control of the dimensions and the proportions of the scenic space was a priority parameter in the design of the stage space. Thus, Guidobaldo proposes a second procedure, which primarily establishes the dimension and the proportions of the trapezoid of the stage, i.e.

Fig. 4. Method used for the construction of perspective traces (graphic elaboration by the authors) and comparison with Egnatio Danti's method [Barozzi da Vignola 1583].



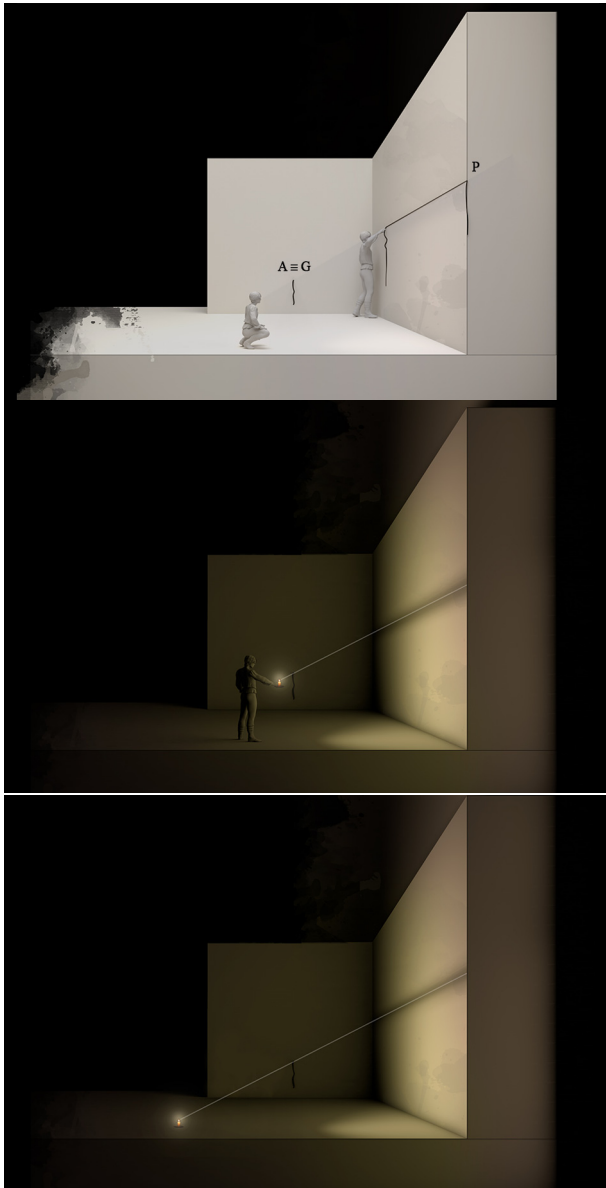


Fig. 5. Guidobaldo del Monte's projective method executed by the view and by the source of light (graphic elaboration by the authors).

the foreshortening of perspective. Although the operational relevance of this procedure is highlighted in the treatise, the passage is synthetic and cryptic, in fact it is not explained, contrary to what happens for the other constructions.

Once the trapezoid of the stage is traced: 'the AG must be moved up or down, providing that it always occupies the center of the scene (as has been said) and that it is located equidistant from the horizon, until, once on the side of ED, we look at the BD line through the AG line, and the AG e BC lines appear in a single line [...]; and once the AG line is found, the AG must then be immobilized; and in this way the A eye position will be determined' [Del Monte 1600, p. 291; Sinisgalli 1984, pp. 222, 223].

In this type of construction there are two variables, the height of the auxiliary observer and that of the AG rope which we conveniently imagine this time as a rigid shaft. The simultaneous movement of the shaft and the observer cause the evident operational difficulties in the procedure, antithetical to the spirit with which Guidobaldo addresses the question. It is possible to hypothesize the separation of the movements and order them according to a logical sequence that simplifies the problem (fig. 6).

We can imagine the AG shaft fixed in a point on the back-drop plane, placed at an arbitrary height from the stage floor. The auxiliary observer is free to move up and down anywhere on the stage and observe, with a single gaze, the AG shaft and the inclined BC edge, placed on the ground on a wing. Once a position is chosen, there is only one observer height in which the AG shaft and the BC edge appear parallel. When this height is established it is possible to vertically move the AG shaft until its image coincides with that of the BC edge and, consequently, find the height of the projection center. The proof is not given in the treatise, but it is possible to imagine that Guidobaldo reasoned again in perspective terms to resolve this particular geometric problem. In fact, the BC edge and the AG shaft are two skew lines. It is always possible to observe a pair of skew lines from a position that makes them appear parallel, a condition experimentally verifiable by turning one's gaze around a pair of wooden sticks.

Construction of perspectives in the scenic space

Once established the contraction of the stage box, Guidobaldo continues his dissertation with the construction of linear perspectives on side wings. This phase is particularly significant because it clarifies again –and in an even more

incisive way— the field of experimental application of the theoretical statements of the work, once again giving rise to the methodical and repeated application of the theory of points of concurrence.

Each wing of the scene, including the scenic stage with its declivity, constitutes a portion of a picture plane to be imagined infinitely extended, on which to draw the perspective of the illusory space that one intends to represent: “given that scene conformation is usually expressed through numerous objects reproduced in various sections appearing in front of one’s eyes” [Del Monte 1600, p. 283; Sinisgalli 1984, p. 219]. Thus, the overall illusion of the setting of the scene was entrusted to the representation of perspectives painted on the wings, picture planes that delimit the contracted scenic box, all seen from the same projection center. These two-dimensional painted backdrops were able to be installed directly in the theater or to be used as a support for the construction of three-dimensional wooden units, such as those made by Vincenzo Scamozzi for the Teatro Olimpico in Vicenza.

In this perspective system, the observer recognizes the illusion by gathering together the multiplicity of the picture planes, as is done by observing the architectural perspectives painted in fresco on the walls of a single hall, on which perspective images that allude to the same illusory space are represented.

Then, Guidobaldo must resolve the problem of representing straight lines having a generic position in space on different picture planes so that the set of perspectives turns out to be coherent. The solution lies in the universal nature of the theory of points of concurrence, thanks to which it is possible to construct scientifically, that is, with a repetitive procedure, sets of straight lines in any direction and in whatever position. The treatise accompanies the reader through increasing levels of complexity that lead to the maximum generalization of the method. The classes of lines in question can be summarized in the following order:

- straight lines perpendicular to the front of the scene;
- horizontal lines parallel to the front of the scene;
- horizontal lines but oblique with respect to the front of the scene;
- straight lines in a generic position.

The first class to be examined concerns those that must appear perpendicular to the front of the scene. In the role of a Renaissance scenographer focused on representing, on a lateral wing, a regular building with openings on the facades, Guidobaldo observes that the perspectives of straight lines with this direction, such as window sills, converge towards the

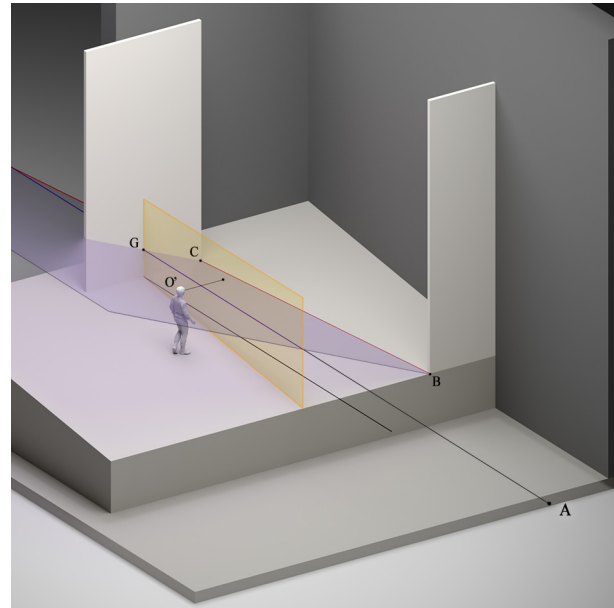


Fig. 6. Reconstructive hypothesis of the design process of the scene starting from a pair of skew lines (graphic elaboration by the authors).

point of concurrence already determined in the phase of contraction of the scenic box.

The problem is then the depth measurement of these lines by proposing two distinct procedures. The first, operative, performed directly in place to divide the lengths into proportional intervals; the second, graphical, which instead permitted the measurement of the line’s lengths, represented on paper through the ichnography of the scene.

The first procedure leads to obtaining an opening placed at the center of the wall that appears perpendicular to the front of the scene, which is used to construct the $P'Q'$ perspective of a horizontal PQ edge (fig. 7). From the Q' point, a straight line parallel to PQ was traced. This line is the edge of the actual building ideally positioned beyond the wing, which meets the visual PA ray in point S . Two points, T and V , were thus detached on the $Q'S$ segment in such a way that the ST and VQ' the segments were the same length, representing a centrally arranged opening with respect to the facade. Finally, by means of the visual AV and AT rays, door width was estimated in proportional terms and reported on the $P'Q'$ segment.

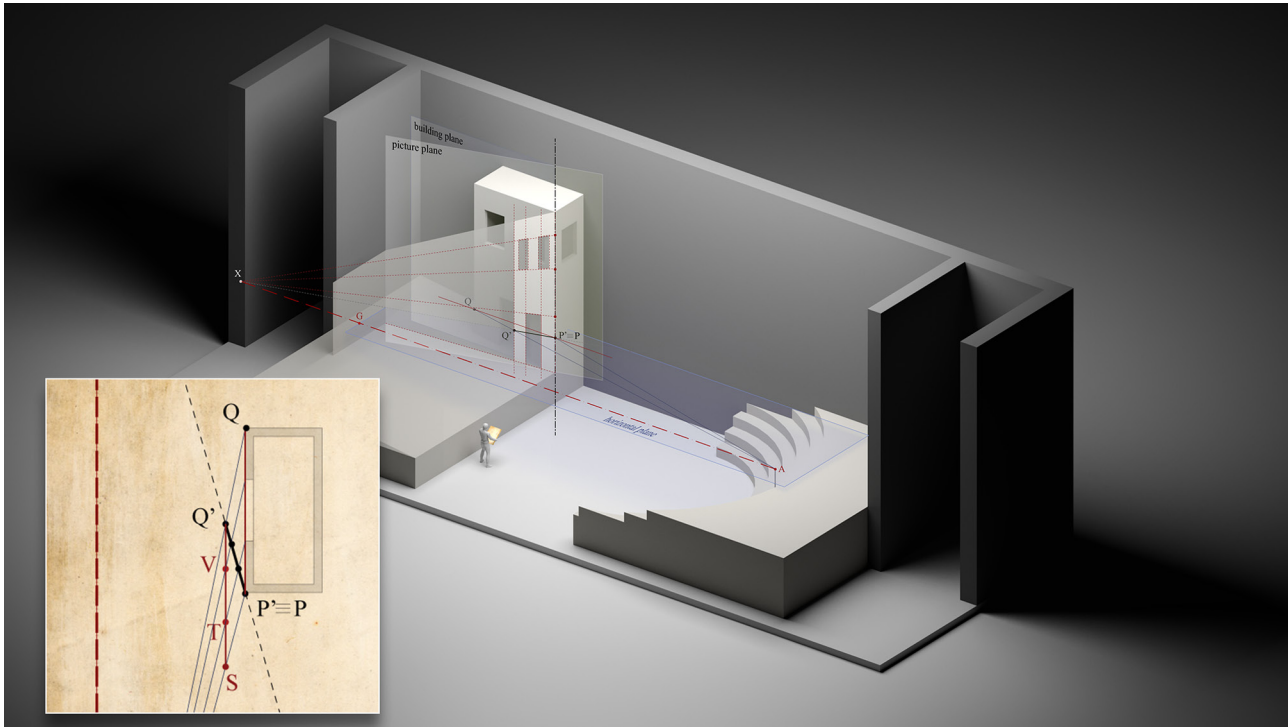


Fig. 7. Method of proportional division of lengths on lateral wings (graphic elaboration by the authors).

On the other hand, if a metric check were to be done in the subdivision of the same segment, the second method would be used. In Guidobaldo's time, perspective procedures based on the use of distance points had already dealt in perspective treatises [6]. However, the proposed method once again follows the logic of a purely projective nature operating directly in the scenographic space, appropriately reduced in scale on paper without resorting to the use of distance points (fig. 8). Thus, Guidobaldo projects the intervals on paper where, unlike the previous procedure, he obtains the measure in the true shape of the real edge. Then, dividing this edge according to measured lengths, the latter were projected onto the picture plane through the visual rays.

Once the representation of the straight lines perpendicular to the picture plane has been resolved and the position of the vertical edges of the buildings have been defined, the problem

is complicated by the construction of a second class of straight lines: those horizontal and parallel to the front of the scene, whose perspective lies on the lateral wings. In particular, this is the representation of the wall thickness of doors and windows. This passage is especially significant in that it highlights Guidobaldo's *modus operandi*, which experiments the potential of the points of concurrence theory with surprising ease. Consider the plan of the side wing on which we want to represent the thickness of the openings. The problem is resolved again with the determination of the point of concurrence of this class of straight lines, given by the intersection of the straight line projecting the real straight line, parallel to the front of the scene, with the *producto scilicet* picture plane (fig. 9). In this particular case, the linear perspective that is defined in the plane of the side wing, is generally characterized by a reduced principal distance; it follows that the

entities represented are arranged outside the circle of distance, giving rise to an anamorphic perspective that finds its correct vision only if it is brought back to the condition of restricted view (fig. 10). With this procedure, the construction of all the thicknesses of the walls of the scene is systematically resolved, by evaluating each time the direction of the real edge and the position of the picture plane.

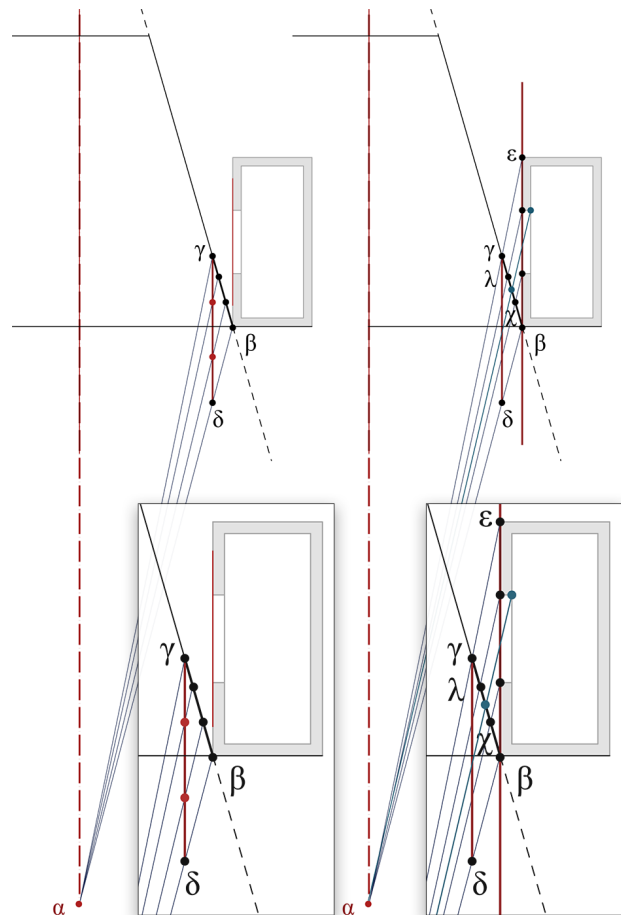
The full control of the representation of straight lines perpendicular and parallel to the front of the scene allows Guidobaldo to increase the level of complexity, teaching us how to represent on a single wing two adjacent faces of the same building: one parallel to the front of the scene, the other perpendicular. The described constructions, where possible, were carried out away from the scene, moving the wing in a position that allowed the materialization of the points of concurrence through the nails to which the taut ropes were fixed. In the event that the wing was irremovable, the proportional division procedures, known at that time, were used to resolve the problem of inaccessible vanishing points [7]. Once the quadrilateral that defined the perspective image of a rectangle was determined, the opposite sides were divided into equal parts. The straight lines passing through the corresponding points of the partitions would therefore share the same point of concurrence. The plane of the backdrop parallel to the front of the scene, is also a *section* on which to continue the graphic construction of buildings in line with those represented up to now. This representation is an opportunity to generalize the proposed method as it introduces the problem of the perspective of horizontal and oblique lines with respect to the picture plane (fig. 11).

The problem is resolved, as usual, with the construction of the points of concurrence. Guidobaldo represents rotated buildings of known angles with respect to the picture plane, resolving with surprising modernity the non-trivial problem of angular measurement in perspective. The reasoning always has a projective character: It resolves the question through the measurement in true form of the angle formed by the visual rays in the center of projection. The visual rays in question correspond to the pair of lines that form the given angle, of which the perspective is intended to be constructed. It is possible, with this method, to draw horizontal lines that form an angle with the picture plane as n-sided polygons to be used as reference structures for the envelope of curved lines such as circumferences.

Finally, the highest level of generalization is reached with the representation of inclined straight lines in a generic position, such as those of maximum declivity of a plane on which a

building can be imagined placed. The reasoning is always the same and it is indispensable for the resolution of the problem in a condition of maximum obliquity, showing, in practice, the absolute generality of the points of concurrence theory: "Thus, it is possible to see how useful and advantageous the real knowledge of the points of concurrence theory is for perspective, which will certainly guarantee maximum facility for painters" [Del Monte 1600, p. 309; Sinisgalli 1984, p. 232].

Fig. 8. Method of measuring the lengths of segments on lateral wings (graphic elaboration by the authors).



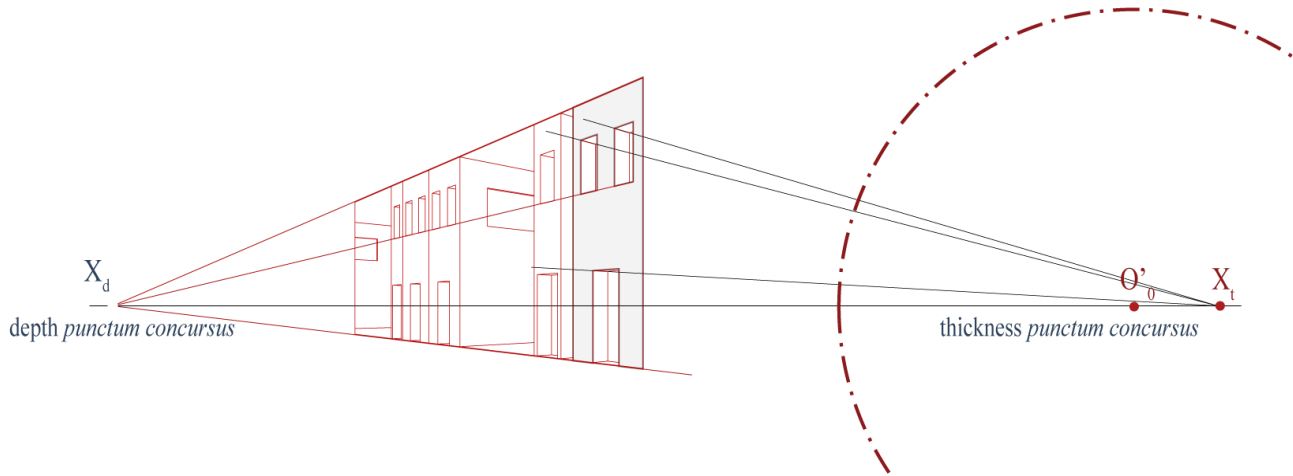
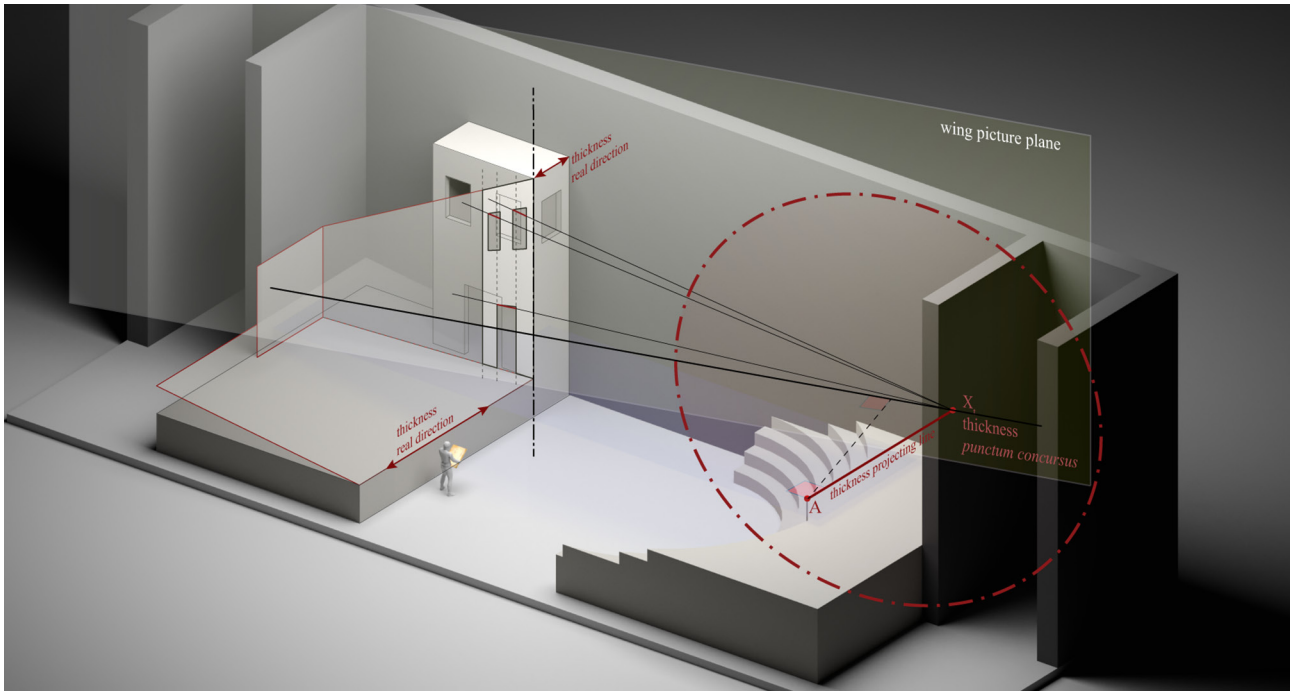


Fig. 9. Method of representation of horizontal lines parallel to the front of the scene with the theory of points of concurrence (graphic elaboration by the authors).

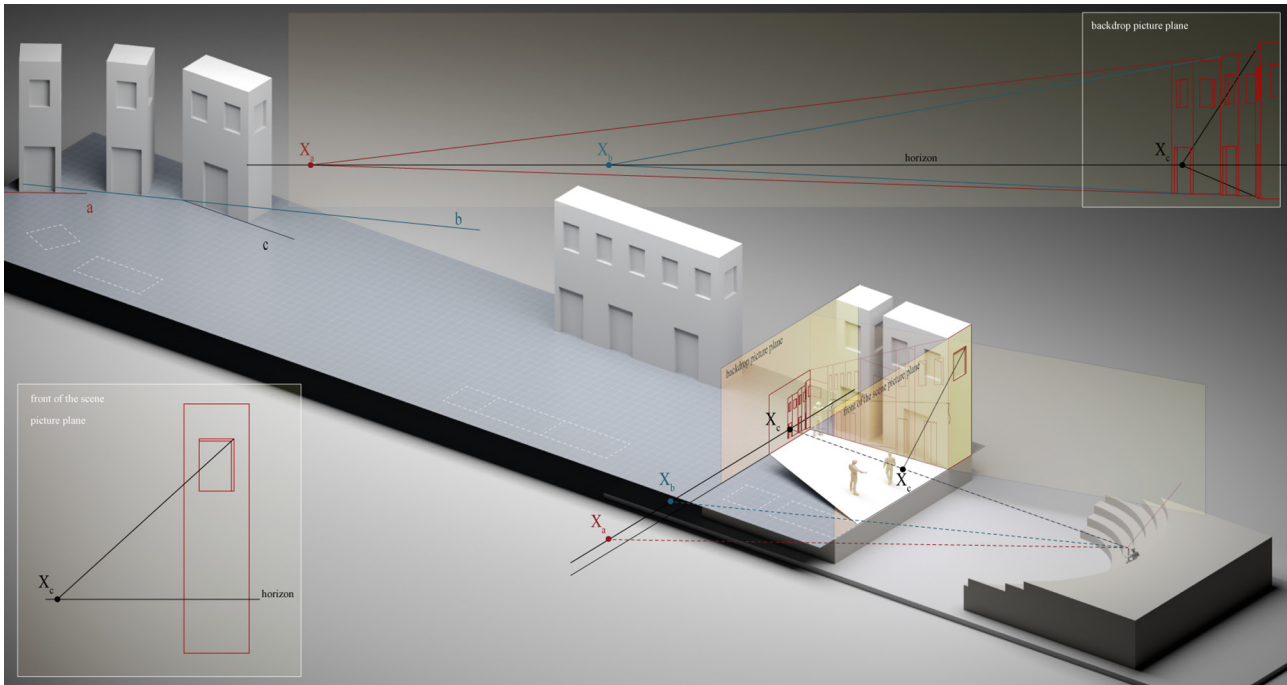
Fig. 10. Anamorphic perspective on the picture plane constituted by the lateral wing (graphic elaboration by the authors).

Conclusions

The points of concurrence theory, elaborated by Guidobaldo Del Monte, constitutes a focal point in the history of perspective. It shows, for the first time, how the perspective of a set of parallel lines is a family of intersecting straight lines, a circumstance which is established empirically, but which, until then, no one had ever even tried to demonstrate [Loria 1921, p. 16]. "Guidobaldo del Monte, using methods to represent three-dimensional figures on a plane reached such a high level, that very little remained to be added to reach the heights where they are today." [Loria 1950, p. 362]. In theatrical scenography, the points of concurrence theory reveals all its theoretical validity and its operative effectiveness, since it allows the representation of sets of straight lines however oriented in space and, at the same time, resolves, with projective reasoning the problem of the measurement of angles and lengths. The protagonists of the constructions are the straight project-

ing lines, then called visible rays, to which, today, as then, the solution of every kind of perspective construction is entrusted. Guidobaldo's scenography inaugurates, in the history of perspective, the treatment of the theoretical principles of a new applied science, or "relief perspective" [Loria 1921, p. 18], a form of immersive representation in which the projective procedures, which generally resolve linear perspective on paper, acquire physical form. In this artificially ephemeral place, between the real space and the projective space, all the operability of theoretical reasoning is revealed. This concretely reproduces, using ropes and candles physically present on stage or using sight only, refined projective reasoning through the reproduction of the visual rays, icon of the universality of the theories enunciated. Guidobaldo is responsible for the development of a scientific method for practicing perspective, based on principles of projective character capable of conferring, through the strength of theory, the dignity of science to the scenographic practice at that time.

Fig. 11. Real space and projective space: set of perspective images painted on the wings of the scene (graphic elaboration by the authors).



Conceived to bring artists closer to the theoretical foundations of perspective, perhaps due to the difficulties of a Latin text or the excess of theoretical content that characterized it, *De scenis* did not immediately receive particular recognition from artists, as his theories were not very accessible to them [Andersen 2007, pp. 264]. Thus, the work by Guidobaldo, which was extraordinarily ambitious in scope with respect to the generaliza-

tion of perspective theories up to that time, formed a solid foundation for subsequent advanced studies. This led, only a hundred years later, the English mathematician Brook Taylor to extend the concept of the point of concurrence, or vanishing point, to the wider definition of the vanishing line, refining theories that will bring perspective to appropriately reach the concept of infinity [Migliari 2012, pp. 116, 117].

Notes

[1] Commandino and Benedetti gave significant contributions to the science of projections.

[2] The reasoning derives from the hypothesis that the scene has a parallelepiped shape and therefore the planes of the wings form ninety degree angles with the front of the scene. The declivity of the stage floor that must be perceived as horizontal by the observer causes the contraction of all the surfaces that delimit the scenic space, which otherwise would appear as an irregular prism [Del Monte 1600, pp. 283-289].

[3] In this regard, see the wooden scenes designed by Vincenzo Scamozzi for Andrea Palladio's Vicenza Olympic Theater in 1584.

[4] Point *R*, constructed in the text through the intersection between the line conducted from the *P* point perpendicular to the *AG* line, can assume a generic position along the *AG* rope.

[5] In the figure it has been hypothesized that in point *P* a second rope was fixed, which could easily be tautened according to the appropriate indications of the auxiliary observer:

[6] The use of the distance points recurs differently in *De Artificiali Perspectiva* by Jean Pelerin, in 1505, and in the *Commentarii* by Egnatio Danti in the *Due regole della prospettiva pratica di Jacomo Barozzi da Vignola*, published posthumously in 1583.

[7] Once some essential lengths were established, the partitions of the façades could be made using graphic constructions based on partitions in proportion to the sides of a quadrilateral with respect to its diagonal; methods of progressive division of the parts of a flat figure were in use in perspective of time, as demonstrated by some of the propositions of the first book of *De Prospectiva Pingendi* by Piero della Francesca (such as propositions IX, X, XI and XV).

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