On the Genealogy of Geometry in Drawing for Design: Primitive Future of a Techno-aesthetic Issue

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Introduction

The acquisition of this historical knowledge –according to the purpose of this issue of *diségno*– should help us to understand what we actually mean when we talk about 'Geometry' in the field of university researches especially applied to Design, Pedagogy and Arts schools. In this field, specifically, the history of geometry should not be mistaken for the histories of mathematics [Chasles, 1837, Loria, 1921], or with the history of art and ancient erudition: these general historiographical issues have other (authoritative) scientific and editorial references.

The 'geometry for the *design*' can be actually regarded as a single issue (topic) provided that at least two conditions are met: 1°) the presence of a common technical subject shared by studies of different disciplines, and 2°) common vocabulary and methods shared by the different points of view.

1°) 'Geometry for the *Design*' means an 'applied science' that studies the 'categories of the object shapes' (eidetic categories), as well as their projective and diagrammatic representations. The history of this 'practical geometry' gathers those studies that, although relying on different points of view, explicitly or implicitly include all the geometric-morphological aspects pertaining to the field of the history and anthropology of 'visual artefacts', especially of those 'visual artefacts' specifically 'designed to represent'. Therefore, the stories of visual artefacts investigate specific 'geometric-morphological issues.'

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Historical studies on 'geometry' intended as such take into account various disciplines –from the history of exemplary (artistic) visual artefacts, to essays or the current parametric modelling, thus offering a multi-faceted overview to be intended as a single thematic area, albeit transdisciplinary.
The 'geometry for the *design*' globally consists of 'paradigms', i.e. a collection of geometric models, a sort of (historical and current) synchronic plurality, which could be considered 'Descriptive Geometries' to point out a genealogy which precedes and still continues after the historical parabola of Descriptive Geometry.
Based on our assumptions, these 'Descriptive Geometries' share the same existence of the 'technical objects' described by Gilbert Simondon [1958; 1992; 2013] and, as such, according to the French philosopher, their specific technical-aesthetic dimension can be grasped.
2°) The unity of this 'collection of geometric models' is made up of two aspects: i) the mutual comparability (translatability) of the models and ii) their adequacy, namely their ability to describe the most significant aspects of the object shapes and their images. Precisely these two conditions –i) comparability of the models and ii) their explanatory adequacy define the thematic unity and the relevance of the historical studies on the 'geometric morphology of visual artefacts.'
This reasoning, however features some criticalities.
Any 'naïve realist' like me would wonder what the 'real explanatory adequacy' of the geometric models actually is. In order to adequation the geometric descriptions to the physical and anthropological (cultural) values that indicate the meaning to the object shapes, it is necessary to gain a historical awareness of the actual technical and aesthetic dimension of geometry.

From Descriptive to Computational Geometry

The internal combustion engine of modern cars still preserve something similar to the ancient steam engine. We realize this fact only if we trace the genealogy of the design of that type of engine, going back to Watt machine. However, the present cars are complex mechatronic artefacts so complex that it is no longer possible to figure out the calculations that a series of algorithms –in spite of us– performs in a fraction of a second, for example, to adapt the braking command to the four wheels according to their specific speed. The consistency of these algorithms –similar to those which control airplanes, the road traffic or even the secretion of hormones– goes well beyond the imagination of the same authors. This happens also with the operation of microprocessor networks in the objects around us, which relies on algorithms going exceeding the visual and object imagination of the ancient 'theatres of machines' and the technical drawing.
The steam engine and the *Géométrie Descriptive* (DG) are both technical objects belonging to the first industrial revolution at the end of the 18th century; while the current mechatronic artefacts, often provided with artificial perception, fall within the computational geometry (CG) and, like the latter, are the result of the third as well as of the incoming fourth industrial revolution.
Descriptive Geometry (DG) and Computational Geometry (CG), albeit in different periods, arise from the same historical development, i.e. the means of technical conception of the artefacts. The DG is associated with the '*l'art du trait*', stereotomy, to mechanical drawing and photogrammetry; while the CG is connected with the CAD and CAM systems, automatic photogrammetry and artificial vision. They are both designed as translation systems for geometric entities from a 'mathematical representation' to drawings, prototypes and models. They both result from the elaboration of new categories of curves and surfaces, intended as mathematical objects to be geometrically transformed and created in workshops. Together with the *Géométrie Descriptive* and the manufacture of cannons, Gaspard Monge, established the Differential Geometry where the 'constant slope surface' is still named after him. In line with the improvement of numerical control machines in the Renault factories, thus laying the foundations for CAD and CAM systems, Pierre Bézier created an *ante-litteram* CG thanks to the invention of 'polynomial curves and surfaces' which are performed still today based on the algorithms of Paul de Casteljau and are named after him.
DGs and CGs –surfaces of Monge and surfaces of Bezier– both result from the same two-century development of design and industrial manufacture, moreover they arise from the same thousand-year-old development of the science of vision.
The DG actually emerged some centuries before its establishment through the invention of the Renaissance theory of perspective –i.e. the development of geometric optics in the *perspectiva artificialis*– as well as with the first projective propositions of practical geometry. Conversely, the further progress of the Science of Vision towards the

computational models of the psychology of perception as well as towards artificial (robotics) vision has promoted the development and applications of the CG, as will be discussed in the conclusion.
Then –shifting from the scientific background to the technical practices– it is worth stressing that DG and CG both refer to the techniques of survey of the surfaces of real bodies in the space. The representation method of the DG par excellence is the 'bicentric projection', namely a kind of 'stereoscopic vision' arising from the ancient topographic assessment system based on 'forward intersection', a scheme similar to the geometric model of stereo-photogrammetry.
The CG can be also regarded as an enhanced development of photogrammetry which, however, leverages the technological evolution of digital sensors and their sensitivity to a wider range of radiations and vibrational phenomena. After the spread of digital imaging systems, thanks to the CG algorithms, the DG photogrammetric processes and, above all, Epipolar Geometry, are now fully available to everybody through software working on personal computers, with images provided by simple cameras or shared on the web. Moreover, the CG grew in the Eighties in line with the great success of 3D data optical acquisition systems –from 'active triangulation' to 'structured light'– by applying the constant invention of new sensors and scanning solutions of natural objects, artefacts, minerals or living subjects to the traditional topographic and photogrammetric schemes.
It is impossible to summarise the amazing technological development of the spatial geometric data collection systems over the last thirty years, together with their numerous applications in various fields –from biology to astronomy, from the manufacturing to the entertainment industry, from the medical image to the robotic vision– up to our everyday life through, e.g., smartphones and means of transport.
For example, the evolution of tomography –starting from the first machines of Godfrey Hounsfield to X-ray scanning devices– can help us understand how the CG has extended the DG applications to bodies and dimensions previously inaccessible to the human eye and imagination. From the molecular scale –for example in the study of protein vibration phenomena– to astronomy –in the study of the spongy form of matter in the cosmic space– the CG is fostering the development of different '*Descriptive Geometries*'. However, these new 'Computational

Fig. 1. Survey of double pendulum trajectories through long exposure photographs (graphic elaboration by E. Calore, F. Giordano, E. Pettenà, IUAV University of Venice, Course on 'Morphology of artefacts', prof. Fabrizio Gay, academic year. 2016-2017).

Descriptive Geometries' can investigate the object shape also through a reverse mechanism with respect to the traditional DG.
In the DG, as happens with the CAD modelling, the form is given '*a priori*' compared to the concrete representation. On the contrary, in the CG, the form is derived '*a posteriori*', and is implied as a geometric structure underlying a large amount of spatial data.
The CG is a morphological instrument which proposes to study patterns, stochastic regularities, dots, corrugations and un-dulations, the morphology of organic or geographic tissues, the tessellations of the *alveoli*, cracks, marbling, stripes, zebra patterns in animal and mineral pigmentations, ramifications, etc.
While the DG mainly acted as a representation instrument, the CG is a form of 'aesthesic' geometry. Today the CG helps to develop instruments for the perception and categorisation of bodies in the space as well as of image networks, reconstructing processes similar to those leading living beings to the recognition and aesthesic knowledge of the objects of the world.

The (historical) translation among geometries and the primordial concept of 'distance'

Computational Geometry is the title of the final dissertation of Michael Shamos dealing with "the Issues that arise in solving geometric problems by machine at high speed and the fact that such devices have only recently been built obliges us to consider aspects of geometric computation that simply do not occur in classical mathematics, and new methods are required." [Shamos, 1978, p. I]

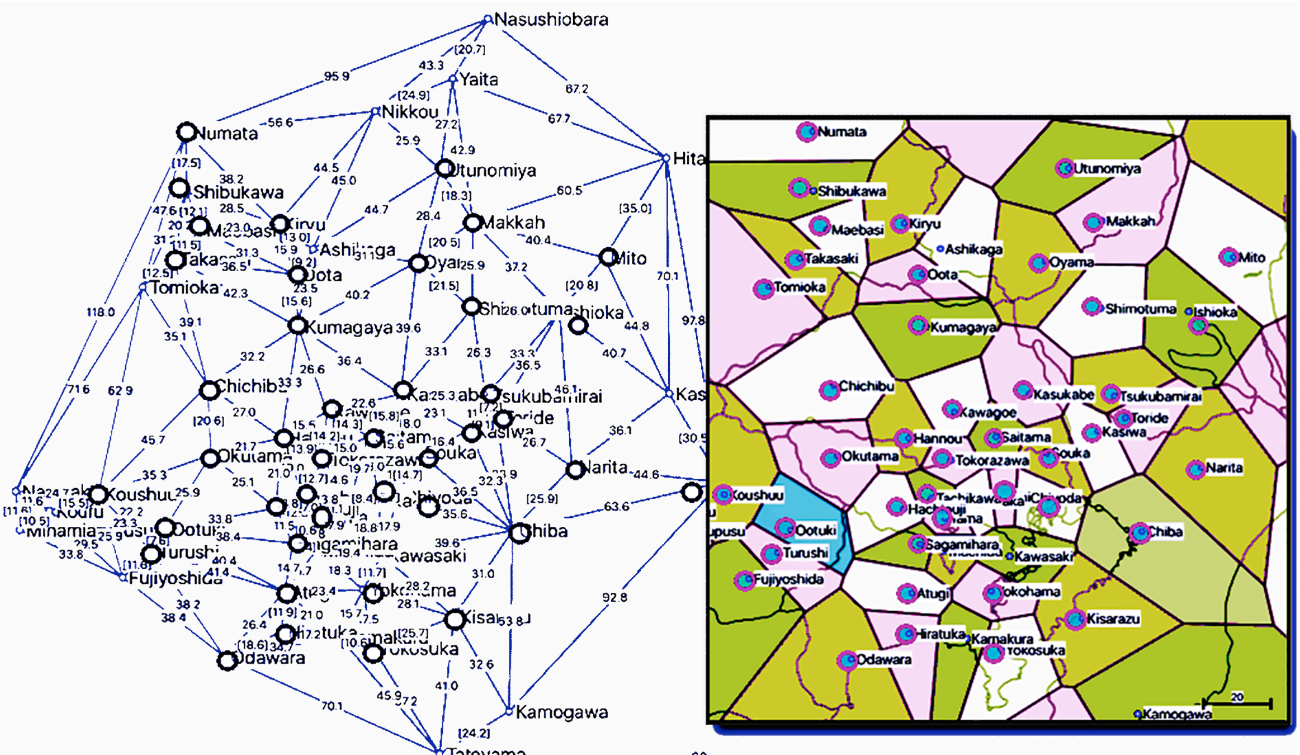


Fig. 2. Example of two-dimensional Voronoi diagrams.

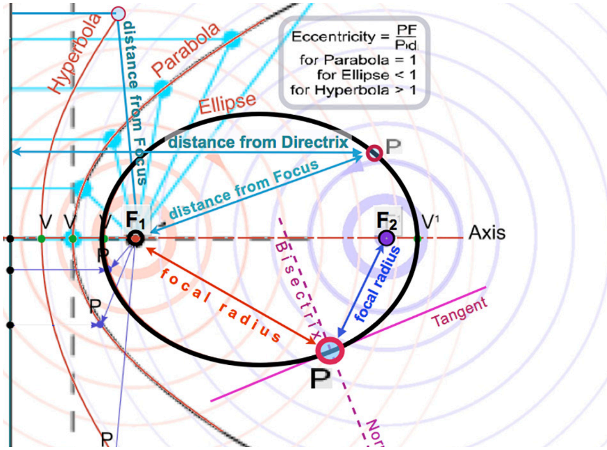


Fig. 3. Eccentricity and focal properties of conics.

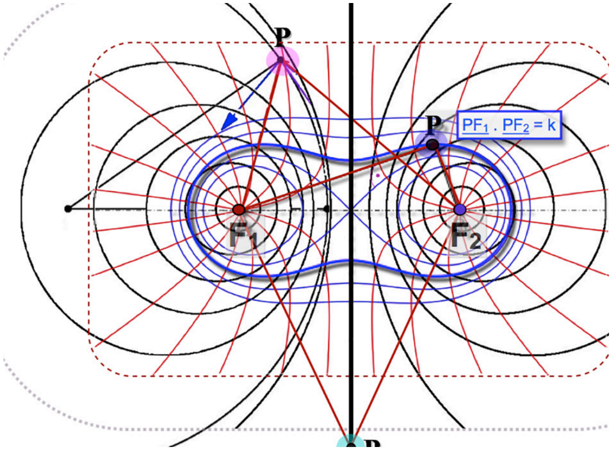


Fig. 4. Circles of Apollonius and lemniscates of Bernoulli with the hyperbolas, their orthogonal trajectories.

An example of these issues is the so-called "Closest-Pair problem", which requires the identification of the two closest points in a certain series of *n* points. What we are trying to roughly assess is, geometrically speaking, the result of an efficient calculation. However, if a machine measured the distance of each $m = n \cdot (n - 1) / 2$ pairs of the *n* points and then arranged and compared the distances, it would carry out a $O(n^2)$ computational complexity task. Conversely, the author [Shamos 1978, p. 163] suggests a recursive algorithm based on the *divide-et-impera* principle, which reduced complexity to $O(n \cdot \log n)$. Basically, the calculation time on set of ten million points drop from one week to one second.
The above-mentioned scattered sequence of *n* points can be considered by the CG only through discrete mathematical models and the figure which best expresses the discretisation of space is the Voronoi diagram. It consists (fig. 2) of the division of a certain space into distinct regions (called Voronoi cells) in such a way that each of them contains only the points closest to a given point (seed) referring to other points (seeds.) The boundaries of the Voronoi cells are places equidistant from two or more points (seeds.)
The qualitative meaning of 'distance between two points' redefines, in computational terms, the traditional categories of the geometric entities. Thus the 'distance' define not only the circle and the sphere –intended as a locus of points equidistant from the centre– but also the straight line and the plane, since they are a locus of points equidistant from two given points. Therefore, the straight line and the plane are the Voronoi diagram only for two points. The locus of points equidistant from a point and a straight line (or a plane) is obviously represented by a parabola (or a paraboloid of revolution.) In general, the metric definition of the conic sections proves that 'equidistance' is just a special case of the 'distances ratios' ($= 1$). In fact, conics are defined by their 'eccentricity', i.e. as is a loci of all points *P* whose distances *PF* from a given point *F* (focus) and *Pd* from a given straight line *d* (directrix) have a constant ratio ($PF / Pd = k$). Obviously, the presence of an ellipse or hyperbole depends on whether *k* is bigger or smaller than one. After all, even circles or spheres can be regarded as loci of points *P* whose distances from two given points (F_1 and F_2) have a constant ratio ($PF_1 / PF_2 = k$). Actually they are the circles (and spheres) of Apollonius, which degenerates into a straight line (and in a plane) when $k = 1$. (fig. 4)
The metric property of the eccentricity ($PF/Pd = k$) defines all the conics and is directly reflected by their focal properties. (fig. 3) Based on their definitions, the ellipse and the hyperbole can be intended, respectively, as loci of the points of a plane whose distances from two other given points (focus F_1 and F_2) have an unchanged sum (the ellipse $PF_1 + PF_2 = k$) and unchanged difference (the hyperbole $PF_1 - PF_2 = k$). When *k* is equal to the F_1F_2 distance, the ellipse

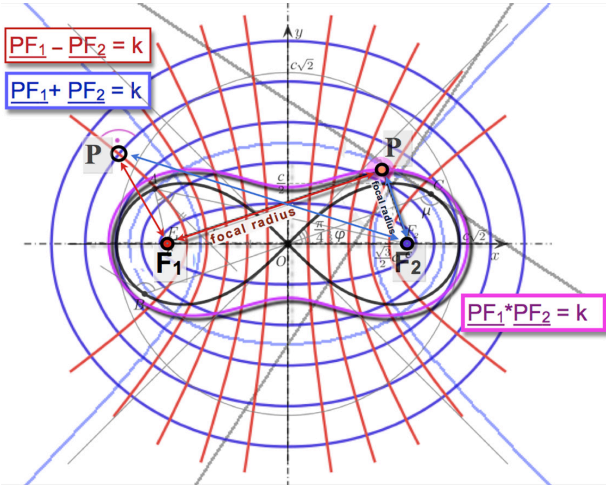


Fig. 5. Confocal Conics and Cassini ovals.

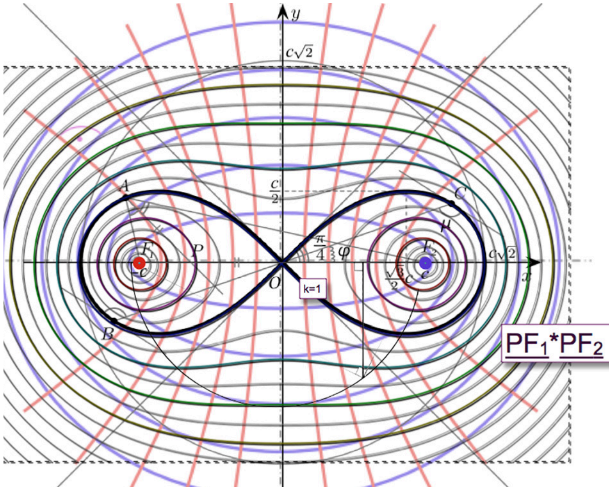
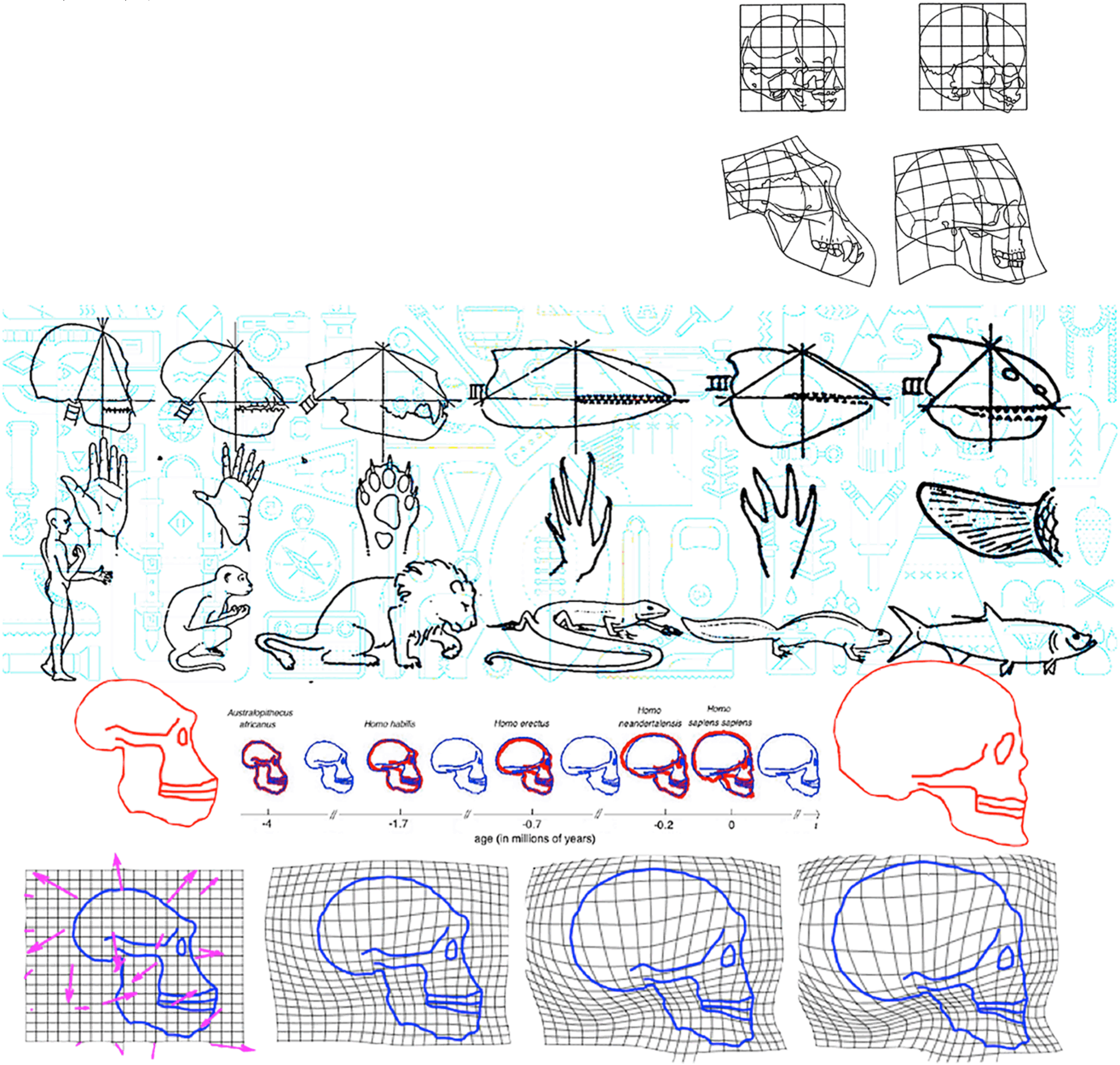


Fig. 6. Different shapes of confocal Cassini ovals.

degenerates into the finite F_1F_2 straight segment, while the hyperbola degenerates into the infinite F_2F_1 segment.
The eccentricity and focal properties of the conic sections, by analogy, help to define other curves and surfaces. For example, Conchoidal curve can be regarded as set of the points whose product of their distances from a given point and straight line is constant ($PF \cdot Pd = k$). Conversely, (fig.4) the lemniscate which Jakob Bernoulli defined in 1694 by hybridizing the construction of the ellipse and circle of Apollonius ($PF_1 / PF_2 = k$) (fig. 5), can be intended as a locus of points of the plane whose product of their distances from two focuses ($PF1 \cdot PF2 = k^2$) is constant ($= k^2$). The traditional eight-shaped lemniscate (fig. 6) result from $k = 1$; while when $k < 1$ the curve degenerates into two distinct branches, two quartic ovals, or, when $k > 1$, it takes on the various shapes of Cassini Ovals.
These examples prove that the emergence of new definitions of curves and surfaces over the history of geometry transforms the previous definitions. After the invention of the conic sections, even the circle, the straight line and the coplanar pair of straight lines have become special cases: 'degenerate conics'. The most important fact in the historical development of the theory of conics is that the new properties streamlined the previous ones and, above all, defined physical properties.

The word 'focus' is physically and historically connected with the renown optical properties of the conics. In fact, considering (fig. 3) that each pair of 'focal radii' meeting in one point of the ellipse is always such that: 1*) their bisecting line is the normal of the curve (orthogonal to the tangent), and 2*) their extensions have a constant sum, then these geometrical properties physically lead to the fact that all the radius originated by a focus are reflected in the other focus (due to 1*) at the same time (due to 2*.)
Therefore, the energetic interpretation of 'distance' is shared by physics and geometry. Actually the Archimedean line –the shortest among the lines that have the same extremes– and the Euclidean one –a line that lies equally between its points (a curve that coincides with every tangent)– were already defined *ab-antiquo* in 'energetic' terms, like the circle of Aristotle, intended as the form of perfect motion.
From the Burning mirrors of Archimedes to the development of mechanical and optical curves in the 17th century, up to the study of the patterns of electromagnetic fields in the 19th century, physics and geometry reflect a figurative conception of the science of extension.
Due to the imaginal nature of geometry there is not a single formalised way to categorize and figure out an entity or a geometric figure. The conceptualized and formalised 'eidetic categories' rarely have a hierarchical network. The

Fig. 7. Sequence of limbs and cranial vault of vertrebrates by Leroi-Gourhan [Leroi-Gourhan1986] and application of diffeomorphisms to statistical analysis of skull shapes and profiles.



fact that what we know about things is tied to what we know about other things also applies to geometry, although it is conceived as a self-referential, symbolic and axiomatic language.
Actually, we can paraphrase correctly, for example, the definition of 'sphere': locus of points equidistant from a given point. In terms of 'distance ratios' it can be regarded as:
1) locus of points whose ratio of their distances from two given points is constant (the above-mentioned sphere of Apollonius);
2) locus of the points P whose 'distance' PX from a given segment AB is always and only the square root of $AX \cdot (AB - AX)$;
3) two-axis ellipsoid with coincident focuses.
Recalling the right angle –or the isoptic curves– the sphere can be regarded as:
4) the locus of the vertices of all the right-angled triangles having the same hypotenuse AB; or, the set of the vertices of the right angles whose sides pass through two given points A and B; or the set of the points from which a given segment AB is always seen under a right angle.
From a differential point of view the sphere is:
5) the only surface of constant positive curvature;
6) the only surface with Geodesic curves all closed and congruent with one another;
7) the only surface (in addition to the plane) made up only of umbilical points.
Conversely, from the opposite 'integral' point of view, the sphere can be intended as:
8) the regular polyhedron with an infinite number of infinitesimal faces;
9) the envelope of the possible polygons whose apothems have the same extension and reach out to the same point;
Then there are many variants of the kinematic genesis of the sphere: surface of revolution of a circle around its diameter; for example:
10) surface 'of revolution' in infinite ways, in every point;
11) the only surface that a plane always intersects into circles;
12) the simplest surface with a constant width, whose parallel tangents are always equidistant.
Eventually, the sphere could be effectively intended as a soap bubble, according to the physical principle of the stress minimisation, namely as
13) the littlest surface that covers a given volume.
These definitions –which somehow recall Queneau's *Exercises in Style*– shape different concepts of the sphere. A bubble blown into a perfectly elastic membrane (def. 13) is very different from a ball produced with a lathe (def. 4 and 10) or shaped (def. 5, 10 and 12), or woven with interlocked rings (def. 6 and 11), or by stacking homothetic disks whose radius varies in consistency with the cosine of the spherical radius (paraphrase of def. 2.)
In order to mutually and geometrically reproduce these conceptualised images, it is necessary to apply the energetic notion of 'distance'. This notion is 'primitive' in three senses: logical, historical and psychological. It is logically preliminary to others and is older. It goes back to times when no distinction was made between physics and geometry, when the 'distance' ratios corresponded to the 'force' ratio and the simplest figures –straight line and plane, circle and sphere– were just the least probable elements among the 'disputed' spaces between opposite forces.
On the one hand, the notion of 'distance' recalls the sense of ancient arithmetic operations between segments traced with a ruler and compass. On the other hand, it refers to the phenomenal and cultural properties of the physical objects. In this sense, it deals with a 'figurative conception' which has always been intrinsic in the history of geometry, emerging especially in the psychological genesis of the axioms [Enriques 1906, pp. 174-201.] The idea that geometry is basically a figurative science –and not abstract– has been shared by the science of extension 'realist' school up to a project of a Semiophysics [Thom 1988.]

Conclusion: geometries and categorisation of the objects

The above-mentioned figurative conception of geometry leads her back to the field of the natural philosophy and the techniques [Thompson 1945.] It also helps better grasp the historical continuity and discontinuity of geometry for the Design in the shift between the DG and the CG. From this point of view, the DG and CG are episodes which both stand out along the composite genealogies of two important chapters of natural philosophy:
1) the 'Science of Form' which flourished especially in naturalistic morphology, from ancient comparative anatomy to the morphogenetic theories of the 19th and 20th century;
2) the 'Science of Vision', from the Euclidean optics to the Renaissance *perspectiva artificialis*, from the psychology of perception of the 19th-20th century up to the current computer and robotic vision.

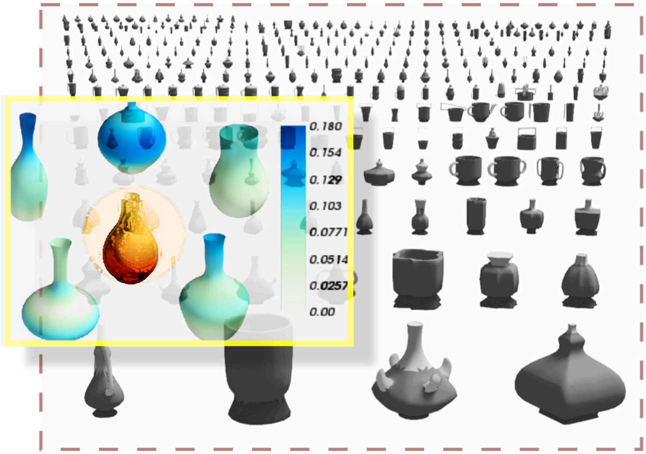


Fig. 8. Statistical shape analysis of the Karcher mean of vase shaped objects; The mean shape is displayed in the centre [Bauer, Bruveris, Michor 2014.]

The 'Science of Form' and the 'Science of Vision' are very different from each other, but they have often acted as the 'objective' and 'subjective' sides of analogous 'morphological questions'. They both share some problems and methods, some geometric and mathematical instruments that, after the third industrial revolution, were confronted with a quantum shift towards the techno-sciences and the current digital dimension of the world.
1) Obviously, from the second half of the 19th century, morphometric and statistical comparative methods were applied to the natural sciences. They helped develop typologies taken from comprehensive *corpora* of *exempla* –mainly consisting of samples, calques or graphical representations– categorised and parametrically arranged by degree of typicality. Starting from these corpora, each naturalistic typology has always been based on the correlation of analogous characters of homologous samples of different bodies. Therefore, every typology presupposes a measurable taxonomy, i.e. a paradigm shared by a comparative specimens, and a mathematical criterion for the measurement of their differences. From a technical point of view, the geometric, metric and differential properties –'distance' and continuity of curvatures in one point– are compared. These geometric transformations are called 'diffeomorphisms' – somewhere between differential geometry and topology – and were predicted in the famous *On Growth and Form* by D'Arcy Thompson [1945]. In his works the English naturalist [Thompson 1945, pp. 1026-1090] introduces the use of diagrams called 'transformation graphs': (fig. 7) graphs that describe the morphological variability of the bodies in terms of the 'deformation' of a reference grid made up of homologous lines in organisms and parts of different organisms represented at the same metric scale. Obviously, the 'transformation diagrams' can be performed only if there is a paradigm shared by the comparative bodies. In fact, they depend on the choice of the pairs of places (ontogenetically) homologous in different organisms. Therefore, this transformation network should have identified the closest parameter system to the 'real form line growth' (ontogenetic lines) as well as to those organism parts featuring (phylogenetic) speciation differences.
D'Arcy Thompson's 'science of form' –in the revolution towards the current digital world and techno-sciences– has turned in comparative Biometrics and morphometrics which –since the Sixties– have been accompanying the amazing development of morphogenetic models in the field of theoretical chemistry, physics and, above all, theoretical biology.
2) The geometric models which explain the emergence of forms from matter are extended also to neurosciences. Today, the *Neurogeometry* [Petitot 2008] studies the functional geometry of the perception system, with special reference to the low-level visual perception processes. It focuses on the translation of visual information from a 2D proximal (retinal) stimulus to the perceived form processed by the first visual cortex (V1). Obviously, these studies fall within the broader framework of the geometric models used in the study of the different stages of the visual perception process:
a) the extraction of the first morphological structures from the retinal image (positions, dimensions, directions, colours, contrasts, distances, motions),
b) the emergence of visual forms,
c) the extent of the articulation of surfaces in the environment,
d) the identification of regions and objects as 'things' of the actual environment,
e) the attribution of the size and position of things,
f) the perceptive segmentation of the 'things' into their 'parts',
g) the recognition of the objects,

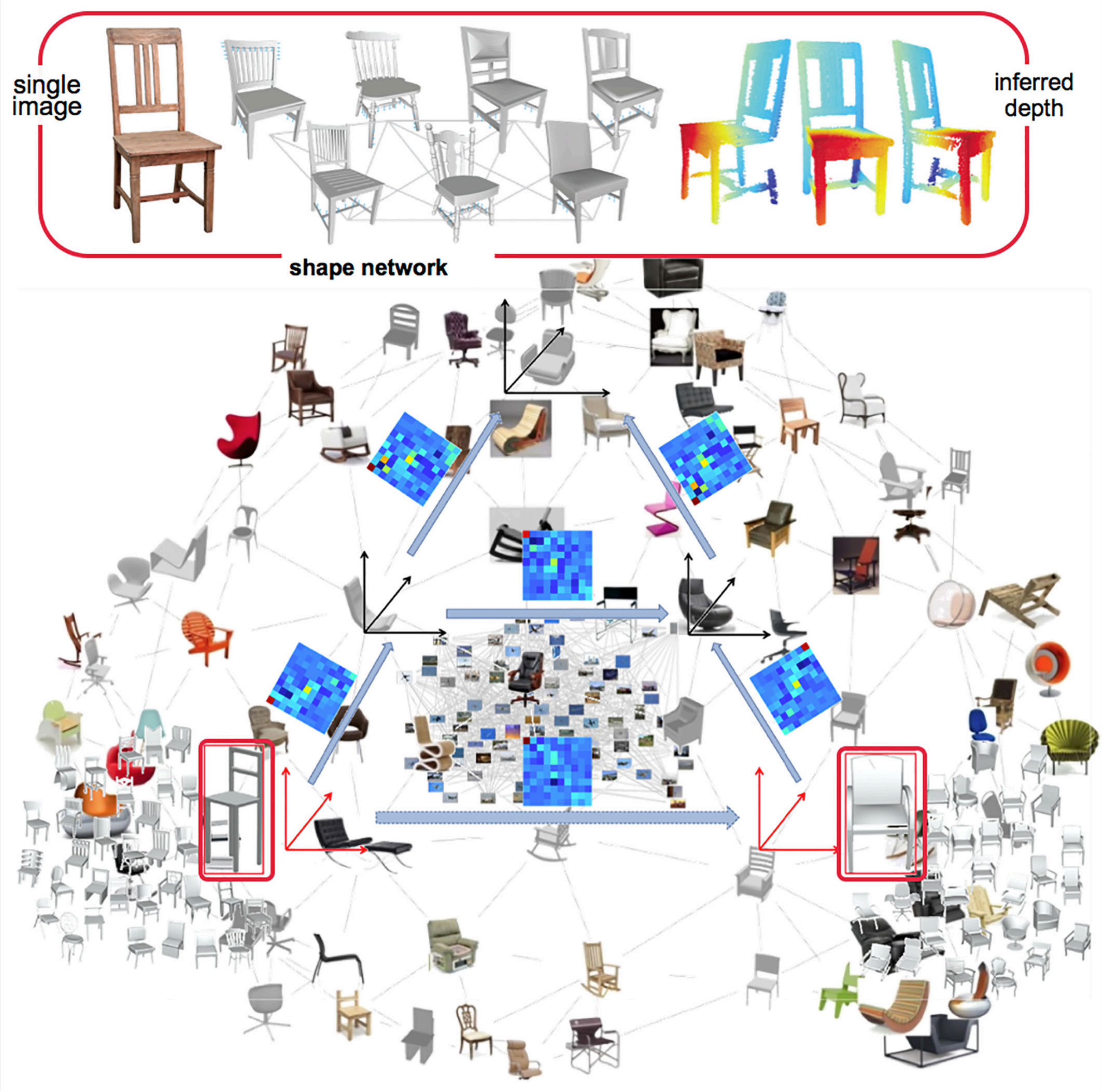


Fig. 9. Automatic recognition and categorization of an object through the inference of depth of a single image and the extraction of features from image repertoires [Yi et al. 2017].

h) the attribution of (functional and cultural) categories to the perceived objects and situations.
For example, with reference to the 'e' stage, vision is similar to a 'photogrammetric' and 'stereoscopic' restitution of the seen scene according to the so-called "inverse (representation) problem" developed by the DG.
As regards the 'c' stage, a differential geometry is used which describes the luministic behaviour of the surfaces [Palmer 1999, pp. 243-246.] The perceptual computation of the curvatures is supposed to depend on the isophote lines, intended as an variation index point by point of the normals to the surface.
Differential geometry is applied also to the 'f' stage. For example, based on the tendency [Hoffman 1998] to attribute 'names' only to the convex parts of the objects, all the perception curvature indices of a surface are decisive [Koenderink 1972.]
The search for geometric models able to explain the perceptive extraction of the most significant characteristics of the objects also involves the study of higher cognitive processes ('h' stage), although a perceptive categorisation already takes place in the early perception stages. According to the main assumption, vision is guided by categorisation through computational economics strategies, as happens with the perceptive organization criteria already theorised by the *Gestaltpsychologie* which identify psychology and geometry of the image-form (*Gestalt*).
The shift of the computational paradigm included both geometry and the psychology of perception [Marr 2010.] In fact, in the same years, they are fully incorporated in the neural networks theory developed by Minsky and Papert [1990.]
The history of this convergence is long and well known. But what does the geometry developed within the *design studies* have to do with the convergent genealogy in the computational model of the various 'morphologies' arising from natural sciences?
The first answer is historical. Since the second half of the 19th century, the morphological measurement of phylogenetics had involved also anthropology, then regarded as 'the natural history of men.' The comparative geometry of *naturalia* led to the *artificialia* ones, i.e. the diffeomorphism measurement method was also extended to the study of the historical and archaeological species of human artefacts, appropriately divided into model corpora and typologies. (fig. 8) Since then, methods similar to statistical biometrics have been applied to analytical archaeology [Clarke, Pinnock 1998] and today they have reached their fully accomplishment by working (on line) on digital model corpora deriving from the 3D scan of huge collections of findings.
The GC has significantly expanded the technical possibilities of the morphology of the findings and of the collections. In the era of big data, the CG allows for the extraction of geometries starting from various types of data –from physical bodies, measurements, image web collections [ex. Heath et al. 2010] and network models, … (fig. 9)– thus performing semiotic elaborations well beyond the human possibilities [Stiegler 2016.]. The algorithms of the GC exceed even the possibilities of human imagination, but not of their traceable history, where the technical and aesthetic genealogy cannot be separated.
As 'morphology' (science of form), Geometry is part of the aesthetic knowledge invested in the construction of objects; especially, the 'History of DG', the 'genealogy of the methods of projective representation' and the 'morphology of curves, surfaces and patterns' are relevant points of view in the study of the evolution of (artistic and technical) 'visual artefacts'; therefore these topics must refer to the thematic area of 'geometry in drawing for design': a field that, from a historical point of view, stands out against the background of the millennial mutual exchange between 'science of form' and 'science of perception'.
This thesis offers a unified –retrospective and prospective– vision of the historical events related to the 'Geometry for Design'. This is the thesis that we have shown, starting from the replacement –half a century ago– of DG with CG and indicating their continuity and discontinuity.
In retrospect, we have highlighted the discontinuity between the mechanical-projective paradigm and the computational-informational one, superimposed on the deep continuity of a geometry intended as a natural science: a kind of knowledge that is always negotiated between 'morphology' and 'theories of perception'.
In prospect, we have proved the thesis by recognizing that today's applications of CG are articulated following –step by step– the chapters of the psychology (and semiotics) of vision: from the elaboration of the proximal stimulus to the processes of perceptual, cognitive and cultural categorization.

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